The Relationship Between CKM and PMNS Matrices in the CP-Violation Sectors

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Abstract: There are two minima in the global distribution for the mixing angle (23) in the PMNS matrix. It suggests that there are two different phenomena. We described them within the Scale-Symmetric Theory (SST). The two minima lead to two invariants for a certain relationship between the CKM and PMNS matrices in the CP-violation sectors. We also showed how SST is related to the orthodox PMNS matrix and we calculated the maximum allowed CP violation in the leptonic sector parameterized by the Jarlskog determinant – our value is 0.03291. Presented here model allows us to understand the electroweak interactions of baryons.

1. Introduction

The different size scales in Nature and the CP violation lead to the very simple initial conditions in the Scale-Symmetric Theory (SST) [1]. Such initial conditions lead to the atom-like structure of baryons and to the internal structure of bare fermions [1] which is neglected in the Standard Model and General Theory of Relativity. Bare baryons (i.e. their core) contain a torus/electric-charge/spin-1/2 and central condensate of the Einstein spacetime (ES) which is responsible for the nuclear weak interactions of baryons with coupling constant equal to $\alpha_{w(proton)} = 0.0187229$ [1]. The ratio of the poloidal speed to toroidal one of the torus leads to the Jarlskog invariant in the quark sector $J = 2.996 \cdot 10^{-5}$ – it defines the poloidal time characteristic for the weak interactions [2]. Outside the core of baryons, due to the electroweak interactions, there are created the Titius-Bode (TB) orbits but they are embedded in the nuclear strong field – their radii are: $R = A + dB$, where $A = 0.6974425$ fm, $B = 0.5018395$ fm, and $d = 0, 1, 2$ and 4 for the last orbit [1]. The range B is associated with virtual particle with a mass of $M_B = 750.292$ MeV while A with a virtual mass equal to 4 masses of neutral pions ($4\pi^o$) [1]. Notice that ranges are inversely proportional to masses so $M_B/(4\pi^o) = A/B = 1.3898$. On the TB orbits can be relativistic pions. All real and virtual pions are produced inside the torus on orbit with a radius of $R_\pi^{creation} = 2A/3$. Due to the weak decays of charged pions inside the core of baryons, due to the poloidal speeds for left-handed tori (baryons have left-handed internal helicity), the preferred direction of decay is antiparallel to spin of the torus i.e. the CP phase is $\delta_{CP} = \pi$ rad (180^o) and such a value we use in the SST PMNS matrix in the paper [3] and in this paper. But due to the poloidal speed, such CP phase can change from 360^o to zero. On the other hand, the pions have spin equal to zero so their decays in some distance of the torus (i.e. for $d = 1, 2$ and 4) have not a preferred direction of weak decay i.e. it changes in relation to the spin of the torus from 0^o to 180^o so the mean is $90^o = \pi/2$ rad and such a value we use in the SST CKM matrix in the paper [4] and here. Such value causes that in nuclear strong interactions there do not appear CP-violation. But emphasize that the poloidal motion of the torus can decrease a little the CP phase for the left-handed nucleons even in bigger distances from the torus.

Within SST we derived the CKM matrix and PMNS matrix and showed the true origin of the mixing angles [3], [4]. The mixing angles in SST and in the orthodox matrices have similar values so the two very different descriptions are equivalent. There is only one difference which causes that in the orthodox PMNS matrix, values of the elements have very low statistical significance. SST shows that $\Theta_{23}[^o] = 5\Theta_{13} = \underline{41.3250}$ [4] while experimental data lead to two minima in the global distribution: $\sin^2\Theta_{23} \approx 0.44$ i.e. $\Theta_{23}[^o] = 41.55$ which is close to the SST value, and $\sin^2\Theta_{23} \approx 0.55$ i.e. $\Theta_{23}[^o] = 47.87$ – see Figure 2 in [5]. Can we explain such duality? In SST, masses of the electron-neutrino and muon-neutrino are the same so Δm_{21}^2 should be close to zero and it is satisfied because $\Delta m_{21}^2 \approx 7.5 \cdot 10^{-5}$ $(eV)^2$. On the other hand, mass of the tau-neutrino is three times higher but it is the electron-type neutrino [3]. This causes that the measured value $\Delta m_{32}^2 \approx 2.5 \cdot 10^{-3}$ $(eV)^2$ can be much lower than it follows from formula $\Delta m_{ij}^2 \equiv \Delta m_i^2 - \Delta m_j^2$. The SST value for the pure Δm_{32}^2 state is about 10 times higher than the experimental one but we can see that we can explain it within our model. In SST, mass of all species is 5 times higher than mass of the electron-neutrino (or muon-neutrino) – it leads to the formula $\Theta_{23} = 5\Theta_{13}$ [3]. But there are 6 different neutrinos (3 species) so the following solution is possible as well: $\Theta_{23}[^o] = 6\Theta_{13} = 49.59$ (it is for the perfectly neutral object which can be created inside the core of baryons because of decay of three different pions) – it solves the problem of duality of Θ_{23}. Duality and broadening of the δ_{CP} phase cause that for normal ordering at 3σ range is $38.4 < \Theta_{23} < 52.8$ [5].

