Refutation of a modal logic system for reasoning about the degree of blameworthiness

Abstract: We evaluate a modal logic system for the blameable coalition of an outcome if there is a strategy to prevent it and where the degree of blameworthiness is measured by costs of prevention or sacrifice. Of eight axioms, three are not tautologous, hence refuting the approach. We do not consider the claimed technical result of a completeness theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, lc_phi, lc_psi, Blameable, cost, u, v, w, x, y, z:
c coalition (small), d coalition (large), Statement (N), x, y, z;
¬ Not, ¬; + Or, ∨, ∪; ¬ Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ℨ, ↞; < Not Imply, less than, ⊂
≡ Equivalent, ≡, :=, ↔, ⇔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) F as contradiction, ∅, Null;
(%z<=#z) C as contingency, Δ, ordinal 1;
(%z=#z) N as non-contingency, ∇, ordinal 2;
¬( y < x) ( x ≤ y), ( x ⊆ y).

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3 Axioms: In addition to the propositional tautologies in language Φ, our logical system contains the following axioms:

1. Truth:
   Nφ → φ and B*Cφ → φ,
   \[(w&p)>p) & (((r&s)&(u&p))>p) ; \quad TTTT TTTT TTTT TTTT \quad (1.2)\]

2. Distributivity:
   N(φ → ψ) → (Nφ → Nψ),
   \[(w&(p>q))>((w&p)>((w&q)) ; \quad TTTT TTTT TTTT TTTT \quad (2.2)\]

3. Negative Introspection:
   ¬Nφ → N¬Nφ,
   \[¬(w&p)>(w&¬(w&p)) ; \quad (3.1)\]
4. None to Blame:
\[ \neg B^s \phi, \]  
\[ \neg((r \& s) \& ((z \& z) \& p)) = (p = p); \]  
(4.1)  
(4.2)

5. Monotonicity:
\[ B^s \phi \rightarrow B^t \phi, \text{ where } C \subseteq D \text{ and } s \leq t, \]  
(5.1)

\[ \neg(v < u) \& \neg(t < s) > ((r \& s) \& (u \& p)) > (r \& t) \& (u \& p) \] ;  
(5.2)

6. Joint Responsibility:
\[ \text{if } C \cap D = \emptyset, \text{ then } NB^s \phi \land NB^t \psi \rightarrow (\phi \lor \psi \rightarrow B^{s+t} \cup \psi), \]  
(6.1)

\[ (((u \& v) = (z \& z)) > ((w \& ((r \& s) \& (u \& p))) \& (w \& ((r \& t) \& (v \& q))) > (p \lor q) > ((r \& ((s + t) \& (u \lor v))) \& (p \lor q))) ; \]  
(6.2)

7. Blame for Cause:
\[ N(\phi \rightarrow \psi) \rightarrow (B^s \psi \rightarrow (\phi \rightarrow B^t \phi)), \]  
(7.1)

\[ (w \& (p \lor q)) > (((r \& s) \& (u \& q)) > (p > ((r \& s) \& (u \& p))) ; \]  
(7.2)

8. Fairness:
\[ B^s \phi \rightarrow N(\phi \rightarrow B^t \phi). \]  
(8.1)

\[ (((r \& s) \& (u \& p)) > (w \& (p > ((r \& s) \& (u \& p)))); \]  
(8.2)

For Eqs. 1.2-8.2 as rendered, the three 3.2, 5.2, and 8.2 are not tautologous, hence refuting the proposed system. The claimed technical result of a completeness theorem is not evaluated.