Toward unification

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Abstract

A universe based on a fully deterministic, Euclidean, 4-torus cellular automaton is presented using a constructive approach. Each cell contains one integer number forming bubble-like patterns propagating at the speed of light, interacting and being reissued constantly. The collective behavior of these integers is conjectured to form patterns similar to classical and quantum physics, including the mass spectrum. Although essentially non-local, it preserves the non-signaling principle. This flexible model predicts that gravity is not quantized. Being a causal theory, it can potentially explain the emergence of the classical world and macroscopic observers.

Keywords: unification, cellular automaton, graviton, nonlocality, beyond Standard Model

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1 Introduction

Cellular automata are mathematical idealizations of physical systems in which space and time are discrete. The idea of modeling our universe using cellular automata is not new, discreteness is seen by many authors (Refs. [18] form a small list) as a solution for the divergences of the Standard Model (SM), and is supported by the existence of a fundamental Planck volume, suggesting that structures smaller than this tiny volume should not be relevant to the theory.

Quantum mechanics (QM), despite its resounding success, gives us a somewhat blurred image of the universe because of its base on the uncertainty principle, point particles, and its most accepted interpretation is based on probabilities. Recent results from experimental physics, which far surpass the precision achieved by QM predictions, require a new model of the universe in which QM is only a limiting case.

Can nature be modeled as a cellular automaton? The model described here is designed to investigate this possibility. The emergence of a unified theory of physics is the ultimate goal of a final version based on this approach. Here the automaton is a couple of simple cubic grids closed on themselves as a 4-torus where one brick (formatted integer number) is attached to each cell. The cell has a processor, or logical circuit, and interacts with its eight nearest neighbors only (von Neumann convention). Preons are modeled under the tick of a central clock. The Planck length is the natural candidate to be used as the distance between the automaton cells.

The approach adopted in this work is a constructive one [9, 10]. Whenever possible, I try to emulate directly the laws of physics, probing the adequate heuristics. On the other hand, I’m not saying that the Universe is a vast computer, in fact, I’m attempting to model Planck scale physics using a cellular automaton.

2 Theory

2.1 Ontology

**Definition 1.** Property formats: SI, signed integer; UI, unsigned integer; SV, signed 3d-vector, with $N_D = \pi (SIDE/2)^2$ possible directions. The default length is $SIDE$.

**Definition 2.** Brick is a formatted $(p_1, p_2, \ldots)$ N-integer (see Table 1).
**Definition 3.** The cellular automaton is a dual Euclidean lattice 4-torus of dimension $\text{SIDE}$, where a single brick is attached to each cell. The distance between cells is $L$ and the clock period ($p_1$) is $T$. Each lattice is alternatively principal (read-only) or dual (draft). $D$ is the main diagonal of the lattice. Three dimensions are spatial and the fourth corresponds to internal degrees of freedom.

**Definition 4.** A preon$^1$ is a spherical wavefront of bricks occupying the same $w$ address, expanding at the speed of light $c = L/LIGHT$ (one light step is $LIGHT = 2D$ clock ticks). It is considered real or virtual ($p_8 = \pm 1$).

**Definition 5.** Graviton ($G$) is a brick that propagates in a straight line at the speed of light. It vanishes after traveling the distance of $\text{SIDE}/2$ units in the direction of its spin vector.

**Definition 6.** A burst is a cubic wavefront occupying the same $w$ address, expanding at the maximum speed $s = L/T$. The burst duration is $\text{BURST} = 3\text{SIDE}/2$.

**Definition 7.** Unpaired ($U$) is a non-overlapping preon. It works like a charge fragment.

**Definition 8.** Pair ($P$) are two overlapping preons. The components of the pair are identified by the upper indices $P$ and $P'$, respectively.

**Definition 9.** A vacuum $P$ ($P_0$) has trivial net properties $\rightarrow p_3$, $\rightarrow p_4$, $p_6$, $p_7$, $p_9$, $p_{15}$, $p_{17}$, and $p_8 = -1$.

**Definition 10.** The input parameters are $\text{SIDE} = 2^{208}$, $L = \text{Planck length}$, $T = \text{Planck time}/3 \cdot \text{SIDE}$, and $\text{EXCESS}$. They are used for mapping to the real world.

### 2.2 Auxiliary functions

▷ PWM mask

```
procedure pwm(n) begin
    return n \mod \sqrt{\text{SIDE}} < n/\sqrt{\text{SIDE}}
end.
```

▷ Charge conjugation test

```
procedure conj(p) begin
    if $p_9 \&38 \neq 0$ then return +1  ▷ matter
    if $p_9 \&07 \neq 0$ then return -1  ▷ antimatter
    return 0  ▷ neutral
end.
```

▷ Variable $f$ is a 3-bit field, $n = \{1, 2\}$

```
procedure rot(f, n) begin
    rotate $f$ by $n$ bits to the right
end.
```

