Algebraic Invariants of Gravity

Hans Detlef Hüttenbach

Abstract. Newton's mechanics is simple. His equivalence principle is simple, as is the inverse square law of gravitational force. A simple theory should have simple solutions to simple models. A system of n particles, given their initial speed and positions along with their masses, is such a simple model. Yet, solving for $n > 2$ is not simple.
This paper discusses, why that is a difficult problem and what could be done to get around that problem.

1. Problem Statement

Classical mechanics is essentially a linear, "first order" theory in which the dynamical quantities describe properties of the particles themselves, such as the law of inertia, $F = ma$, as well as energy and momentum conservation etc.
The graviational force, $F = (const)\nabla\frac{m_1 m_2}{|x_1 - x_2|}$, is the exception to that theory: it is a product of quantities, namely the mutual interaction the masses, disguised as a linear first order quantity F. That makes it complicated to even deal with a gravitational interaction of two particles, necessitating elliptic integrals, Legendre polynoms, Bessel functions, and all that, in order to derive its solutions. But it can be done, and it involves some beautiful mathematics and calculations, which explains, why it's done in physics first hand up to this day. The result is that the particles move (with their reduced masses) around the center of mass in all curves given by the intersection of a plane with a cone.
That is mathematically interesting, as it allows to describe the set of solutions through a hyperbolic, quadratic equation, namely that of the cone itself. And it straight leads to the question, if not a quadratic approach to the dynamics might be simpler to describe gravitational interaction.

2. The Cone

The picture of that cone is always that of a two-dimensional surface in three dimensions, because it is easy to visualize, but, even given the fact that one angular, cyclic coordinate can be eliminated, this is still inconvenient:
it is mathematically the product of two non-parallel intersecting lines and a circle. And the circle is well-known to yield us the conservation of angular momentum, which the Hamiltonean does then not depend on. So, let's drop it. We are then left with the two non-parallel lines in a two dimensional (Euclidean) coordinate system, which by proper scaling, transform into the diagonals of $\mathbb{R}^2$, that intersect in the origin. Let a, b denote horizontal and vertical axes. Then the original condition that the path of motion is to be the intersection of a plane with the cone reduces to the intersection of a line with the diagonals, i.e.: $a^2 - b^2 = 0$. That is an invariant, in fact *the* invariant of a mass point moving in a constant gravitational field. I am now free to scale a^2, which I choose to be the square of total energy E^2 of the system, up to an additive constant of motion. Then $E^2 - b^2 = (Const)$ is a constant of motion. And, as E is a constant of motion, E^2 is conserved, so b^2, and b must be conserved, too. Since a is now measured in units of energy, b will have to be of the same dimension. Now, for a closed system, total momentum and total angular momentum are invariants of motion, which therefore can be subtracted. Now, suppose for that closed system, there was a notion of a state, in which which all particles are at rest with respect to eachother. Let me call it "the" rest system and denote it by E_{rest}. Then

$$Q^2 := E^2 - E_{rest}^2$$

was an invariant, either. Q^2 would then capture the square of all motional energy in the system, namely the (square of) kinetic energy, including the potential energy into which it converts to and from. So, it is a function of the kinetic energy T and the potential energy V.
Let's inspect the bounded 2-particle system: Because the angular momentum is preserved (and hence an invariant of motion), the polar coordinates are irrelevant, and consideration restricts to the radial coordinate r. If the reduced mass stays at constant $r(t) = r_0$ from the center of mass for all t, it moves in a circle, which makes it a perfect rest system. Generally, however, in a stable solution of the 2-particle system, moving and bounded (reduced mass oscillates in r between the minimal radius $r_{min} >= 0$ and a maximal radius $r_{max} > 0$. With the exclusion of $r_{min} = 0$, the velocity is zero at both r_{min} and r_{max} and must have maximum absolute value of velocity in between, which is at that radius, where the centrifugal force cancels the gravitational attraction, i.e.: $(Const)\frac{mM}{r^2} = \frac{L^2}{2mr^3}$, where L is the angular momentum of the (reduced) moving mass m, and M is the total mass. This yields $r_0 = \frac{L^2}{(Const)2Mm^2}$ as the radius of maximal velocity and therefore maximal kinetic energy in the radial direction and suggests to define the rest system of a bounded moving (reduced) mass to be the energy of the mass m on a circle of radius r_0, and clearly, it would make sense to set the potential energy to zero at r_0.)

(Note that that system is not defined for $L = 0$, in case of which $r_0 = 0$, for which the kinetic energy T undefined.)