2. The true origin of the elements of the CKM and PMNS matrices

Orthogonality of both matrices follows from the fact that their elements are defined as the square root of energy $E_{ij}^{1/2} = m_i^{1/2} v_j$ and from the fact that the total speed of the components of the Einstein spacetime, from which the particles are built, is $c = \text{constant}$ and that in the rows of the matrices the components of the mass of nucleon are normalized to unity, while in columns the components of the velocity $\boldsymbol{c}$ are normalized to one. The mixing angles parameterize both matrices in such a way. We present here the origin of the mixing angles and of the components of the mass of nucleon and of the components of the velocity $\boldsymbol{c}$. The components of the velocity $\boldsymbol{c}$ are the following three velocities: the radial velocity which is the strong velocity, the toroidal velocity which is the electromagnetic velocity, and the poloidal velocity which is the weak velocity. The atom-like structure of baryons described within the Scale-Symmetric Theory shows that the three velocities are orthogonal.

Particle in baryons are placed or are created on the allowed orbits with radii equal to [1]

$$R \equiv R_\pi^{creation} = 2A/3, A, A + B, A + 2B, \text{ and } A + 4B \,. \quad (1)$$

SST shows that on the orbit with radius $R_{\pi}^{creation} = 2A/3$, which lies inside the central torus of the baryons, are produced the virtual or real large loops (m_{LL} = 67.54441 MeV) and pions which contain two such loops [1].

To simplify the description assume that a baryon emits particle from the equator of the torus which has the radius A. Then decay due to the nuclear strong interactions is defined by the radial velocity in the plane of the torus equator

$$v_s = v_{strong} = v_{radial} . \tag{2a}$$

Decay due to the electromagnetic interactions is defined by the toroidal velocity

$$v_e = v_{electromagnetic} = v_{toroidal} . \tag{2b}$$

Decay due to the electromagnetic interactions is defined by the toroidal velocity

$$v_w = v_{weak} = v_{poloidal} . \tag{2c}$$

The three velocities are orthogonal and their vectorial sum is equal to $\boldsymbol{c}$

$$(v_s / c)^2 + (v_e / c)^2 + (v_w / c)^2 = 1 . \tag{3}$$

Such is the origin of orthogonality of the Standard-Model (SM) interactions.

On the other hand, when we neglect the binding energy of the proton $p \approx 938.3$ MeV ($\Delta E \approx 15$ MeV) then proton consists of three elements: of the torus with a mass of X ≈ 318.3 MeV, of the relativistic pion with a mean mass of W ≈ 212.2 MeV on the d = 1 orbit with a radius of $A + B$, and of the central condensate with a mass of Y ≈ 424.1 MeV [1].

There is satisfied following relation

$$X / p + W / p + Y / p \approx 1 . \tag{4}$$

But notice that the kinetic energy of an object is related to its momentum p by the equation:

$$E_k = p^2 / (2m) . \tag{5}$$

On the other hand, when we detect particles from interactions with nucleons then the electromagnetic interactions inside baryons slow down the weak decays of neutrons because they force the exchanges of the lepton pair $e^-\nu_{e,anti}$ (or pair $e^+\nu_e$) between the relativistic pion W in the d = 1 state and the core of baryons, so number of observed neutrinos should be lower than expected according to following relation

$$f = (N_{Observed} / N_{Expected})_{SST} = (\alpha_{w(proton)} - \alpha_e) / \alpha_{w(proton)} = 0.6102 . \tag{6}$$

where $\alpha_{w(proton)} = 0.0187229$ is the coupling constant for the nuclear weak interactions and $\alpha_e = 1/137.036$ is the fine-structure constant [1].

