Graviton Theory, Part 3: Gravitonic StatMech and Thermodynamics

The Theories Of The Graviton
Part Three: The Statistical and Thermodynamical Applications to Gravitonic Mechanics

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Gravitons are the quanta of gravity that, if proven to exist, would potentially connect quantum mechanics with gravitation. The third (and supposedly last) part of the Graviton Theory entity focuses on the thermodynamic and statistical applications to the theories that were proposed in Parts One and Two. This analysis will look at gravitonic influences in black holes and during the Big Bang, as well as interacting gravitons in two extreme systems: as a boson gas and as a Bose-Einstein condensate. The analysis will also explore the thermodynamic and statistical influences on the nature and mechanics of gravitons.

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INTRODUCTION

The physics of gravitation and thermodynamics, in the classical framework, have not been thought to be dependent on each other. In the scale of massive objects, i.e., stars, its strength of gravity and internal thermodynamic interactions due to the Pauli Exclusion Principle are rather seen as independent. However, astronomical masses such as black holes (where gravity overpowers the Pauli Exclusion Principle) behave like thermodynamic systems. Therefore, as a thermodynamic system, black holes have their own set of thermodynamic laws, which are in the pretense of quantum mechanical interactions. Assuming that black holes store and distribute gravitons in a well, and disperse them as gravitational waves, gravitons in various systems - as stable particles in the energetic lattice, as particles oscillating within a mass-energy system, or as rest particles in free space - are subject to thermodynamical interactions.

Thusly, this proposes that the graviton particles, as an entity, shall have their own laws of thermodynamics. The two main applications of thermodynamics towards graviton particles are: 1) the behavior of gravitons during the Big Bang, and 2) the theoretical construction of a gravitonic condensate: a boson condensate purely made of gravitons. The paper, therefore, supports the Theory of Entropic Gravity, stating that classical gravity is an entropic phenomenon conceived from small bits of spacetime information[1]. The small bits of spacetime information, hypothesizing that the gravitonic lattice compasses all of spacetime, are the gravitons themselves; this was first proposed through the Gravitonic Oscillation Hypothesis in Part One [2].

The paper begins with the evaluation of gravitons in black holes and in the big bang. For gravitons are thought of to be emitted as gravitational waves after a black hole binary collapse, a distribution function is derived to calculate the probability of “graviton storage,” postulating that larger black holes provide small values of probability to prove that a graviton is stored within. Smaller black holes, such as quantum black holes, instead provide large values of probability, providing analytical evidence that quantum black holes store gravitons within.

The second part of the paper analyses the interacting gravitons within a collapsing binary system, considering the system to be a degenerate boson gas. Using the ideal gas law \( PV = Nk_B T \), it is verified that the pressure and volume of the graviton mesh is proportional to the number of interacting gravitons, providing a dynamic equation that provides a constant result. Afterwards, the entropy of the degenerate gas is derived, proving that the decrease in graviton particles leads to the decrease in entropy.

The third part of the paper considers the formulation of a theoretical boson condensate purely made of gravitons: a gravitonic condensate. Hypothesizing that such a condensate is the beginning stage of creating a Kugelblitz black hole, the graviton condensate is given its own set of Four Thermodynamic Laws that is dependent on energy transfer and, naturally, alterations in temperature. The zeroth law considers the stability of the graviton lattice, the first and second laws consider the formation and stability of the graviton condensate, and the third law considers the alterations of gravitonic mechanics influenced by changing temperatures.
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GRAVITONS IN BLACK HOLES AND THE BIG BANG

Based on the doctoral thesis of Stephen Hawking\[^3\], the universe began as a singularity (i.e. a black hole). If the Big Bang was indeed a catastrophic thermodynamic event from a quantum black hole, it is theoretically understood that gravitons, among other particles, were dispersed to illustrate the basis of universal gravitation. The Big Bang itself would be seen as the universe’s first ever gravitational wave.

Therefore, it is proposed that all black hole singularities are the wells and distributors of gravitons residing in our set of dimensions. Furthermore, each black hole singularity has a specific distribution probability to which that singularity is storing gravitons. This distribution probability shall be called the Schwarzschild Distribution Probability.

Schwarzschild Probabilities

The one proposition as to why gravitons are very hard to detect, other than higher-dimensional travel, is that black holes are simply too large for particle detection.

Referring to the Heisenberg Uncertainty Principle, as long as the graviton’s velocity is known (which is understood to be the speed of light \(c\)), its position is unknown if the black hole’s Schwarzschild Radius \(r_s\) is greater than or exactly 3 solar masses \((1\odot = 2 \times 10^{30}\text{kg})\).

However, if the Schwarzschild Radius were the radius with a length of half the Planck length \(l_p = \sqrt{\hbar G/c^3}\), the uncertainty of the probability becomes far more certain, thus having the graviton’s velocity relaxed and position known.

That is to say, quantum black holes (black holes with a mass of Planck mass \(m_p = 2.2 \times 10^{-5}\text{kg}\)[⁴] and a diameter of Planck length \(l_p = 1.6 \times 10^{-35}\text{m}\)) are the keys to verify whether or not gravitons are stored within their singularities.

In deriving the function for Schwarzschild Probabilities, the probability must be proportional to the ratio between half of the Planck length and the Schwarzschild Radius:

\[
Pr_s \propto \frac{l_p}{2r_s}
\]

For black holes, a decrease in radius size (which corresponds to a decrease in mass and an increase in black hole temperature due to Hawking Radiation) would make the probability approach the value of 1, resulting to concrete certainties of graviton storage.

However, an increase in mass/radius and decrease in temperature result to uncertainties.

Therefore, a new distribution function shall be derived based on the statements. This distribution, or the Schwarzschild Distribution (adopting the Bose-Einstein distribution, for gravitons are bosons), is seen as follows:

\[
F_s = \left( \frac{\alpha \varepsilon}{k_B T_{BH}} - 1 \right)^{-1} \Rightarrow (\alpha \varepsilon - 1)^{-1}
\]

where \(\alpha = 8\pi G m/\hbar c^3\).

As for the energy \(\varepsilon\), it must be the energy of one stored graviton within the black hole. Although there would be an infinite number of gravitons stored inside the black hole, if all gravitons are confined to the singularity as a large super-boson, it would behave as one graviton.