Next, we look into Q: The cone equation demands: $T^2 - V^2 = 0$; but if we set $E_0 := V + T$ and demand E_0 to be constant, as we would in classical mecanics, then $E_0 = (1/2)T$ follows, so the oscillating particle can never reach y_{min} and y_{max} with constant $T+V$. That said, Q^2 cannot be $(T+V)^2$. However, $-V^2 = (iV)^2$, the imaginary factor maps the cone equation into a circle, and $Q^2 := (|T|^2 + |V|^2)$ does fit. So, I take that as given. Lastly, the square energy of the center of mass at rest itself is a dynamic invariant. I can subtract that and get for that moving particle of reduced mass in the gravitational field of center of mass:

$$E^2 = E_{rest}^2 + Q^2,$$

where $Q^2 := (|T|^2 + |V|^2)$ is itself a dynamic invariant.

Remark 2.1. Elastic (headover) collisions follow the same rules as elliptic oscillation: the only difference is that the minimal radius of return for the lighter (reduced mass) particle is not behind the center of mass, but before, or at the surface of the center of mass itself.
Now, suppose that the center of mass is within a compound solid body of diameter greater zero. Then, in the frame of reference, of zero total angular momentum, the body at the center of mass will be rotating in the opposite direction to the outer reduced mass, and that reduced mass may collide eventually with some edge of the solid body at some radius $r > 0$. By addition of a proper total mamumentum and angular momentum that collision can again be transformed into a headover collision (where the sum of momentum of the collision is zero). Given that the collision is elastic, the outer mass is again just reflected, and while $Q^2 = (|T|^2 + |V|^2)$ is invariant w.r.t. space inversion (a.k.a. parity), the effect of the collision is a symmetry transformation w.r.t. $E^2 = E_{rest}^2 + Q^2$ and does not change the overall dynamics.

As has been shown, the cone invariance allows to separate the outer, moving particle (of reduced mass) from the mass center: their energy squares simply add. (For a 1-particle system that clearly also holds.) Then, by induction, given a system of $n + 1$ particles, we can replace the 1^{st} n particles by its own center of mass and reduce it to a 2-particle problem and separate the square of energy for the last particle (with reduced mass) out, leaving the square energy of the center of mass for the first n particles as a summand. So, that principle of additivity holds for all n-particle (elastically colliding) systems. Finally, for a large number of fairly equally heavy particles, the reduced masses are approximately the particles' masses, so we get for such an n-particle system:

$$E^2 = \sum_{1 \le k \le n} E_k^2 = \sum_{1 \le k \le n} E_{rest,k}^2 + Q_k^2 = \sum_{1 \le k \le n} E_{rest,k}^2 + (|T_k|^2 + |V_k|^2), \quad (2.1)$$

in which all terms and in particular $Q = \sqrt{\sum_k (|T_k|^2 + |V_k|^2)}$ are invariants. We define Q to be the *heat* of the system. The fact that it is an invariant of the closed system is important, because that means that heat can be transfered deliberately from one closed mechanical system to another, which is what we observe. (This does however not answer the question of why heat would flow from hot to cold systems, only.) And, as an invariant, that is why it was and can be neglected in the Newtonian, gravitational mechanics.

Remark 2.2. What was done, was to diagonalize the quadratic form: Any quadratic form is mathematically defined through a linear operator, that is itself the square of a normal operator, and this square operator rewrites into the sum of three linear symmetric operators a positive, a negative, and a zero-operator, where the zero-operator delivers the invariants, and the negative operator can be inverted to a positive operator by taking its absolute values.