Applying formula (6) we can rewrite formula (4)

$$[X / (f\,p)]^2 + [W / (f\,p)]^2 + [Y / (f\,p)]^2 \approx 1\,. \quad (7)$$

From formulae (5)-(7) results that when we measure momentums of particles after their interactions with nucleons then the elements of the considered here matrices are proportional to ratios of masses

$$V_{ij} \sim m_1 / m_2\,. \quad (8)$$

Here we are considering cases related to formula (8).

Formulae (3) and (7) show that the SST CKM matrix V_{SST}(CKM) with elements $V_{ij}(X, W, Y, s, e, w)$ is orthogonal (Fig.1).

$$V_{SST}(\mathrm{CKM}) = \begin{bmatrix} V_{ud} = V_{Xs} & V_{us} = V_{Xe} & V_{ub} = V_{Xw} \\ V_{cd} = V_{Ws} & V_{cs} = V_{We} & V_{cb} = V_{Ww} \\ V_{td} = V_{Ys} & V_{ts} = V_{Ye} & V_{tb} = V_{Yw} \end{bmatrix}$$

Fig.1
The CKM mixing matrix in Scale-Symmetric Theory

Similar is for the SST PMNS matrix V_{SST}(PMNS) with elements $U_{ij}(2m_{LL} \approx \pi^o = P\ m_{LL} = G\ m_{LL}/2 = N, s, e, w)$, where $m_{LL}/2$ is the characteristic energy/mass of neutrinos created inside the core of baryons, m_{LL} is the characteristic energy/mass of gluons/photons, and $2m_{LL} \approx \pi^o$ is the mass of neutral pion – such matrix is orthogonal as well. We have

$$[P / (f\,M_{\pi\mu})]^2 + [G / (f\,M_{\pi\mu})]^2 + [N / (f\,M_{\pi\mu})]^2 \approx 1\,, \quad (9)$$

where $M_{\pi\mu} = \pi^{+,-} + \mu = 245.23$ MeV [7].

The SST PMNS matrix looks as follows (Fig.2).

$$U_{SST}(\mathrm{PMNS}) = \begin{bmatrix} U_{e1} = U_{Ps} & U_{e2} = U_{Pe} & U_{e3} = U_{Pw} \\ U_{\mu 1} = U_{Gs} & U_{\mu 2} = U_{Ge} & U_{\mu 3} = U_{Gw} \\ U_{\tau 1} = U_{Ns} & U_{\tau 2} = U_{Ne} & U_{\tau 3} = U_{Nw} \end{bmatrix}$$

Fig.2
The PMNS mixing matrix in Scale-Symmetric Theory

The SST mixing angles define values of elements of the two matrices and are consistent with the orthodox ones.

In all denominators of the SST CKM mixing angles is the mass of the torus X [4] which structure leads to orthogonality of the SM interactions. On the other hand, in numerators are masses concerning the nuclear Titius-Bode orbits i.e. m_{LL} which relates to the first orbit with a radius of $2A/3$, M_B which relates to B, and mass of the b quark which relates to the last orbit with a radius of $A + 4B$ [4], [1].

In all denominators of the SST PMNS mixing angles is the characteristic energy/mass of the neutrinos $m_{LL}/2$ produced inside the torus of baryons [4]. On the other hand, in numerators is mass of the pair composed of the negative and positive pion (in their weak decays are produced the three different neutrinos) multiplied by number of lightest neutrinos in the characteristic objects in Nature – i.e. 1, 4, and 5 or 6.

Approximate absolute values of elements in the two matrices are as follows.

The approximate values of the elements U_{ij} in the SST PMNS matrix for the sequence 1, 4, and 5 defines formula (10) and the matrix A_{ij} in Fig.3

$$U_{ij} = 1 / (1 + A_{ij}) . \quad (10)$$

$$A_{ij} = \begin{bmatrix} \frac{2}{3\pi} & \frac{8}{3\pi} & 2\pi \\ 2 & \frac{1}{2} & \frac{1}{2} \\ 1+\frac{2}{3\pi} & 1 & \frac{1}{3} \end{bmatrix}$$