Letting \(\varepsilon = m_g c^2\), where \(m_g\) is the mass of a stored graviton (rewritten in terms of its Compton wavelength \(m_g = \hbar/\lambda_g c\), where \(\lambda_g = 1.6 \times 10^{10}\text{m}\) [⁵] - the Compton wavelength of a stored graviton is the same wavelength of the gravitational wave that was first detected by LIGO, which were emitted from the binary GW170104), then

\[
\alpha \varepsilon = 8\pi^2 \frac{r_s}{\lambda_g}
\]

Therefore,

\[
F_s = \left( e^{8\pi^2 \varepsilon r_s/\lambda_g} - 1 \right)^{-1} \quad \text{(1)}
\]

In quantum mechanics, the integral to derive the probability from a wave function is the following:

\[
Pr(x) = \int |\psi(x)|^2 dx
\]

In this case, a certain wave function will not be evaluated in the probability integral. Instead, it will be the Schwarzschild Distribution \(F_s\), evaluated with respect to the energy \(\varepsilon\) (which is ultimately with respect to the Schwarzschild radius \(r_s\)).

Therefore, with the main aspects of the probability established, the beginning of the derivation is as follows:

\[
Pr_s = \frac{l_p}{2r_s} \int F_s^2 \cdot d\varepsilon
\]

And only acknowledging the particular solution that accepts the physical reality,

\[
Pr_s = \frac{\hbar^2}{16\pi^2 r_s^2} \left[ \ln \left( F_s^{-1} \right) + F_s \right] \quad \text{(2)}
\]

The derivation of the probability function supports the proposition that the decrease in the Schwarzschild Radius contributes to more certain probabilities. Planck units are in effect.
Above is a graph of the Schwarzschild Probability for a black hole with the minimal mass of three solar masses $3 \odot \approx 6 \times 10^{30}$ kg. At its maximum radius of $r_s = 8.89 \times 10^3$ m, the probability of finding a graviton within is infinitely small. But as the radius decreases, and approaches the Planck length, the probability increases drastically to where it is certain, at $P_{r_s} = 1$.

Solved by computation, it is hypothesized that certain probabilities exist within the coefficients of $r_s$ less than or equal to 0.20247, no matter the value of the radius itself. This is proven after taking 100 derivatives of the Schwarzschild Probability with respect to the changing radius:

Shope’s Extension

This extension on Schwarzschild Probabilities is named after fellow collaborator Ashten Shope, who recognized that gravitonic transitions into higher sets of dimensions would also contribute to lower probabilities.

It is recognized in Part Two that oscillating gravitons at the central node of spacetime curvature obtain an infinite amount of energy to transition into any higher set of 4d spacetime\cite{6}. However, at nodes of extreme spacetime curvatures, such as in black hole singularities, the gravitons are welled, and are expected to be “trapped” until being emitted out as a gravitational wave. Nevertheless, being at a central node of spacetime, the graviton within a black hole can “escape” by means of dimensional transitioning, but it must transition into a corresponding black hole in that higher set of dimensions.

These connections between black holes of varying dimensional sets are very much like Einstein-Rosen Bridges (a.k.a. wormholes). But instead of making shortcuts within the same 4d spacetime manifold between two non- or low-massive singularities, they are theoretical bridges between two sets of dimensions, between two massive black hole singularities. They shall be called “graviton belts.”

This would mean making a slight revision to the previously-derived Schwarzschild Probability Function to satisfy this condition - applying gravitonic dimensional transitions. The coefficient of the function (taking account of the Planck units) is in units of inverse meters $m^{-1}$. For “dimensional fluidity,” the coefficient must be unitless.

The leading coefficient of the Probability Function, calling it $N$, will be morphed into a unitless parameter, calling it $\bar{A}$, such that

$$\bar{A} = aN$$

where $a$ is the transformation.

$N$ is expanded by rewriting $r_s$ as $\frac{2Gm_{BH}}{c^2}$, resulting to

$$\bar{A} = a \frac{(hc)^2}{32\pi^2 Gm_{BH}}$$

Using Plank units, where $h = c = G = 1$, the units of $N$ are now inverse-kilograms $(kg^{-1})$. Therefore, the transformation $a$ must be a particular value of mass to make $\bar{A}$ unitless - and the mass of an oscillating graviton (the one type of graviton that was focused on the previous part when dealing with dimensional transitions) will be used.

Instead of using the oscillating graviton mass $m_\gamma$ that was derived in Part One\cite{7}, the mass will be instead be $p_\gamma / c$, where $p_\gamma$ is the momentum of an oscillating graviton\cite{8}. Therefore, $a = 2\pi h / pc$, where $\rho$ is generally the displacement for gravitonic oscillation. For gravitons
confined in the black hole singularity, \( \rho = n l_p^2 / 2 \) (where \( n \) is the energy state of the graviton, which is related to the set of dimensions the graviton is currently taking place.

\[
\tilde{\Lambda} = \frac{\hbar^2 c}{8\pi G m_{BH}} \frac{\hbar}{n l_p^2}
\]

Using Planck units, a single \( \hbar \) can be used as the square of the Planck length \( l_p^2 \). Having \( c = 1 \),

\[
= l_p \frac{\hbar}{8\pi n G m_{BH}}
\]

And after simplification, \( \tilde{\Lambda} \) is derived as follows:

\[
\tilde{\Lambda} = l_p \frac{\hbar^2}{8\pi^2 n r_s}
\] (3)

When considering Shope’s Extension, the Schwarzschild Probability must have \( \tilde{\Lambda} \) as its leading coefficient, which in actuality should be used as a permanent revision. This is written because \( \tilde{\Lambda} \), when using Planck units, is more identical to the initial proportionality of \( Pr_s \propto l_p / 2r_s \).

\[
\tilde{\Lambda} = l_p \frac{\hbar^2}{n 2r_s}
\]

and having \( \hbar = 1 \),

\[
\tilde{\Lambda} = l_p \frac{1}{2r_s n}
\]

The Mass of the Graviton

Upon discovering gravitational waves through the black hole binary GW170104, assuming that gravitons are massive particles (which was theorized in Part One\(^9\)), the mass of a graviton dispersed from a black hole after binary collapse was calculated to be \( m_g \leq 7.7 \times 10^{-23} \text{eV}/c^2 \) \(^5\).