3. Parity

The next step is to from a system of n disrete particles to a continuum of particle densities, which will replace the mass squares m_k^2 with a square mass densitiy $\rho_k^2(t, \vec{x})$, and the Q_k^2 would become a density, representing the square of heat, and the sum over the k particles will be replaced by the integration over the spatial volume $4dx^3$. But there is a technical problem in the flow of particles $j(t, \vec{x}) := \rho(t, \vec{x})$, which is to replace the momenta $\vec{p}_k$: Wheras in the discrete finite n-particle model collisions occur only sporadically, more exactly: at each time t on a set of measure zero, and the momenta are observable to the outside almost everywhere for all $\vec{x} \in \mathbb{R}^3$, in the continuous model, the fluxes superimpose destructively: in fact, within a solid body or a liquid in a container, the internal flow of particled cancels out completely to the ouside, and macroscopically, all particles appear to stay at rest: the total momentum of the particles cancels out, even locally.
To resolve that problem, notice that locally, both energy and momentum are constant right before and after an elastic collision, which - as is well-known from the theory of an ideal gas - means that the particles behave equivalently to ones that pass through freely instead of colliding and bouncing back. That way, we can model the situation by splitting the flux $\vec{j}$ into two component $\vec{j}_+$ and $\vec{j}_-$ of opposite parity. When both are equal, then their superposition will cancel completely, but that would not mean that the system was without motion, because the Euclidean square, of the components, $|j^2| := |j_-|^2 + |j_+|^2$ would be greater zero.
That leads straight to set $j = j_1\sigma_1 + \sigma_2 j_2 + \sigma_3 j_3$, where the σ_k are Hamilton's quaternions or Pauli's sigma matrices (both differ from eachother by a factor i): if we would go in with a unit vector $2^{-1/2}(1, 1)$ on the r.h.s., as equal pairing of positive and negative parity, then that would lead to destructive superposition. The other extremes are the unit vectors $(1, 0)$ and $(0, 1)$, in which case all motion will be synchronous, either to the left or to the right,

like waves in a water glass that shake seemingly synchronously to either side: Evidently, $j(t,\vec{x}) \neq 0 \implies \rho(t,\vec{x}) \neq 0$, in other words: in the absence of masses the heat is zero. Other contraints for the scalar components j_1, j_2, j_3 then obviously are that their absolute sqares need to be well-defined and finite, and that they all vanish on the system's boundary. That again gives two further extremes: j_1, j_2, j_3 could be constant within the confined body's region, in which case the whole motional energy within the body was kinetical (although at the boundary this energy will have to be converted instantly into potential energy and to be released as kinetical energy thereafter), and it also can be totally potential energy, in case of which nothing would move at all.

That said, could we gain energy from a system for which all kinetic energy has been converted into potential energy? It appears that we need to convert that energy into a kinetical energy, which then can be used to drive mechanical machines.
There is another side to it: When the momenta cancel out locally within the body, then the T_k^2 and therefore V_k^2 are stationary, so the V_k^2 can be included into the square rest energy $E_{rest,k}^2$.

4. Laplace and the Inverse Square Law

Consider a mass m_1 located in the origin, say. Then the square of its gravitational potential is $V^2(r) = G^2 \frac{m_1^2}{r^2}$, and if we integrate over any sphere around the origin of radius $r > 0$ we trivially get $4\pi G^2 m_1^2$, and that then holds for any (decently smooth) boundary over a stars-haped region containing the origin. Hence. integrating over such a region containing n masses $m1, \ldots, m_n$ we get out $4\pi G^2 \sum_{1\leq k\leq n} m_k^2$, and if we'd want to take its root, then that would be the square root of that. Simple. The only problem is that we conceive mass to be additive, rather than its square, so the common standpoint is that the elliptic Euclidean geometry has to be flat, disregarding the fact that two masses m_1 and m_2 at locations $\vec{x}_1$ and $\vec{x}_2$ both have their own, linearly independent 3-dimensional location coordinates.
The demanded additivity can be enforced by introducing an additional attractive potential between masses:
Given a closed system of n masses $m_1, \ldots, m_n$ with their center of mass at the origin with total mass $M = m_1 + \cdots + m_n$, the square of its gravitational potential is given by
$V^2(r) = 4\pi G^2 \frac{\sum_k m_k^2}{r^2} = 4\pi G^2 \frac{M^2}{r^2} - 4\pi G^2 \frac{2\sum_{k\neq l} m_k m_l}{r^2}$, so
$V(r) = \sqrt{4\pi} G \frac{M}{r} \sqrt{1 - 2\sum_{k\leq l} \frac{m_k m_l}{M^2}} \approx \sqrt{4\pi} G \big(\frac{M}{r} - \frac{\sum_{k<l} m_k m_l}{Mr}\big)$, where the total mass $M \gg m_k$ is assumed to be much larger than each of the m_k.
For each $k = 1 \ldots, n$ then $\mu_k := \frac{\sum_{k<l} m_k m_l}{M}$ sums up to the reduced mass for m_k, so the needed gravitational potential between two (gravitational) masses

m_1 and m_2 at location $\vec{x}, \vec{y} \in \mathbb{R}^3$ to ensure mass additivity turns out to be $V_{grav}(|\vec{x}-\vec{y}|) = \sqrt{4\pi} G \frac{m_1 m_2}{|\vec{x}-\vec{y}|}$.

