Refutation of sabotage modal logic (revisited)

Abstract: We evaluate six equations for a definition and properties of sabotage modal logic. The special symbols ■ and ♦ act as functions, so we assign them variable names. Because none is tautologous, sabotage modal logic is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ, ■, ♦;
~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⟺, ↔, ≙; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤; (z@z) **F** as contradiction, ∅, Null, ⊥;
(%z<#z) C as contingency, Δ, ordinal 1;
(%z>#z) N as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆y); (A=B) (A~B).

From: Aucher, G.; van Benthem, J.; Grossi, G. (2017).
Modal logics of sabotage revisited. Journal of Logic and Computation, 2018. 28:2. 269–303.
academic.oup.com/logcom/article/28/2/269/4774578?guestAccessKey=ae079e7e-fe35-449d-af50-9e34493a615c
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Remark: Because the modal sabotage notations of ■ and ♦ act as functions, we assign them variable names.

... we define the usual abbreviations:

■ϕ≙¬ ♦¬ϕ (2.1.1)

(r&p)=(~s&~p) ; **F**T**F**T **FFFF** TTTT T**F**T**F** (2.1.2)

2.2 Some notable validities and expressible properties
We list some validities of SML that demonstrate how the deletion modality works:

p→■p (2.2.3.1)

p>(r&p) ; T**FTF** TTTT T**FTF** TTTT (2.2.3.2)

$$p\rightarrow\blacksquare(\Diamond\top\rightarrow\Diamond p) \quad (2.2.5.1)$$

p>(r&(%(p=p)>#p)) ; T**F**T**F** TNTN T**F**T**F** TNTN (2.2.5.2)

$$\Diamond\phi\wedge\Diamond\neg\phi\rightarrow\blacklozenge\blacklozenge\top \quad (2.2.6.1)$$

(%p&%~p)>((s&s)&(p=p)) ; NNNN NNNN TTTT TTTT (2.2.6.2)

The fact that we are using propositional atoms instead of variables for formulas in the first five of the above validities is not accidental. Surprisingly, many prima facie valid-looking principles fail for SML in their full schematic form with all complex substitution instances once we realize that under a deletion modality, ordinary modalities can change their truth values. A good example is principle (2.2.5.1). Consider its schematic formulation

$$\Box\phi\rightarrow\blacksquare(\Diamond\top\rightarrow\Diamond\phi) \quad (2.2.7.1)$$

#p>(r&(%(p=p)>%p)) ; TCTC TTTT TCTC TTTT (2.2.7.2)

which states that if every accessible state satisfies ϕ, then after any link deletion, if the evaluation state still has a successor, it still has a ϕ-successor. The formula may fail if ϕ is modal, since deletion may happen deeper in the model and disrupt the truth of ϕ at successor states. ... In the above list, only the last item (2.2.6.1) is a schematic validity.

Another sign of strength for SML is its power to define frames up to isomorphism. For instance, it is a simple exercise to show that the formula

$$\Diamond\top\wedge\Box\Diamond\top\wedge\blacksquare\Box\bot \quad (14.1)$$

(%(p=p)&(#%(p=p)))&(r&#(p=p)) ;
FFFF NNNN **FFF**F NNNN (14.2)

is true in a model if and only if its underlying frame consists of one reflexive point.

Because the six Eqs. above are *not* tautologous, sabotage modal logic is refuted.