This mass greatly differs from the mass of a stable graviton, which is theorized to be \( m_g = 6.53 \times 10^{-5} \text{eV}/c^2 \), and the mass of an oscillating graviton \( m_T = 2\pi \hbar / \gamma^* pc \) (\( \gamma^* = 26.889 \)).

According to the Part One paper, gravitons must be dictated by General Relativity as well as quantum mechanics. Therefore, gravitons are relativistic particles, meaning having the tendency to increase in mass. If the gravitons with a mass of \( m_g \) were rest particles that are not subject to energy (unlike the stable gravitons in a lattice connected by strong energy bonds, illustrating the fabric of spacetime), then the masses that were theorized in the Part One paper remain true, for they signify relativistic and energetic interactions.

The Transition from Quantum Phenomena to the Classical Framework

Considering the history of the universe, the universe expanded from an origin that was once in the quantum scale. This fact may draw the conclusion that everything in the classical world had once followed the laws of quantum mechanics shortly after the Big Bang.

At that particular quantum state, the universe was first composed of pure energy (in the form of photons and heat) until more complex particles began to form. One of the first particles to be formed, next to the photon, was the graviton, in order for the building and gathering of matter and fields to take place.

This makes a link between the graviton and an essential process that must have taken place for quantum objects to become classical bodies. This process shall be given a Latin name, calling it the Facto mensio-atticus.

To describe the Facto mensio-atticus mathematically, the quantum and classical Hamiltonian operators are equal to each other. As they are distinctly differently in their own respectful frameworks, the Hamiltonians are never seen as identical within a common framework. However, in such a moment in the universe’s history where quantum gravity begins a transition into classical gravity, the quantum and classical Hamiltonians shall be equal:

\[
\left( \sum_{i=1}^{N} -\frac{\hbar^2}{2\mu_i} \nabla_i^2 + V \right) \phi(x_1 \rightarrow N, t) = \sum_{i=1}^{N} \partial_t \Phi_i p_i - \mathcal{L}
\]

where \( \phi(x_1 \rightarrow N, t) \) is the “procedure function,” an arbitrary function that “transforms” a quantum wave function into a classical path function, \( \mathcal{L} \) is the Lagrangian \( T - U \), and

\[
\Phi_i = \int \phi(x_1 \rightarrow N, t) \ dx
\]

which turns the procedure function into a classical function of displacement.

Because the procedure itself requires energy, the initial formulation is rewritten as an expression of kinetic energy:

\[
T_{\phi} = \sum_{i=1}^{N} \left( \partial_t \Phi_i p_i + \frac{\hbar^2}{2\mu_i} \nabla_i^2 \phi(x_1 \rightarrow N, t) \right)
\] (4)

The sum of the quantum and classical Hamiltonians produce the “procedure energy,” while the sum of that sum creates the “kinetic energy of the procedure.”

For graviton particles, explicitly,

\[
T_{\phi, \gamma} = \sum_{i=1}^{N} \left( \frac{4\pi^2 \hbar}{\Phi_i} \partial_t \Phi_i + \frac{\hbar c}{8\pi^2} \Phi_i \nabla_i^2 \phi \right)
\]
If the kinetic energy $T_{\phi,T}$ is equal to zero, meaning there is no work done during the *Facto mensio-atticus*, then the exchange of energy is instead a process of diffusion between frameworks

$$4\pi^2 \partial_t \Phi = \frac{-c}{8\pi^2} \Phi \nabla^2 \Phi$$

This then produces Fick’s Law of Diffusion for the *Facto mensio-atticus*, which is given as follows:

$$\partial_t \Phi = -D \nabla^2 \Phi$$

where

$$D = \frac{c}{32\pi^4} \phi(x_{1\to N}, t) \int_0^t \phi(x_{1\to N}, t) dx$$

### IDEAL GAS LAW FOR OSCILLATING GRAVITONS

The one and only scenario where oscillating gravitons behave like an ideal gas within a system is where interacting gravitons with conflicted intensities are within a mesh encompassed by two binary masses. Reassessing the traditional ideal gas law $PV = Nk_b T$ in the framework of gravitons, the thermodynamic system is the pressurized, volumetric mesh of the interacting gravitons.

The pressure-volume product $PV$ is essentially energy, which is the sum of all systematic energies of each interacting graviton within the mesh. The pressure of the mesh is dependent on the number of interacting graviton particles Υ.

**Ideal Gas Law for a Degenerate Graviton Gas**

The law of gravitons as a degenerate ideal gas, in a simplified form, is the following:

$$\sum_{n=1}^{\Upsilon} \frac{2\pi hv_n}{\eta \sqrt{1 - \frac{v_n^2}{c^2}}} = k_b T$$

where $\eta$ is a degenerate length parameter of either of the two masses from the center of mass in a collapsing binary. For mass $m_1$, $\eta = \lambda$, and for mass $m_2$, $\eta = \gamma$.

Dynamically, the ideal gas law for gravitons confirms that temperature is based on the amount of the kinetic energies of all particles. Provided that temperature is constant, the variable nature of the Boltzmann energy $k_b T$ would have to come from the changing pressurized volume.

The force of the pressure $P = F/A$ would be derived from the Gravitonic Field Energy $E_\beta$ \[^{[10]}\] by using the Lagrangian $\mathcal{L}$ and the Lagrange-Euler Equation $\partial_\mu \mathcal{L} - d_i \partial_\mu \mathcal{L} = 0$. The area will be the surface area of an ellipsoid, using the following parameters: $\lambda$, $\gamma$ and the sum of the two radii of the masses $R_1 + R_2$.