Remark 4.1. The extra factor $\sqrt{4\pi}$ can of course be integrated into G. Note also that $V^2(r) = G^2 \frac{m^2}{r^2}$ represents the intensity of a signal that is emitted continually from the mass source m at the origin and spreads radially at a constant velocity c. To get to the relativistics, all it needs is to restrict time to the local time of the mass source, which means that $V^2(r)$ will be replaced by $V^2(t, r = ct) = G^2 \frac{m^2}{t^2 - r^2}$. Finally, note the difference between $V(r)$ and $V_{grav}(r)$: While $V(r)$ is the potential energy of the mass source m itself, in which m is a first order factor in the nominator, $V_{grav}(r)$ factors the mass m with the external mass m_2, so seen from the perspective of V_{grav}, $V(r)$ looks like representing the potential energy per mass. This is however treacherous: $V(r)$ is exactly the (static) field representation of the particle, which is an equivalent view to that mass, but it is not the potential for any other particle m_2.

As was ssen in section 3, in order to describe the internal heat of a bounded, closed system of particles, we need the matrix-valued flux $j = j_1\sigma_1 + \sigma_2 j_2 + \sigma_3 j_3$ that operates on component pairs of positive and negative parity, where the flux is converted to energy by factoring it with the speed of light c. According to equation 2.1, $T^2 := j^2 c^2$ plays an equivalent role as V^2 in the dynamics: So, when the mass densities ρ lead via $V(r)$ to some gravitational field $V_{grav}(r)$, then cj should be yielding a pseudo-vector field $A(\vec{x}) = A_1(\vec{x})\sigma_1 + A_2(\vec{x})\sigma_2 + A_3(\vec{x})\sigma_3$ with an analogous effect as V_{grav}, that is: we expect a vector field $\vec{A} := (A_1, A_2, A_3)$ to take effect on external particles m_2 just like V_{grav} does on m_2. Not surprisingly, such a field that does transmit heat (i.e.: a mechanical quantity) is known to exist. But surprisingly, that field is the electromagnetic field, which is based on electrical charges rather than masses. That leads straight to:

5. Action and Charges

The next step will be to notice the similarity of the relation for E^2 with the energy equation of a free system in special relativity: Let's set $E_{kin} := cp_1\sigma_1 + p_2\sigma_2 + p_3\sigma_3$, and likewise $\mathcal{E} = jc = j_1\sigma_1 + j_2\sigma_2 + j_3\sigma_3$), with c being the speed of light, and replace c by r/t. If now we multiply the energy by the time t, we get out the action, the two coordinates x and y of the cone section above transform up to constants into time and space, the invariance of the cone equation w.r.t. the inversion of x and y becomes the invariance of time and space inversion, and the derived energy equation turns into the constancy of the absolute square of the action, i.e.: the action becomes the unitarity of action in space and time.
We already dealt with parity, the space inversion, and we saw that fluxes with positive and negative parity can superimpose destructively. On the same line,

time inversion leads to energy inversion, but that means equivalently mass inversion. We therefore should have negative and positive masses, to which the parity rules then will carry over. And we do have, namely electrical charges: With the exception of neutrinos, of which is currently not known whether thse have a mass greater zero, down to the very quarks every particle of mass greater zero is charged, plus: it is known that the negative charges match with the positive ones. So, this strongly points to mass as being the absolute values of charges, i.e.: we may conceive the mass m as being equivalent - up to a factoring coupling constant ϵ - to the absolute value $\sqrt{q_+^2 + q_-^2}$ of a charge pair (q_+, q_-) of a positive charge q_+ and a negative charge q_-.
Therefore, fields V, A_1, A_2, A_3 should follow the covariant Maxwell equations,

$$\Box A^\mu = \frac{4\pi}{c} j^\mu, (0 \leq \mu \leq 3),$$

where $A^0 = V, j^0 = \rho$ is the charge density, j_1, j_2, j_3 are the (relativistic) charge fluxes, A_1, A_2, A_3 are their vector field components as above, $\Box := \frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_1^2} - \cdots - \frac{\partial^2}{\partial x_3^2}$ is the d'Alembert operator, and $(x_0 := ct, x_1, \ldots, x_3) \in \mathbb{R}^3$ are the components in 4-dimensional time and space.
Because we have two components for either parity and two components for either charge, it follows that we can and should state these equations as one equation in terms of the four Hermititian Dirac matrices $\alpha_1, \ldots, \alpha_3$ as

