Refutation of a universal operator for interpretable deep convolution networks

© Copyright 2019 by Colin James III  All rights reserved.

Abstract: We evaluate a universal operator then apply the specified parameters to form operators for AND, OR, XOR, and MP (modus ponens). None are tautologous. This refutes the universal operator as proposed for interpretable deep convolution networks.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x, y, z:
P, ϕ, A, B, α, β, γ, b, x, y, z;
~ Not, ¬; + Or, ∨, ∪; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →, ⊃; < Not Imply, less than, ⊂
≡ Equivalent, ≡, :=, ↔, ⇔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ∅, M: # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) F as contradiction, ∅, Null;
(%z<#z) C as contingency, ∆, ordinal 1;
(%z>#z) N as non-contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ≤ y).


Table 1: Comparison between our proposed universal logical operator (ULO) and four classical probabilistic logical inference rules (AND, OR, XOR, MP) under independence assumption.

Note that x = P(ϕx), y = P(ϕy); MP stands for modus ponens, for which P(ϕx) = P(A); P(ϕy) = P(B|A) [ie, conditional probability meaning P(A And B)/P(A)]; and P(ϕc) = P(B).

Inference rule: Output: P(ϕc) = P(U(ϕx, ϕy)) Logical operator parameters:
ULO (ϕx, ϕy) αxy + βy + γx + b α, β, γ, b to be optimized (1.0)
P(B)=αP(A)P(B)+βP(B|A)+b (1.1)
(p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s))))(p&r)|w)) ;

FFFF FFFF FTEE FTEE FTEE (2),
FFFF FFFF FTEE FEEE (1),
FFFF FFFF FTEE FTEE (3),


Table 1: Comparison between our proposed universal logical operator (ULO) and four classical probabilistic logical inference rules (AND, OR, XOR, MP) under independence assumption.

Note that x = P(ϕx), y = P(ϕy); MP stands for modus ponens, for which P(ϕx) = P(A); P(ϕy) = P(B|A) [ie, conditional probability meaning P(A And B)/P(A)]; and P(ϕc) = P(B).

Inference rule: Output: P(ϕc) = P(U(ϕx, ϕy)) Logical operator parameters:
ULO (ϕx, ϕy) αxy + βy + γx + b α, β, γ, b to be optimized (1.0)
P(B)=αP(A)P(B)+βP(B|A)+b (1.1)
(p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s))))(p&r)|w)) ;

FFFF FFFF FTEE FTEE FTEE (2),
FFFF FFFF FTEE FEEE (1),
FFFF FFFF FTEE FTEE (3),
Remark 1.2: Eq. 1.2 serves as the antecedent along with the specified parameters to imply the consequent of the designated operator.

AND \((\phi_x, \phi_y)\) \(xy\) \(\alpha=1, \beta=0, \gamma=0, b=0\) (2.0)

Eq. 1.1 and specific parameters imply \(P(B)=P(A)P(B|A)\). (2.1)

\[(((p&s)=(t(&(p&r)(&(p&s)))+((u&(p&(r&s)))(p&r)+w))&
 (((t=(z>#z))&(u=(z@z))&(v=(z@z))&(w=(z@z)))))>
 ((p&s)=((p&r)&((u&(p&(r&s)))(p&r)))
 ;
 TTTT TTTT TCTC TTTT (1),
 TTTT TTTT TTNM TTTT (1),
 TTTT TTTT TTTT TTTT (14) (2.2)\]

OR \((\phi_x, \phi_y)\) \(x + y - xy\) \(\alpha=-1, \beta=1, \gamma=1, b=0\) (3.0)

Eq. 1.1 and specific parameters imply \(P(B)=P(A)+P(B|A)-P(A)P(B)\). (3.1)

\[(((p&s)=(t(&(p&r)(&(p&s)))+((u&(p&(r&s)))(p&r)+w))&
 (((t=(z>#z))&(u=(z@z))&(v=(z@z))&(w=(z@z)))))>
 ((p&s)=((p&r)+((u&(p&(r&s)))(p&r))-(p&r)&(p&s)))
 ;
 TTTT TTTT TTTT TTTT (1),
 TTTT TTTT TCTC TTTT (1),
 TTTT TTTT TTTT TTTT (14) (3.2)\]

XOR \((\phi_x, \phi_y)\) \(x + y - 2xy\) \(\alpha=-2, \beta=1, \gamma=1, b=0\) (4.0)

Eq. 1.1 and specific parameters imply \(P(B)=P(A)+P(B)-2P(A)P(B|A)\). (4.1)

\[(((p&s)=(t(&(p&r)(&(p&s)))+((u&(p&(r&s)))(p&r)+w)))&
 (((t=(z>#z))&(u=(z@z))&(v=(z@z))&(w=(z@z)))))>
 ((p&s)=(p&r)+((u&(p&(r&s)))(p&r))-(p&r)&(p&s))-
 (%z=#z)((p&r)&((u&(p&(r&s)))(p&r)))) :
 TTTT TTTT TTTT TTTT (1),
 TTTT TTTT TTTT TTTT (1),
 TTTT TTTT TTTT TTTT (14) (4.2)\]

MP \((\phi_x, \phi_y)\) \(xy + (1-x)/2\) \(\alpha=1, \beta=0, \gamma=-0.5, b=0.5\) (5.0)

Eq. 1.1 and specific parameters imply \(P(B)=P(A)P(B|A)+(1-P(A))/2\). (5.1)

\[(((p&s)=(t(&(p&r)(&(p&s)))+((u&(p&(r&s)))(p&r)+w)))&
 (((t=(z>#z))&(u=(z@z))&(v=(z@z))&(w=(z@z)))))>
 ((p&s)=((p&r)+((u&(p&(r&s)))(p&r))-(p&r)&(p&s))-
 (%z=#z)((p&r)&((u&(p&(r&s)))(p&r)))) :
 TTTT TTTT TTTT TTTT (1),
 TTTT TTTT TTTT TTTT (1),
 TTTT TTTT TTTT TTTT (14) (5.2)\]
Eqs. 1.2-5.2 as rendered are not tautologous. This refutes a proposed universal operator for interpretable deep convolution networks.