The “Gravitonic Field Force”:

$$F_\beta = \frac{1}{2} G \sum_{n=1}^{2} \frac{m_n^2 O_n}{R_n N_n}$$

The surface area of the binary ellipsoid:

$$A = 4\pi \sigma$$

where

$$\sigma = \left( \frac{1}{3} \left[ (\lambda \gamma)^{1.6} + (\lambda (R_1 + R_2))^{1.6} + (\gamma (R_1 + R_2))^{1.6} \right] \right)^{1/1.6}$$

making the mesh pressure to be

$$P_\beta = \frac{G}{8\pi \sigma} \sum_{n=1}^{2} \frac{m_n^2 O_n}{R_n N_n}$$

Using the volume of the same ellipsoid

$$V = \frac{4\pi}{3} (\lambda \gamma [R_1 + R_2])$$

The expanded pressurized volume is now

$$PV = \frac{G}{6\sigma} \left[ \frac{m_1^2 \lambda^2 \gamma}{\lambda^2} \left( 1 + \frac{R_2}{R_1} \right) + \frac{m_2^2 \gamma^2 \lambda}{\Gamma^2} \left( 1 + \frac{R_1}{R_2} \right) \right]$$

$$= \frac{G}{6\sigma \eta} \sum_{n=1}^{2} A_n m_n^2 O_n^2$$

where

$$A_1 = \left( 1 + \frac{R_2}{R_1} \right)$$

and

$$A_2 = \left( 1 + \frac{R_1}{R_2} \right)$$

These coefficients depend on the ratio between the radius of the lighter second mass $R_2$ and the radius of the heavier first mass $R_1$. If $R_1 = R_2$, then $A_1 = A_2 = 2$. If $R_1 > R_2$, then $A_1$ converges to 1, while $A_2$ diverges into a number greater than 2.

Having the equivalence

$$\sum_{n=1}^{\Upsilon} \frac{2\pi hv_n}{\eta \sqrt{1 - \frac{v_n^2}{c^2}}} = \frac{G}{6\sigma \eta} \sum_{n=1}^{2} A_n m_n^2 O_n^2$$

and having the following revision

$$\sum_{n=1}^{\Upsilon} \frac{2\pi hv_n}{\eta \sqrt{1 - \frac{v_n^2}{c^2}}} = \Upsilon B \frac{2\pi h(v)}{\eta \sqrt{1 - (\frac{v}{c})^2}}$$
(for the summation is central around all interacting gravitons, it can be rewritten as the instantaneous number of particles within the collapsing binary $\Upsilon_B$ times the average of the indexed variable), then

$$\Upsilon_B \frac{2\pi \hbar \langle v \rangle}{\sqrt{1 - \left(\frac{\langle v \rangle}{c}\right)^2}} = \frac{G}{6\sigma} \eta_\mu \eta_\nu \sum_{n=1}^{\infty} A_n m_n^2 O_n^2$$

Attempting to simplify the expression further, the leading coefficient on the right hand side of the expression will be a dimensionless parameter

$$\frac{G}{6\sigma} \eta_\mu \eta_\nu = K_{\lambda\gamma}$$

where $K_{\lambda\gamma}$ is the variable gravitonic mesh parameter, letting $\eta_\mu \eta_\nu = \lambda\gamma$.

The degenerate ideal gas law for interacting gravitons in a mesh is therefore:

$$\Upsilon_B \frac{2\pi \hbar \langle v \rangle}{\sqrt{1 - \left(\frac{\langle v \rangle}{c}\right)^2}} = K_{\lambda\gamma} \sum_{n=1}^{\infty} A_n m_n^2 O_n^2$$

This supports the initial hypothesis that the pressurized volumetric system is in direct relation with the instantaneous number of interacting gravitons of conflicted intensities.

**Entropy of a Degenerate Graviton Gas**

The degenerate ideal gas law depicts the balance between the pressure of the collapsing binary ellipsoid region and the number of interacting gravitons within, but it does not depict the changing entropy of the mesh as the binary masses come closer.

Treating the interacting, conflicted gravitons as if they are thermal photons in a blackbody, the absolute entropy of $k$ particles in a specific volume $V$ (Eq. [7.89] in Daniel Schroeder’s *Introduction to Thermal Physics* [12]) is

$$S = \frac{32\pi^5}{45} V \left(\frac{k_b T}{h c}\right)^3 k$$

In the scenario of a collapsing black hole binary, both the volume and the number of interacting particles are variable. Using the ellipsoid volume and the number of gravitons within a collapsing binary, and assuming the temperature to be constant at $T_U = 2.725K$, the absolute entropy is expanded:

$$S(\lambda, \gamma) = \frac{32\pi^5}{45} \left[ \frac{4\pi}{3} \left(\frac{k_b T_U}{h c}\right)^3 \Upsilon_B \right]$$

$$\Rightarrow \frac{\pi^3 c^2 (k_b T_U)^3}{864 G^2 h^4} \frac{[\lambda\gamma(R_1 + R_2)(\lambda + \gamma)^3]}{(m_1 + m_2)}$$

The equation above, therefore, is the entropy of the degenerate graviton gas. In this revision of the absolute entropy, higher degrees of entropy reside in large ellipsoidal regions, where there is a “crowding” of conflicted gravitons, and/or in higher temperatures, exciting the gravitons and increasing the probability for the particles to interact with one another.

However, larger masses result to lower degrees of entropy, as well as in lower temperatures, where the conflicted gravitons have a stronger attraction with the masses, reducing the “crowding,” and that the particles are not as excited to interact with the others.

**GRAVITONIC THERMODYNAMICS**

Thermodynamics may be generally known as the motion of heat confined to a system. But in a broader sense, it is the physics of energy transfer, and the creation of a gravitonic condensate requires energy.

The proposition of creating a gravitonic condensate is the following: waves of radiation perfectly in phase collide with each other at a common rendezvous point in un bent spacetime. Continuous, repetitive bombardment of radiation onto this singular point in spacetime disrupts the fabric, as it begins to curve.

In terms of gravitons, the bombardment of radiation distorts the energy bonding of stable gravitons, causing a small graviton cluster to form. This begins the formation of a self-gravitating condensate as more gravitons cluster - the basis of a Kugelblitz singularity.

A theoretical illustration of such a condensate is seen as follows:
As there are thermodynamical laws for black holes and systems of heat, there shall be four laws that contribute to the formation of a gravitonic condensate in a system of constant and variable temperatures.

I. THE ZEROTH LAW

The zeroth thermodynamical law to gravitons describes the stability of gravitons in a system of independent temperatures:

*Stable gravitons remain stable as long as spacetime itself remains unbent by a source of mass and energy.*

For temperature is the average kinetic energy of a system of stochastic particles, in a part of the universe whose temperature is that of the whole universe ($T_U = 2.725$K), the stable energy bonds between gravitons will remain as a stable energy bond.

If the temperature of the universe $T_U$ were to increase, the energy bond would fluctuate in length, and would not be the supposed bond length at the current universal temperature of $0.019$m.