$$\Box A = \frac{4\pi}{c} j,$$

where $A := A_0\alpha_0 + \cdots + A_3\alpha_3$ and $j := j_0\alpha_0 + \cdots + j_3\alpha_3$. This matrix equation operates on a complex space $\mathbb{C}^4$, in which - up to $SU(4)$-equivalence - the first two components represent positive and negative parity of positive charges and the last two the positive and negative parity of negative charges. (Up to the fact that we track charge and parity of either sign, instead of destructively superimpose their values, the last equation is nothing but an equivalent rewrite of the Maxwell equations.)
These equations say that charge sources and their motion can equivalently be rewritten as radial waves of energy that are steadily emitted from these sources and spread at the speed of light. Up to an additional coupling constant, that is exactly what we would expect for the first energy component $V = A_0$ for masses, and it boils down to the Laplace equation for the gravitational field in the non-relativistic limit. The other three components are analogous extensions to the fluxes. And, if there was no other energy than gravitational interaction, then the three other components were sheer invariants, as by principle, gravity does not depend on the speed of its mass sources. So, as to gravity alone, we would demand the vanishing of the vector components $A_1 = A_2 = A_3 \equiv 0$ and would not need to care about parity and mechanical heat.
The point now is that in electrodynamics, it's just the other way round: Note that the 1^{st} component A_0 always is a longitudinal energy wave (i.e. in the direction of the spreading wave). But according to electrodynamics, the

waves have to be transversal. That can only be fulfilled, if and only if $A_0 \equiv 0$. And it is clear, why: over a distance larger than the atomic scale, particles are neutral, and a superposition of positive and negative charge densities will destructively cancel itself, while the sum of absolute squares of positive and negative charges still is non-zero.

So, the Maxwell equations suffice to describe both gravitation and electromagnetism. Note that a 4-dimensional vector field $(A_0, \ldots, A_3)$ is needed for this, so the symmetry (Lie) group of this is $U(2) = SU(2) \times U(1)$ (instead of just $U(1)$). In here, $U(1)$ comes in by the the association of the absolute square of charge pairs with the absolute square of mass, in particular, the coupling constant ϵ attaches to that.

It was also shown that a mathematically satisfactory formulation needs the distinction of either sign of parity, which leads to map the quadrupel $(j_0, \ldots, j_3)$ to $j := j_0\alpha_0, \ldots, j_3\alpha_3)$. And these 4-vector fluxes now have $U(4)$ as their symmetry group, which is defined as the group of all unitary mappings on the parity-charge quadrupels $(\lambda_1, \cdots, \lambda_4) \in \mathbb{C}^4$. $U(4)$ is a super group of $U(2)$, more exactly, we have: $U(4) = SU(3) \times SU(2) \times U(1) \times U(2)$. Now $SU(3)$ is known to be the symmetry group of strong interaction, $SU(2)$ that for weak interaction, while the group $U(2)$ is our symmetry group of long ranged photonic interaction of charge and mass. So, what's that additional group $U(1)$ about? It couples the square of baryons and leptons with the square of masses, in other words: we have another coupling constant (aside of ϵ), through which baryons and leptons get assigned a mass. This explains on mathematical grounds, why protons are thousand time heavier than the electron. Keeping with contemporary physics, we might equvalently associate this group $U(1)$ as the symmetry group for the Higgs particle.

6. The Mass Dilemma

According to Special Relativity, the total energy for a free particle system is given by $E^2 = m^2c^4 + p^2c^2$, where E is its total energy, m the total inertial mass of that system, p the absolute value of total momentum, and c is the speed of light. When, in particular the mass system is at rest (which means that the total momentum vanishes), then $E = mc^2$ is termed rest energy and m the rest mass. Still, that rest mass can consist of any number of not massless particles, and we may add a heat Q to that resting system. Then $E_{heated}^2 = m^2c^4 + Q^2 = m_{heated}^2 c4$. Because the principle of equivalence of gravitational and inert mass mandates the ratio of inert and gravitational mass to be constant $G = m/m_{grav}$, we run into the conflict with the principle of velocity independence of masses in the gravitational field: the square m_{grav}^2 has to be proportional to its square of heat, either, in order to maintain mass equivalence! If that was the case, gravity and termodynamics as well as mechanics and electrodynamics would only decouple from eachother for the temperature limit $T \to 0^0K$ - contrary to what was assumed before.