Fig.3

Notice that the elements of the matrix A_{ij} we can define using 9 particles with 8 different masses: $2/(3\pi) = m_{LL}/X$, $2\pi \approx Y/m_{LL}$, $2 \approx Y/W$, $1 = \nu_e/\nu_\mu$, $1/3 = \nu_e/\nu_\tau$, where masses of the electron-neutrino and muon-neutrino are the same but their internal structure are not the same [1]. We can see also that mass of neutral pion is about 4 times higher than the characteristic energy of neutrino: $\pi^o \approx 4m_{LL}/2$, so we have $8/(3\pi) = 4m_{LL}/X$. These considerations show that it is untrue that the PMNS matrix leads to the neutrino oscillations – SST shows that the PMNS leads to the atom-like structure of baryons (X, W, Y, π^o=P, m_{LL}=G, and $m_{LL}/2$=N) and to three different neutrinos (ν_e, ν_μ, ν_τ) with two different masses. In reality, the neutrino oscillations are the exchanges of neutrinos on the neutrinos in the cosmic neutrino background (CNB) or on neutrinos in the neutrino-antineutrino pairs the Einstein spacetime and all observed particles consist of.

Formula (10) and the matrix A_{ij} leads to an approximate PMNS matrix $U^{approx.}$(PMNS) – see Fig.4.

$$U^{approx.}(PMNS) = \begin{bmatrix} 0.8249 & 0.5409 & 0.1373 \\ 0.3333 & 0.6667 & 0.6667 \\ 0.4520 & 0.5000 & 0.7500 \end{bmatrix}$$

Fig.4

Discrepancy from normalization to 1 of the $U^{approx.}$(PMNS) is from –0.013 to + 0.026.

The approximate values of the elements V_{ij} in the SST CKM matrix defines formula (11) and the matrix B_{ij} in Fig.5 (it is by an analogy to the SST PMNS matrix)

$$V_{ij} = 1 / (1 + B_{ij}) . \quad (11)$$

$$B_{ij} = \begin{vmatrix} f & g & h \\ g & j & k \\ h & k & a \end{vmatrix}$$

Fig.5

The elements in B_{ij} are defined as follows.

$f = \alpha_{w(proton)} + \alpha_e$, where $\alpha_e = 1/137.036$ is the fine-structure constant,

$g = (X + Y) / W$; notice that this value is close to the ratio of masses of the b and c quarks calculated within SST: $b = 4190$ MeV and $c = 1267$ MeV [1],

$h = X / (e^{+}_{bare} + e^{-}_{bare})$, where $e^{-}_{bare} = 0.510407$ MeV is the bare mass of electron [1],

$k = m_{LL} / (H^{+} - H^{o})$, where $(H^{+} - H^{o}) = 2.663$ MeV is the mass distance between the charged core and neutral core of baryons [1],

$a = \alpha'_{w(electron-proton)} = 1.12 \cdot 10^{-5}$ [1],

$j = f + (e^{-} - e^{-}_{bare}) / e^{-}_{bare} = f + 0.001159652$.

$$V^{approx.}(CKM) = \begin{bmatrix} 0.97464 & 0.22229 & 0.00320 \\ 0.22229 & 0.97354 & 0.03793 \\ 0.00320 & 0.03793 & 0.99999 \end{bmatrix}$$

Fig.6

In such a way defined B_{ij} and the formula (11) lead to an approximate CKM matrix $V^{approx.}(CKM)$ – see Fig.6. Discrepancy from normalization to 1 of the $V^{approx.}(CKM)$ is from –0.0014 to + 0.0014 so is very low.

In the CKM matrix is

$$V_{Xs} = \alpha_s / (\alpha_s + \alpha_e + \alpha_{w(proton)}) = 0.97464,$$

where $\alpha_s = 1$, $\alpha_e = 1/137.036$, and $\alpha_{w(proton)} = 0.0187229$ [1]. We see that this element defines in very good approximation the share of nuclear strong interactions in the three other interactions.

The W pions interact first of all electromagnetically so V_{We} should be close to 1.

The Y condensate first of all interacts weakly so V_{Yw} should be close to 1.

We showed that the atom-like structure of baryons leads to the CKM matrix so quarks are not needed but emphasize that within SST we derived the masses of quarks [1].

3. The relationship between CKM and PMNS matrices in the CP-violation sectors derived from the SST mixing angles [3], [4]

First we will describe the problem by using the SST matrices [3], [4] – see Fig.7 and Fig.8 here. The CP-violation sectors we marked in red and created expressions as in the figures.