For the equation of the bond length is

$$\lambda_b = \frac{8hc}{3k_bT}$$

as the temperature increases, the bond length would decrease until a graviton cluster forms, creating a condensate.

If the temperature were to cool, and approach absolute zero, the bond length would increase slightly to where the bond length equation can no longer be applied.

II. THE FIRST LAW

The first law of gravitonic thermodynamics describes the theoretical formation of a gravitonic cluster:

*In a system of constant temperature, the rest energy of a gravitonic condensate made of stable gravitons shall be equal to the collective energy of the colliding in-phase waves of radiation reacting to the elasticity of the stable graviton lattice.*

A. The Theorem

If the temperature of the universe remains 2.725K, then gravitonic condensates are only formed through the bombardment of in-phase radiation waves at a single point. For a condensate is a rest particle of its own, its rest energy appears as so:

$$E_C = m_Cc^2$$

where the mass of the condensate $m_C$ is equal to the combined mass of a certain number $N$ of stable gravitons,

$$m_C = Nm_o$$

where $m_o = 6.53 \times 10^{-5}\text{eV}/c^2$.

As for the in-phase radiation waves colliding at a single point, the energy that is being produced is related to the sum of all energy wave functions belonging to these radiation waves:

$$\varepsilon_Z = \sum_{\eta=1}^{n} |\psi_{\eta}\rangle$$

Examples of such radiation waves are mainly gravitational and electromagnetic. For gravitational radiation, the $\varepsilon_Z$ energy remains at the sum of energy wave functions, as does the radiation composing of particles with mass. For electromagnetic radiation, or photonic interactions with gravitons and other photons, this summation of all wave functions becomes a summation of all photonic energies:

$$\varepsilon_Z = \sum_{\eta=1}^{n} \frac{hc}{\lambda_\eta}$$

However, for a condensate to form in a system of constant temperature, the bombardment energy $\varepsilon_Z$ would have to overcome the elastic potential of the stable energy bonds between the interacting gravitons, for forming a condensate will distort the stable lattice. This elastic potential is seen as:

$$\epsilon_k = \frac{3k_bT}{2N}$$

Therefore, the energy of condensate formation is given as:

$$Nm_oe^2 \equiv \varepsilon_Z - \frac{3k_bT}{2N}$$

This theorem supposes that both numbers of stable gravitons $N$ and colliding in-phase radiation waves $n$ are dependent on forming a gravitonic cluster, assuming the temperature is constant at $T_U$.

A single Joule of rest energy ($6.242 \times 10^{18}\text{eV}$) must be composed of $9.56 \times 10^{22}$ interacting stable gravitons, and requires $5.03 \times 10^{19}$ perfectly in-phase colliding energetic microwaves (using the test wavelength of $\lambda = 10^{-5}\text{m}$), considering the cosmic microwave background.

B. The Function of Condensate Growth

Reordering the energy equation of the condensate so that all of the energies are on one side

$$0 = \varepsilon_Z - \frac{3k_bT}{2N} - Nm_oe^2$$
the equation above best resembles the Groß-Pitaevski Equation (a.k.a. the nonlinear Schrödinger Equation). It will be used to draft a function that best describes the growth of a gravitonic condensate.

The zero holds the place of the energy operator \( i\hbar \partial_t \psi \), which indicates that the growth function is NOT a wave. This makes physical sense, for the growth of any object or system over time is graphed by using exponential functions.

The Groß-Pitaevski Equation, using the corrections in this scenario, is the following:

\[
\hbar \partial_t \varphi = -\frac{\hbar^2}{2\mu} \partial_{xx} \varphi + V \varphi + \Gamma_o(N) |\varphi|^2 \varphi
\]

Because the desired function is time-dependent, the wave equation \( \partial_{xx} = c^{-2} \partial_{tt} \) will be used on the kinetic energy operator. Having \( |\varphi|^2 = 1 \), the potential \( V \) and the condensate operator \( \Gamma_o(N) \) will be summed into a new energy operator: the Groß operator \( G_o \).

Using quadratic formula to solve for the eigenvalue, it results to

\[
\lambda = \frac{3\pi}{\hbar} k_b T g_o
\]

where \( g_o = \sqrt{G_o / k_b T} \). Because \( G_o \) is energy, the quotient between \( G_o \) and the Boltzmann energy is a dimensionless parameter.

This partially solves the growth function to be

\[
\varphi(t) = A_0 \exp \left[ \frac{3\pi g_o}{\hbar} k_b T t \right]
\]

The final step is normalization. Although the function is not a wave function, it is nonetheless a quantum mechanical function.

After normalization, integrating the square of \( \varphi \) from zero to \((3k_b T g_o / \hbar)^{-1}\), the coefficient is solved as

\[
A_0 = \sqrt{\frac{3k_b T}{1.71828 \hbar}} g_o
\]

which slightly complicates the derivation.

Instead, let there be this equivalence:

\[
\frac{3k_b T}{1.71828 \hbar} g_o = G(n - 1)\nu
\]

where \( G = 6.67 \times 10^{-11} \), \( n \) is the number of radiation waves acting upon one lattice graviton in attempt to create a condensate, and \( \nu \) is the collective frequency of these radiation waves.

This leads to a slight revision of:

\[
\frac{6\pi k_b T}{1.71828} g_o = G(n - 1)h\nu
\]

\[
= G(n - 1)\varepsilon_Z
\]

In order to make this substitution into the normalized coefficient, Planck units will be used to place a \( \hbar \) in the denominator.

Therefore, the function of condensate growth is derived as follows:

\[
\Phi(t) = [G(n - 1)\varepsilon_Z]^{1/2} e^{f(n-1)t}
\]

where \( f = 0.85914 \text{s}^{-1} \).

The function of condensate growth is graphed, using \( n = 2 \) for two in-phase colliding waves, as follows:
where the black curve has \( n = 2 \), the gray curve \( n = 3 \), the black dashed \( n = 4 \), and the gray dashed \( n = 5 \).

III. THE SECOND LAW

The second law is the internal stochasticity and entropy of a developed gravitonic condensate, assuming the difference results to a positive value of energy:

\[
\Delta S = \frac{\Delta Q}{T}
\]

For heat is a form of energy transferred between systems, the heat can be seen instead as the energy needed to form a gravitonic condensate, for the bombardment “outside” energy is transferred into the resulting condensate as its rest energy.