Simple experiments could test for that: if the (mathematical) pendula of two masses of different temperature, suspended at a rod of same length (measured from suspension point to the center of mass) have the same period, then the weight of the masses have to increase with temperature. On the same line, one could prepare three identical metallic spheres with valves to pump air in or out; while the first one is unchanged, the second one gets some of its air pumped out, which is weighed, say 10 (gravitational) grams of weight (at normal pressure), and the missing 10 grams replaced by spraying 10 grams of lacquer to its surface. The third sphere then is brushed off 10 grams of weight, which are replaced by 10 grams of air pumped into the shell. Then measure the weight of the three shells as well as their ratio of inert and gravitational mass (via mathematical pendulum). In case the mass equivalent proves to be correct, the 2^{nd} and 3^{rd} spheres are to be of the same weight and weigh more than the first shell.

7. Heat Flow

Given two neutral dynamical systems S_1 and S_2 at different temperature, what Maxwell's equations predict is that heat "under normal conditions" the heat will flow from the hotter System, S_1, say, to the cooler one: the more heat $S1$ has over S_2, the larger the impact $(A_1, \dots, A_3)$ will be on the target sources of S_2 and vice versa. This implicitly preliminates the elastic scattering of matter with the electromagnetic field. Nothing will hinder S_2, however, to follow inelastic scattering, according to which kinetic energy is converted into potential energy, and this inelastic scattering may even lower ihe temperature, even though heat is delivered by another system.

8. Lorentz Transformation of Mass and Charge

As is well-known, the relativistic energy of a free particle of mass m_0 and momentum $\vec{p} \in \mathbb{R}^3$ is given by $E^2 = m_0^2 c^4 + p^2 c^2$. Setting $\vec{p} = m\vec{v}$, Einstein concluded: $m^2 c^4 = m_0^2 c^4 + m^2 v^2 c^2$, and therefore: $m^2 = \frac{m_0^2}{(1-(v/c))^2}$, so $m = +\frac{m_0}{\sqrt{1-(v/c)^2}}$.

The last step is a simplification of the complete algebraic solution: The square root of E is $E = \alpha_0 m_c^2 + mc \sum_k \alpha_k v_k$, where the $\alpha_0, \dots \alpha_3$ are the 4×4 α-matrices that - as already discussed above - operate on a vector space $\mathbb{C}^4$, for which each and every unit vector $\lambda \in \mathbb{C}^4$ is a solution of the square equation. In the standard Dirac representation, α_0 is the diagonal matrix with eigenvalues $+1, +1, -1, -1$, and α_1 is the interchange operator $\alpha_1 : (\lambda_1 \dots, \lambda_4) \mapsto (\lambda_4, \lambda_3, \lambda_2, \lambda_1)$. It is this the reason, why for $(\lambda_1, \dots, \lambda_4)$ the first two components could be associated with positive charge or mass of positive and negative parity and the last two of a negative mass or charge with positive and negative parity.

Another implication of $E = \alpha_0 m_0 c^2 + mc\sum_k \alpha_k v_k$ then is that the operator $\mathcal{C}$ which interchanges the first two components with the last ones, namely $\mathcal{C} : (\lambda_1, \ldots, \lambda_4) \mapsto (\lambda_3, \lambda_4, \lambda_1, \lambda_2)$ is the charge inversion, while $\mathcal{P} : (\lambda_1, \ldots, \lambda_4) \mapsto (\lambda_2, \lambda_1, \lambda_4, \lambda_3)$ is the parity inversion, and $\mathcal{T} = \mathcal{CP} : \lambda \mapsto -\lambda$ is the energy (or time) inversion. In particular, $\mathcal{TPC} = \mathbb{I}_4$ is the identity.
Given all that, let's see what the Lorentz transformation of charges rewrites in terms of alpha matrices: We have analogously $m = m_0(1 - \vec{\gamma} \cdot (\vec{v}/c))$, where $\gamma_0 = \alpha_0$ and $\gamma_k = \gamma_0 \alpha_j$, $(1 \leq k \leq 3)$ are the Dirac matrices. And because $m\alpha_0 \vec{v} \cdot \vec{\alpha}/c = (-m)\alpha_0(-\vec{v}) \cdot \vec{\alpha}/c = \mathcal{CP} m\alpha_0 \vec{v} \cdot \vec{\alpha}/c$, although the absolute value $|m|$ of the charge increases (according to Einstein's relation), the net charge stays constant, in other words, the additional charge is neutral. This additional charge is then the source of magnetic fields, and of course it can be interpreted as being made of circular currents.
Moreover, because of that neutrality of that magnetic field source, heat could increase the gravity of the mass sources without affecting the atomic light spectra of their contained, heated particles.

Hans Detlef Hüttenbach