The PMNS-CKM invariant K is the ratio of such expressions

$$K = U_{CP,SST}(\text{PMNS}) / V_{CP,SST}(\text{CKM}) = 3.602 / 2.602 = 1.384 \,. \qquad (12)$$

$$U_{SST}(\text{PMNS}) = \begin{bmatrix} V_{e1} = 0.8294 & V_{e2} = 0.5399 & V_{e3} = -0.1438 \\ V_{\mu 1} = -0.3301 & V_{\mu 2} = 0.6812 & V_{\mu 3} = 0.6535 \\ V_{\tau 1} = 0.4507 & V_{\tau 2} = -0.4945 & V_{\tau 3} = 0.7432 \end{bmatrix}$$

$$U_{CP,SST}(\text{PMNS}) = \left| \frac{V_{e2} V_{\mu 3}}{V_{e3} V_{\mu 2}} \right| = 3.602$$

Fig. 7
The PMNS neutrino-mixing matrix in Scale-Symmetric Theory

$$V_{SST}(\text{CKM}) = \begin{bmatrix} V_{ud} = 0.97372 & V_{us} = 0.22774 & V_{ub} = 0.00370 \\ V_{cd} = -0.22769 & V_{cs} = 0.97287 & V_{cb} = 0.04113 \\ V_{td} = 0.00587 & V_{ts} = -0.04089 & V_{tb} = 0.99915 \end{bmatrix}$$

$$V_{CP,SST}(\text{CKM}) = \left| \frac{V_{us} V_{cb}}{V_{ub} V_{cs}} \right| = 2.602$$

Fig. 8
The CKM quark-mixing matrix in Scale-Symmetric Theory

On the other hand, we have [1]

$$A / B = (\alpha_{w(proton)} + \alpha_{em}) / \alpha_{w(proton)} = 1.3898 \,, \qquad (13)$$

where $\alpha_{em} = 1/137.036$ is the fine-structure constant. Coupling constant is inversely proportional to distance so A corresponds to nuclear weak interactions while B corresponds to electroweak interactions.

From formulae (12) and (13) follows that the invariant K for the two CP-violation sectors concerns both the atom-like structure of baryons and the electroweak interactions.

The first SST PMNS-CKM invariant K is associated with the electroweak interactions, characteristic masses and the constants in the TB formula for baryons

$$K \approx A / B = M_B / 4\pi^{o} = (\alpha_{w(proton)} + \alpha_{em}) / \alpha_{w(proton)} = 1.39 \,. \qquad (14)$$

Now we can consider the orthodox matrices.

The values in the CP-violation sector in the orthodox CKM matrix are as follows [6]:

$V_{us} = 0.22534(65)$, $V_{ub} = 0.00351^{+0.00015}_{-0.00014}$,
$V_{cs} = 0.97344(16)$, $V_{cb} = 0.412^{+0.0011}_{-0.0005}$.

Applying formula from Fig.8 we obtain

$V_{CP}(CKM) = 2.717^{+0.198}_{-0.151}$,

so it is consistent with the SST result.
The values in the CP-violation sector in the orthodox PMNS matrix are as follows [5]:

$U_{e2} = 0.516 – 0.582$, $U_{e3} = 0.144 – 0.156$,
$U_{\mu 2} = 0.467 – 0.678$, $U_{\mu 3} = 0.639 – 0.774$.

Applying formula from Fig.7 we obtain

$U_{CP}(PMNS)$ is the interval (3.12; 6.84) ,

so it is consistent with the SST result.
Notice that the mean values of U_{ij} lead to $U_{CP}(PMNS)^{mean} = 4.5623$ which is close to $(\pi^{+,-} - \pi^{o})/(e^{-} + e^{+})_{bare} = 4.500$ – it suggests that the electroweak interactions in the orthodox PMNS matrix with the mean values do not concern real particles (as it is in the SST matrix) – there dominate the electroweak interactions of the virtual fields around baryons [2].
We can see that there can be the orthodox invariant

$$K^{orthodox} = 1 + 1/K \approx U_{CP,SST}(PMNS)^{mean}/V_{CP,SST}(CKM) = 4.500/2.602 = 1.73. \quad (15)$$

This value is close to the ratio $H^{+} / Y \approx 1.72$, where H^{+} is the mass of the charged bare baryons whereas Y is the mass of central condensate which is responsible for the weak interactions. For the orthodox PMNS-CKM invariant, we have

$$K^{orthodox} \approx (A + B) / A \approx H^{+} / Y \approx 1.72 . \quad (16)$$

Calculate now the second SST PMNS-CKM invariant for $\Theta_{23}[^{o}] = 6\Theta_{13} = 49.59$.
The values in the CP-violation sector in the SST PMNS matrix for $\Theta_{23}[^{o}] = 49.59$ are as follows:

$U_{e2} = 0.5399$, $U_{e3} = -0.1438$,
$U_{\mu 2} = 0.7079$, $U_{\mu 3} = 0.7535$.