Therefore, the entropy of a gravitonic condensate at the temperature of the universe \( T_U = 2.725 \text{K} \) is as follows:

\[
\Delta S = \frac{E_C - \varepsilon_Z}{T_U}
\]  

(18)

For a perfect condensate, the entropy would be zero, where \( E_C = \varepsilon_Z \). However, such a scenario is short-lived. For the rest mass \( E_C \) is a gravitating mass, more gravitons will bulk onto the condensate to form a larger condensate - even a singularity in an extreme case scenario.

In this scenario, the entropy is positive, for the condensate is becoming a center of pseudo-mass and energy.

Entropy is also dependent on temperature, should the system ever vary in temperature. If a graviton condensate were to become a Kugelblitz black hole, the rest mass of the condensate would have to tremendously exceed the temperature in which it is subject to.

The stochasticity of the gravitonic condensate determines its internal stability. Applying the Boltzmann entropy formula, the condensate stochasticity is as follows:

\[
S = k_b \log(W) = k_b \log \left( \frac{\varepsilon_Z + |\langle \varepsilon_Z \rangle|}{|\langle \varepsilon_Z \rangle|} \right)
\]  

(19)

B. The Expected Energy \( \langle \varepsilon_Z \rangle \)

The expected energy is the energy derived from the Bose-Einstein distribution function for all gravitons in the energy state of \( n = 1 \), based on the proof of a constant quantum number for the graviton’s wave function \[^{[14]}\]. In statistical mechanics, the number of particles in a specific energy level is given as follows:

\[
n(\varepsilon) = g(\varepsilon)f(\varepsilon)
\]

where \( g(\varepsilon) \) is the density of the probability of finding the particles in an energy state \( \varepsilon \), and \( f(\varepsilon) \) is the distribution function.

For the probability density, since the quantum number is always equal to one, the density of probability is equal to one, for it is assured that all gravitons are found in a certain energy level of the state \( n = 1 \). To determine this probability density, it is acknowledged that the gravitonic condensate has a density that is composed of a number of \( N \) gravitons, for \( m_C = Nm_0 \). For two colliding energy waves,

\[
P_Z = \frac{m_C}{V_C} = \frac{Gh^2 (\lambda_1 + \lambda_2)^3}{c^4 \lambda_1^3 \lambda_2^3}
\]

If the number of interacting wave functions surpasses 2 waves, \( (\lambda_1 + \lambda_2) \) becomes a summation, while \( (\lambda_1^3 \lambda_2^3) \) becomes a product notation of at most nine wavelengths of each of the colliding in-phase waves:

\[
P_Z = \frac{Gh^2}{c^4} \left( \sum_{n=1}^{9} \lambda_n \right)^3 \left( \prod_{n=1}^{9} \lambda_n \right)^{-1}
\]  

(20)

If the number of waves is greater than 9, then the total density of the condensate \( P_{tot} \) is a superposition of all densities, such that

\[
P_{tot} = \frac{Gh^2}{c^4} \sum_{s=0}^{\infty} \sum_{k=1}^{9} \left( \left( \sum_{n=9s+1}^{9s+k} \lambda_n \right)^3 \left( \prod_{n=9s+1}^{9s+k} \lambda_n \right)^{-1} \right)
\]
If the number of waves is less than 9, then the smallest wavelength will have the highest power.

By default, the density multiplied by its own inverse is equal to one, which is also equal to the probability density \( g(\varepsilon) \).

\[
P_g \frac{V_C}{m_C} = 1 = g(\varepsilon)
\]

Because the probability density is a constant, the function of \( g(\varepsilon) \) becomes a constant of \( \mathcal{G} \)

\[
\mathcal{G} = \frac{G^4 \hbar^2}{e^{10}} \frac{32\pi m_C^2}{3} \left( \sum_{n=1}^{9} \lambda_n \right)^3 \left( \prod_{n=1}^{9} \lambda_n \right)^{-1}
\tag{21}
\]

The distribution function \( f(\varepsilon) \) is the Bose-Einstein distribution, for gravitons are bosons:

\[
f(\varepsilon) = f_{BE}(\varepsilon) = \left( e^{\varepsilon/(k_BT)} - 1 \right)^{-1}
\]

To find the expected energy \( \langle \varepsilon_Z \rangle \), the functions of \( \varepsilon \) must be integrated with respect to the energy state from zero to the expected energy, even if the expected energy is a value equivalent to zero:

\[
N = \mathcal{G} \int_0^{\langle \varepsilon_Z \rangle} f_{BE}(\varepsilon) d\varepsilon
\]

With \( N \) and \( \mathcal{G} \) being constants, the ratio between the two, or \( \beta \) as is follows:

\[
\beta = \frac{N}{\mathcal{G}} = \frac{e^{10}}{G^4 \hbar^2} \frac{3N}{32\pi m_C^2} \left( \prod_{n=1}^{9} \lambda_n \right)^{-3}
\]

\[
\Rightarrow \frac{2\pi}{234N} \left( \frac{m_P r_s}{m_o l_P} \right)^2 |\Psi\rangle
\]

where \( m_P \) is the Planck mass \( \sqrt{\hbar/G} \), \( l_P \) is the Planck length, \( m_o \) is the mass of a stable lattice graviton, \( r_s \) is the Schwarzschild radius and \( |\Psi\rangle \) is the product notation of \( N \) wave functions \( |\psi_n\rangle \) of each for the composing gravitons in the condensate.

