The MOND Limit of the Inverse Square Law

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Abstract

This paper attempts to give a theoretical foundation for the Modified Newtonian Dynamics equations developed by M. Milgrom Ref.1. It will show that there is a cross sectional limit, below which $1/r^2$ asymptotically changes to 1/r. When the cross section of a distant star falls below the cross section of a particle transmitting the gravitational force, such as a graviton, as viewed from a baryon, such as a proton, then the inverse square law is no longer valid. Gravitational attraction then depends on the distance travelled by the particle and will result in the force of gravity varying as 1/r. It is only the relative geometries which limit the inverse square law. The sizes of the suns and particles involved will only result in different limits and will not change the basic outcome of 1/r.

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Introduction

MOND explains the rotation curves of galaxies, at various luminosities, very well. "Milgrom's law fully specifies the rotation curve of a galaxy given only the distribution of its baryonic mass. In particular, MOND predicts a far stronger correlation between features in the baryonic mass distribution and features in the rotation curve than does the dark matter hypothesis (since dark matter dominates the galaxy's mass budget and is conventionally assumed not to closely track the distribution of baryons)." Quoted from Wikipedia Ref 1. MOND is a simpler solution to explain the rotation curves of galaxies.

However, most cosmologists support the Standard Cosmological Model, which includes cold dark matter Ref. 2. "Since the 1980s, it has been widely accepted that the baryonic matter in the Universe would not itself provide enough gravitational attraction to form the observed structures by the present age of the Universe. This problem can be circumvented by the introduction of non-baryonic dark matter, which provides the extra gravitational force to allow structures to form more quickly and is not inhibited by pressure effects. This view has been vindicated by modern precision measurements of the cosmic microwave background." Quoted from Andrew Liddle's Modern Cosmology Ref. 3.

Applicable Laws of Physics and Cosmological Observations

Kepler's Laws of Planetary Motion Ref. 10

Newton's law of Universal Gravitation Ref. 13

Newton's Second Law of Motion Ref. 13

M. Milgrom's Modified Newtonian Dynamics Ref. 1

V. Rubin's mapped rotation velocities of stars Ref. 11

R. Scarpa's Review of Modified Newtonian Dynamics Ref. 15

Gravity is not limited in distance Ref. 14

The graviton travels at the speed of light. Ref. 12

Gravity only pulls, unlike electro-magnetism Ref. 9

The gravitational interactions occur on a particle level. Equations are integrals summing all the particle level interactions of suns, planets, moons, gases and black holes.

A probable gravitational interaction: a graviton is emitted by a proton, on sun A, and is absorbed by a proton on sun B. The graviton is re-emitted and pulls this proton a little closer towards sun A. The graviton travels at the speed of light to sun A and is absorbed by another proton. When it is re-emitted, it pulls this proton a little closer to sun B. Energy is conserved by adjustments in potential and kinetic energies of the protons.

Relative Geometry and Calculations

At $a_0 \approx 1.2 \pm 0.2 \times 10^{-8} cm\, s^{-2}$ (Ref. 4) is an inflection point, where $1/r^2$ changes to $1/r$. What could cause this? Quantum mechanics must be taking over, but there is no broadly accepted law of quantum gravity. Physicists suspect that a particle is involved in transmitting the force of gravity, such as the hypothesized graviton. That means that there is a minimum size limit of the stream of bosons from a distant star. If the image of this distant star falls below this limit, the cone of gravitons becomes a line. The diameter of this line is the diameter of the particle, the graviton. The frequency of interactions depends now on the distance the particle needs to travel. If a very distant star is even further away, the graviton needs to travel further resulting in a lower frequency of interactions. Of course, the particle mode of interaction is also true for a closer star, but then the cone becomes wider and a huge number of streams of gravitons flow between these two stars, which results in $1/r^2$.

$$\left(\frac{1}{r}\right)^2 \rightarrow a_0 \rightarrow \left(\frac{1}{r}\right)^1 \qquad 1.0$$

On the left is the Newtonian law dependence on distance and area of the cone and on the right is MOND regime dependence only on distance of the line of particles. Here distance is proportional to the time that it takes gravitons to travel between the stars. Exponent 2 indicates the area of the cone with many streams of particles at slightly different angles and exponent 1 indicates a line, that is a single stream of particles.