Applying formula from Fig.7 we obtain

$U^{*}_{CP,SST}(PMNS) = 3.9963$.

Applying formula (12) we obtain

$$K^{*} = U^{*}_{CP,SST}(PMNS) / V_{CP,SST}(CKM) = 3.996 / 2.602 = 1.536 , \quad (17)$$

On the other hand, we have

$$K^{*} \approx A / R_{\pi}^{creation} = A / (2A/3) = (X^{+} + X^{-}) / Y = 1.50 , \quad (18)$$

where $X^{+,-}$ = 318.3 MeV is the mass of the torus in the core of baryons and Y = 424.1 MeV is the mass of the central condensate.

Notice that the two SST invariants and the orthodox invariant all are associated with the orbits in baryons and with characteristic masses for the atom-like structure of baryons.

4. The Jarlskog invariants in the quark sector and leptonic sector

In the quark sector is [7]

$$J_{CP}^{quarks} = (3.04^{+0.21}_{-0.20}) \bullet 10^{-5} \,. \quad (19)$$

It is consistent with the SST value $J_{CP}^{quarks,SST} = 2.996 \bullet 10^{-5}$ [2].

In the leptonic sector, the Jarlskog invariant is [5]

$$J_{CP}^{leptons,max} = 0.0329 \pm 0.0007^{+0.0021}_{-0.0024} \,. \quad (20)$$

On the other hand, we showed that CP-violation can concern the weak masses of charged pions [2] so we can assume that following definition is correct (symbols of particles denote their masses as well)

$$J_{CP}^{leptons,max,SST} = \alpha_{w(proton)}(\pi^{+,-} - \pi^{o})/(\alpha_{w(proton)}\,\pi^{+,-}) = (\pi^{+,-} - \pi^{o})/\pi^{+,-} = 0.03291. \quad (21)$$

We can see that both results are the same.

5. Summary

The mixing angles in the SST and the orthodox CKM and PMNS matrices have similar values so the two very different descriptions are equivalent.

Orthogonality of both matrices follows from the fact that their elements are defined as the square root of energy $E_{ij}^{1/2} = m_i^{1/2} v_j$ and from the fact that the total speed of the components of the Einstein spacetime, from which the particles are built, is ***c*** = constant and that in the rows of the matrices the components of the mass of nucleon are normalized to unity, while in columns the components of the velocity ***c*** are normalized to one. The mixing angles parameterize both matrices in such a way. We present here the origin of the mixing angles and of the components of the mass of nucleon and of the components of the velocity ***c***. The components of the velocity ***c*** are the following three velocities: the radial velocity which is the strong velocity, the toroidal velocity which is the electromagnetic velocity, and the poloidal velocity which is the weak velocity. The atom-like structure of baryons described within the Scale-Symmetric Theory shows that the three velocities are orthogonal.

We showed that in experiments, the elements of matrices are directly proportional to ratios of characteristic masses.

We showed that it is untrue that the PMNS matrix leads to the neutrino oscillations – SST shows that the PMNS leads to the atom-like structure of baryons and to three different neutrinos with two different masses. In reality, the neutrino oscillations are the exchanges of neutrinos on the neutrinos in the cosmic neutrino background (CNB) or on neutrinos in the neutrino-antineutrino pairs the Einstein spacetime and all observed particles consist of.
with 3 significant digits.

We showed that the atom-like structure of baryons leads to the CKM matrix so quarks are not needed but emphasize that within SST we derived the masses of quarks [1].

We showed that for the internally left-handed baryons preferred CP phase for PMNS matrix is $\delta_{CP} = \pi$ rad whereas preferred for CKM matrix is $\delta_{CP} = \pi/2$ rad.

Here we described the origin of the two minima in the global distribution for the mixing angle Θ_{23} in the PMNS matrix and we showed how the Scale-Symmetric Theory couples with the PMNS matrix.

We showed that the two SST PMNS-CKM invariants ($K \approx 1.39$ and $K^* \approx 1.50$) and the orthodox invariant ($K^{orthodox} \approx 1.72$) for the two CP-violation sectors all are associated with the orbits in baryons and with characteristic masses for the atom-like structure of baryons. Moreover, the invariant K concerns the electroweak interactions as well.

We calculated the maximum value of the Jarlskog invariant in the leptonic sector $J_{CP}^{leptons,max,SST} = 0.03291$.

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