The \( \beta \) term simplifies into

\[
\beta = \frac{2\pi}{234} N \prod_{n=1}^{N} |\psi_n\rangle
\]

After integrating, the solution to the expected energy is the following equation:

\[
\langle \varepsilon_Z \rangle = k_b T \ln \left( \frac{1}{e^\alpha + 1} \right)
\]

where temperature is expected to be constant at \( T_U \), and \( \alpha = \beta \gamma \), with \( \gamma = 2.657 \times 10^{21} \). Considering that \( \alpha \) is a large number, the expected energy is further simplified into

\[
\langle \varepsilon_Z \rangle = k_b T \ln \left( e^{-\alpha} \right) = -k_b T \alpha
\]

### C. The Entropy-Stochasticity Equivalence Theorem

The stability and structure of a gravitonic condensate is dependent on its internal entropy and stochasticity of the composing graviton particles. The total entropy \( \mathcal{W} \) shall be seen as

\[
\mathcal{W} = \Delta S - S
\]

\[
\Rightarrow \frac{N m_o c^2 - \varepsilon_Z}{T_U} - k_b \log \left( \frac{\varepsilon_Z + k_b T_U \alpha}{k_b T_U \beta} \right) + k_b \log (\gamma)
\]

Considering the whole of the stochasticity portion of the equivalence theorem, \( k_b \log (\gamma) \) is the initial stochasticity of the graviton lattice before condensate formation; this term is the stochasticity of unbent spacetime, the stable graviton lattice, of values 7.135 \times 10^{-22}J \cdot K^{-1} and 4.449 \times 10^{-3}eV \cdot K^{-1}. These non-zero values of initial entropy is considering quantum foaming.

However, considering an absence of gravity upon the stable lattice, \( \varepsilon_Z = 0 \) and \( N = 0 \). With these parameters, \( \mathcal{W} \) would equal zero, ensuring a balance of entropy in the universe.

### IV. THE THIRD LAW

For thermodynamics is centered around heat and temperature, the third law of gravitonic thermodynamics covers the mechanics of gravitons in a system, in which its temperature is variable:

\[
\text{In a system whose temperature is variable, the natural and physical laws of interacting gravitons, as a boson gas and as condensates, are dependent on temperature as the ultimate parameter.}
\]

### A. Temperature-Varient Zeroth Law

If the temperature is varient, the stability of the graviton lattice will be distorted, for the equation of the bond length is

\[
\lambda_b = \frac{8hc}{3k_b T_U}
\]

As the temperature increases from \( T_U = 2.725K \), the bond length would compress until a graviton cluster forms, creating a condensate. Should the temperature decrease from \( T_U \), the bond length should become static, although the equation above would state otherwise. However, stable gravitons would be seperated by an energy bond length of \( \lambda' \) that will expand ever so slightly in temperatures lower than \( T_U \).
Therefore, there shall be a piecewise function for $X'$:

$$X' = \begin{cases} Le^{-aT(-1+e^{bT}-bT)} + \lambda_b(1-\vartheta) & T \leq 1.45K \\ \frac{8hc}{(3k_bT)} & T > 1.45K \end{cases}$$

where $L = -1\text{m}$, $a = 0.0341525\text{K}^{-1}$, $b = 0.1045\text{K}^{-1}$ and $\vartheta$ is the temperature shift parameter

$$\vartheta = \frac{T_i - T_r}{T_r}$$

where $T_i$ is the instantaneous “test” temperature and $T_r$ is the “reference” temperature. In this scenario, $T_i = 0\text{K}$ and $T_r = 2.725\text{K}$, making $\lambda_b(1-\vartheta) = 2\lambda_b$.

The temperature that separates the two functions of the piecewise, 1.45K, is of significance because it is the temperature where the original definition of the bond length begins its explosive slope upwards. To illustrate the hypothesis that the bond length instead stretches slightly until they come to a stop (signifying the moment where the particles cannot move at absolute zero), the redefinition had to be inserted.

To prove that the redefinition is a valid addition to the piecewise, an error percentage was calculated for the two functions at $T = 1.45\text{K}$. The original bond length equation $\frac{8hc}{3k_bT}$ resulted with a value of 0.0264647m, and the redefinition resulted with a value of 0.0265014m. The error percentage, therefore, is a mere 0.139%.

The piecewise function is graphed below, where the x-axis is the temperature in Kelvin and the y-axis is the variable bond length in meters:

$$g_o = \sqrt{\frac{V + \Gamma_o(N)}{k_bT}}$$

for the function to be temperature-dependent, the coefficients must be swapped into the growth function $\Phi$.

With an increasing temperature, $V$ (the potential of the graviton lattice) is expected to increase, letting $\Gamma_o(N)$ be constant.

Although the potential increases with an increase in temperature, the lattice bond length is expected to decrease in length. Envisualizing the Big Bang, the universe was born from a quantum singularity that expanded catastrophically like an explosion. That was when the universe was at its hottest, at Planck Temperature $T_P = 1.417 \times 10^{32}\text{K}$, when the potential of the graviton lattice would have been only the rest energy of a quantum singularity. This explains the dueling concepts of increasing potential and decreasing bond length with an increase in temperature.

Therefore, the parameter $g_o$ simplifies into

$$g_o \Rightarrow \sqrt{\frac{3}{2N}}$$

To express a dependency on temperature, the condensate growth function is revised as follows:

$$\Phi(T, t) = \sqrt{\frac{6\pi k_bT}{1.71828}} \left(\frac{3}{2N}\right)^{1/2} e^{f(n-2)t} \quad (25)$$

Below is the graph of multiple probability curves of the function above. Provided that the number of incoming colliding radiation waves $n$ is equal to 4 (assuming to be gamma rays), and the initial number of interacting gravitons within a condensate $N$ is 3, each curve varies from the increasing amount of temperature of the universe. It is seen as follows:

B. Temperature-Varient First Law

Variable temperatures affect the rate of graviton condensate growth where temperatures higher than $T_U$ make growth easier to accomplish.

Going back to the equivalence that was last reviewed while deriving the condensate growth function, seen as

$$\frac{6\pi k_bT}{1.71828} g_o = G(n-1)\varepsilon Z$$
The black solid curve is under the influence of temperature $T = 0.0125K$, the blue curve $T = 2.725K$, the black dashed $T = 10K$, the red $T = 100K$, and finally the gray $T = 1000K$.

C. Temperature-Varient Second Law

Considering the equation for the total entropy of a graviton condensate

$$\mathcal{N} = \frac{Nm_o c^2 - \varepsilon_Z}{T_U} - k_b \log \left( \frac{\varepsilon_Z + k_b T_U \alpha}{k_b T_U \beta} \right) + k_b \log (\gamma)$$

an increase of temperature would also lead to a zero value of the entropy, provided that neither of the other independent parameters accumulate in value.

For the temperature vs. entropy graph above, the number of gravitons in the condensate $N$ is again equal to three, and the collective energy of the colliding energy waves $\varepsilon_Z$ is equal to four colliding gamma rays.