There is another effect: If the image of a star on a proton is smaller than the diameter of the graviton, then the image of the star changes from an area to a point. This will result in

$$GM_a \rightarrow a_0 \rightarrow \sqrt{GM_a}$$

M_a is the mass generating the field. (The whole equation will be derived in section MOND Basics Derived.) The square root is the result of changing from an area to a point.

In Fig. 1, stars A and B are stars with masses like our Sun. For simple calculations, there are no other stars or gas clouds near stars A and B. Using Newton's law of universal gravitation to find the distance between them when $a_0 = 1.2 \pm 0.2 \times 10^{-10} ms^{-2}$ $Ref. 1.$

$F = \frac{Gm_a m_b}{r^2}$ Newton's law of universal gravitation 2.1

G = 6.674 x 10^{-11} N m^2 kg^{-2} Ref.5 Gravitational constant

$m_a = m_b$ = 1.989 x 10^{30} kg Ref. 6 Mass of our Sun

r = distance between stars A and B

F = ma = $m\frac{dv}{dt}$ Newton's Second law of motion 2.2

Using equations 2.1 and 2.2 $ma = \frac{Gm_a m_b}{r^2}$ $a = \frac{Gm_a}{r^2}$

Solving for r $r = \sqrt{\frac{Gm}{a}}$ The distance between star A and B 2.3

$$r = \sqrt{\frac{6.67\times10^{-11}\times1.99\times10^{30}}{1.2\times10^{-10}}}$$

$r = \sqrt{1.106 \times 10^{30}}$ = 1.05 x 10^{15} m = 0.111 ly

Light year = 9.461 x 10^{15} m Ref. 7. This is a surprisingly short distance.

In Fig. 2 the tangent of the cone is shown (not to scale). Θ = the angle of the cone. The radius of the Sun is 6.957 x 10^8 m Ref. 6.

$Tan\left(\frac{1}{2}\theta\right) = \frac{r_{sun}}{s_{ab}} = \frac{6.96\times10^8}{1.05\times10^{15}}$ = 6.63 x 10^{-7} 2.4

In Fig. 3 a possible cross section σ of a graviton on a proton is pictured (not to scale).

Radius of proton $r_p = 0.88 \times 10^{-15}$m Ref. 8; r_σ = radius of cross section of graviton and σ = cross section of graviton.

$$Tan\left(\frac{1}{2}\theta\right) = \frac{r_\sigma}{r_p} \qquad 3.1$$

$$Tan\left(\frac{1}{2}\theta\right) = \frac{\sqrt{\frac{\sigma}{\pi}}}{r_P} \qquad 3.2$$

$$\sigma = \pi\left(tan\left(\frac{1}{2}\theta\right) r_p\right)^2 = \pi(6.63 \times 10^{-7} \times 0.88 \times 10^{-15})^2 = 1.07 \times 10^{-42}\ m^2. \qquad 3.3$$

Acceleration at any point in space depends on the sum of the integrals from all directions.

$$\vec{a} = \int \frac{GM_a}{\vec{r}^2} + \int \frac{\sqrt{GM_a a_0}}{\vec{r}} \qquad 1.1$$

M_a is the mass generating the field. (Inclusion of a_0 will be derived further on.)

$\int \frac{GM_a}{\vec{r}^2}$ will approach zero as a star moves far away from any star or planet, at least further than 0.111 ly.

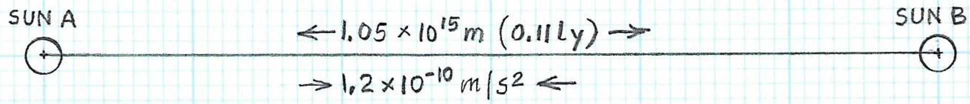
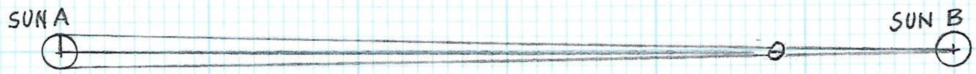