APPLICATIONS OF THE THIRD LAW INTO GRAVITONIC MECHANICS

Although the Third Law has aimed to the temperature-variable interactions between gravitons, individual gravitons are also affected by the variance in temperature. In a system of variable temperatures, where every other parameter (i.e. mass, volume, etc.) remains constant, the gravitons would experience, so to say, energetic surges in their mechanisms.

Dimensional Expansion

As understood in the temperature-variant zeroth law, temperature contributes to the expansion or compression of dimensional parameters. If dimensions, themselves, fluctuate proportional to the changes in temperature, then graviton particles may oscillate along a value of $\rho$ that is expanded or compressed to the change in temperature.

This results to the following:

$$\rho' = \rho \left(1 + \frac{T_t - T_r}{T_r}\right)$$

Hyperactive Oscillations

With the temperature-influenced range of attraction $\rho'$, gravitons can oscillate farther or closer than normal:

$$\chi'(\tau) = \rho' \sin \left(\frac{\tau v}{\rho' \sqrt{1 - \frac{v^2}{c^2}}}\right)$$

Using the sun as an example, it currently has a surface temperature of 5,772K, being that it is a yellow dwarf. As the sun grows, it would become a red giant with a typical surface temperature of 5000K\cite{15}\cite{16}. Assuming it does not gain mass, although it is expected for the growing sun to do so, then the graviton’s oscillation will be altered only by the temperature shift.

Comparing the mechanical wave functions of gravitons oscillating about the sun as a yellow dwarf and as a red giant, the two functions are graphed and seen as follows:

Only relying on the temperature shift, gravitons oscillate with less energy as the temperature decreases. The black curve is the yellow dwarf oscillation (with a maximum displacement value $\rho_{\text{sun}} = 8.35 \times 10^9 m$), and the gray curve is the red giant oscillation (with a temperature-shifted displacement $\rho_{\text{sun}}' = 7.23 \times 10^9 m$).

It is acknowledged that the following example is a slight exaggeration, and that it is understood that the sun would have to increase in mass as it grows, hence strengthening the graviton’s oscillation. However, this principle of temperature shift shall work if there is a fluctuation of temperature with a constant mass.

Increased Energy and Momentum

With the derivation of $\rho'$, the mathematics would expect that the following equations for energy $E_T = 2\pi \hbar c / \rho$ and momentum $p = \mu 2\pi \hbar / \rho$ would
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decrease in value. For temperature is the average of all kinetic energies of particles within a system, it is physically expected for gravitons to have an increased value of energy and momentum in a system of higher temperatures.

For momentum, based on scaling theory, \( p \) is proportional to the length \( l \), for \( p \approx mv \). Therefore, the gravitonic momentum will have a direct temperature shift:

\[
p' = p(1 + \vartheta)
\]  

(28)

For energy \( E_T \), scaling theory proposes the proportionality \( E \propto l^2 \), for \( E \approx mv^2 \). Therefore, the graviton's oscillation energy will have a square-temperature shift:

\[
E'_T = E_T\Theta
\]  

(29)

where

\[
\Theta = (1 + \vartheta)^2 \Rightarrow (1 + 2\vartheta + \vartheta^2)
\]  

(30)

This concurs that during the Big Bang, when the universe was at Planck Temperature, the gravitons that were initially dispersed had a great amount of energy. This, thusly, supports the grand expansion of gravity in the quantum scale, applying that their energies are calculable, respecting the Heisenberg Uncertainty Principle.

A demonstration of the strengths of these temperature shifts, scaling \( p \) and \( E_T \) into 1, is graphed within a range of temperatures from absolute zero to Planck Temperature, and is seen as follows:

The blue curve along the temperature x-axis is the linear temperature shift for the graviton’s momentum, while the black curve is the square-temperature shift for the graviton’s energy.

CONCLUSION

In this attempt to understand the roles of the graviton as a fundamental part of the universe, and how interacting gravitons are understood as a degenerate boson gas and as a self-gravitating boson condensate, gravitons have been postulated to obey the laws of General Relativity as they obey the laws of quantum mechanics.

The paper proposed four main considerations that were theoretically addressed and identified with a mathematical formula.

1.) Each black hole, understood to well and distribute gravitons, has a certain value of probability regarding “graviton storage.” A probability function, called the Schwarzschild Probability Function \( P_{sT} \), was derived to determine the probability density for a certain black hole to store graviton particles. The larger the black hole is in size and mass, it is uncertain to verify whether black holes are storing gravitons. Hence the inability to detect gravitons as gravitational waves are being measured. Shope’s Extension was then derived to account for the graviton’s tendency to transition into higher dimensions.

2.) The pressure of a degenerate boson gas (made of interacting gravitons within a collapsing binary system) is proportional to the amount of interacting gravitons within the system. This is verified by the ideal gas law \( PV = Nk_bT \), that the number of interacting gravitons affects the pressure of the mesh that encompasses a collapsing binary. Although a dynamic but constant relation was formulated, the entropy of the degenerate graviton gas was derived. The entropy of the gas is dependent on the “crowding” and excitations of graviton particles within the mesh.

3.) The gravitonic Bose-Einstein Condensate has its own set of four thermodynamic laws. These laws concern with the transfer of energy, which is essential to the theoretical formation of a gravitonic condensate. Hypothesizing that such a condensate is the quantum beginnings of a Kugelblitz black hole, the graviton condensate is given its own set of thermodynamic laws that is quantum dominant.

4.) The graviton, at the early stages of the universe’s history, is an essential part of making the quantum universe classical. Proven by the Facto mensio-atticus and the square temperature shift of the graviton’s energy, the graviton particle was essential in the widespread expansion of the universe, and the process of making a quantum universe into a classical universe. These postulates are supported by the previous theories of the graviton lattice in Part One and the graviton-gravitino subspace relations in Part Two.

The set-backs of the paper are the following: there is still no complete quantum field theory for gravitational interactions and the graviton, and General Relativity remains as not renormalized. For this paper is the final part of the Graviton Theory entity, it is desired for the reader to be enlightened by the nature and mechanisms of the mysterious particle that is the graviton. Although gravitons have been mentioned across string theory, these series of papers hope to achieve the unity between string theory, loop quantum gravity, and General Relativity through the understanding of the graviton particle.
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