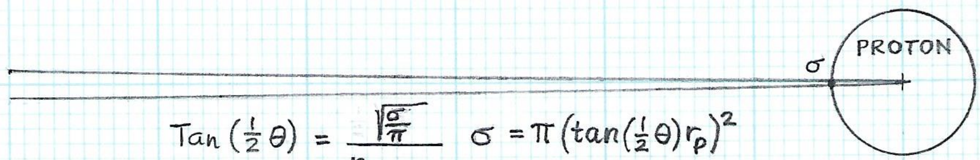
Fig. 1

Inflection point of Inverse Square law



$$\tan\left(\tfrac{1}{2}\theta\right) = \frac{r_{\text{SUN A}}}{\text{Distance } A \to B}$$

Fig. 2



$$\tan\left(\tfrac{1}{2}\theta\right) = \frac{\sqrt{\frac{\sigma}{\pi}}}{r_{\text{PROTON}}} \qquad \sigma = \pi\left(\tan\left(\tfrac{1}{2}\theta\right) r_p\right)^2$$

Fig. 3

Possible cross section of graviton on proton

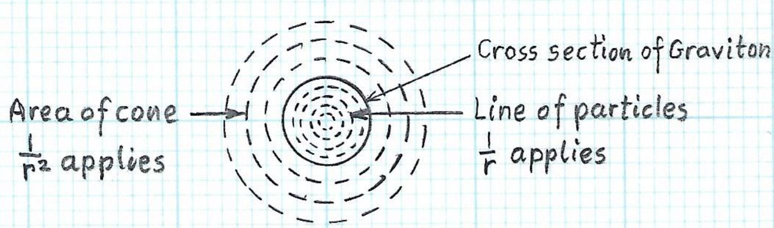


Fig. 4

 Size of image of star on proton

The main assertion of this paper is that the inverse square law changes from analog to digital at the inflection point a_0, where the acceleration due to gravity is equal to or less than 1.2 x 10^{-10} m/s^2. The diameter of the linear beam of particles emanating from a distant star cannot become smaller than the diameter of the particle carrying the gravitational force. Refer to Fig. 4.

From galactic observations of rotational velocities of stars, we know that there is a limit to the inverse square law. The Cosmos already did the "calculations" for us.

MOND Basics Derived

Milgrom's law shows how the Newtonian acceleration a_N needs to be modified in very low acceleration space Ref. 1.

$$a_N = a\mu\left(\frac{a}{a_0}\right) \qquad 1.2$$

a_0 = 1.2 x 10^{-8} cm/s^2, which is a new constant empirically determined by Milgrom.

The interpolation function $\mu(a/a_0)$ results in the asymptotic behavior μ=1 for $a \gg a_0$, to retrieve the Newtonian expression in the strong field regime, and $\mu = a/a_0$ for $a \ll a_0$ in the weak field regime Ref.15

The weak acceleration limit of gravity is

$$a = \sqrt{a_N a_0} = \frac{\sqrt{GMa_0}}{r} \qquad 1.3$$

In a weak field regime, acceleration changes 1/r with distance r from mass M generating field.

Conclusions and Further Research

As the distance between stars increases, a limit will be reached where the cross section of a particle carrying the force of gravity, such as a graviton, will be larger than the image of a star on a baryon, such as a proton. At this point, an upper limit r is reached by the inverse square law, above which gravity decreases by 1/r. The cone of the inverse square law changes to a line of streaming particles. The image of a star changes from an area to a point, which results in the square root of $\sqrt{GM_a a_0}$.

Now there is a theoretical foundation for Milgrom's Law, the MOND limit of the inverse square law.

If there is a limit to the inverse law as claimed above, the following need to be further investigated.

Gravity will decrease only by 1/r in vast spaces of our Universe. This will make gravity much more powerful. The ratio of baryonic matter to dark matter needs to be recalculated.

Just as light only penetrates the top layer of ocean water, gravity between two stars may only involve the top layers of suns facing each other. This needs to be measured and calculated. An experiment with torsional pendulums could be conducted from the surface down into deep ocean trenches, with pendulums suspended every km. The gravitational effect of an orbiting moon may become slightly less with depth since fewer gravitons will reach deeper layers. (Of course, there are many other gravitational influences, such as distance, etc. that need to be considered).

Light is a particle and a wave. If there is a MOND limit for gravity of the inverse square law, is there a LIGHT limit of the inverse square law also?

References

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