▷ Color bits exchange

```
procedure exchange() begin
    if $p_9^{P_1} = p_9^{P_2} \neq 3f_H$ then
        if $p_9^{P_1} = p_9^{P_2} = 0$ then
            $p_9^{P_1} = 20_H$; $p_9^{P_2} = 04_H$
        end if
        rot($p_9^{P_1}$, $p_1 \&01_H + 1$); rot($p_9^{P_2}$, $p_1 \&01_H + 1$)
        rot($p_9^{P_1}$, $p_1 \&01_H + 1$); rot($p_9^{P_2}$, $p_1 \&01_H + 1$)
        rot($p_9^{P_1}$, $p_1 \&01_H + 1$); rot($p_9^{P_2}$, $p_1 \&01_H + 1$)
        if $p_9^{P_1} \&p_9^{P_2} \neq 0$ then
            undo changes
        end if
    end if
end.
```

\(^1\)The word preon was coined by Jogesh Pati and Abdus Salam in 1974.
The brick signature value

procedure signature(p) begin
    return \((SIDE + 1)^2 p.x + (SIDE + 1) p.y + p.z + 1\)
end.

Alignment / anti-alignment test \((a = \pm 1)\)

procedure align(a) begin
    if \(a = -1 \land \vec{p}_3^{P1} \cdot \vec{p}_3^{P2} < 0\) then
        return \(pwm(\vec{p}_3^{P1} \cdot \vec{p}_3^{P2} || \vec{p}_3^{P1} || \vec{p}_3^{P2})\) \(\triangleright \vec{p}_3^{P1} \cdot \vec{p}_3^{P2} \sim -1\)
    else if \(a = +1\) then
        return \(pwm(\vec{p}_3^{P1} \cdot \vec{p}_3^{P2} || \vec{p}_3^{P1} || \vec{p}_3^{P2})\) \(\triangleright \vec{p}_3^{P1} \cdot \vec{p}_3^{P2} \sim +1\)
    end if
end.

Polarization mask

procedure pol(sector) begin
    light = \(|\vec{p}_3^2|\); cycle = SIDE/p_{12}
    if light mod cycle < cycle/sector then
        return \(pwm([p_{11}]^2)\)
    end if
end.

Hash value used for vacuum symmetry breaking

procedure hash(n) begin
    return \(((n + 1) \cdot \text{prime}) \gg (ORDER/2)(SIDE - 1)\) \(\triangleright \text{‘prime’ is a prime number}\)
end.

Changes P to a kinetic P

procedure kineticP(\(\vec{d}, P\)) begin
    \(p_3^{P} = \vec{p}_3^{P'} = \vec{d}; \ p_4^{P} = \vec{p}_4^{P'} = \vec{0}; p_{14}^{P} = p_{14}^{P'} = 0;\)
end.

2.3 Dynamics

Axiom 1. The sinusoidal phase of prons is given by the \(p_{11}\) field, calculated by means of a Direct Form Oscillator cf. [11]. When prons are overlapped, the generator is fired multiple times, and the \(p_{12}\) updated accordingly.

Define the constants

\[
\begin{align*}
    k &= 2 \cos(\omega T), \\
    U_1 &= SIDE \sin(-2\omega T), \\
    U_2 &= SIDE \sin(-\omega T).
\end{align*}
\]

At the beginning of each wave do

\[
\begin{align*}
    u_0 &= 1; u_1 = U_1; u_2 = U_2.
\end{align*}
\]

The evolution law is

\[
\begin{align*}
    u_3 &= k u_2 - u_1, \\
    u_1 &= u_2, \\
    u_2 &= u_3.
\end{align*}
\]

\text{A proof-of-concept C program is under development where very basic operations can be visualized. Its latest version can be accessed on github https://github.com/automaton3d/automaton.git.}
Axiom 2. Interference derives from a track left by the preons on the visited cells (p13 field), inspired by
work of Sciarretta [1]. The value algebraically added by the sinusoidal phase on the cell decays absolutely
and exponentially after each light step [12]. Only entangled preons interfere with each other.

Axiom 3. Decay of P

\[ p_{17}^P = p_{17}^P \triangleq 1 \]

if \( p_{17}^P = 0 \) then

if \( p_{16}^P = \text{BOUND} \) then

\( p_{16}^P = \text{FREE} \) \( \triangleright \) bound P is set free

else if \( p_{8}^P = \text{VIRTUAL} \land \hat{\overrightarrow{p}}_2^P \bullet \hat{\overrightarrow{p}}_4^P = 1 \) then \( \triangleright \) the dot product singles out one brick

end if

\( P \leftarrow P_0; \text{reissue P} \) \( \triangleright \) virtual P is returned to the vacuum

end if.

Axiom 4. Preon interaction is detected by mutual comparisons in the w dimension at the last tick of a
time frame. The interaction type (UxG, PxG, UxU, UxP or PxP) is then calculated. The preons are
reissuued at the contact point by default. If the preon never interacts, it is reissuued by wrapping.

Axiom 5. A preon launches a burst every time it is reissuued. The burst erases the wavefront of the preon,
except a brick seed. Then, its spin (\( \overrightarrow{p}_4 \)) is rotated by the angle \( 2\pi d p_5 / d \), where \( d = |\overrightarrow{p}_2| \mod 2\pi \). If it
is entangled, then the burst will cause its partner to assume the opposite spin direction. Then, it gets
entangled with the preon it is interacting with: \( p_{10}^P = p_{10}^P = w^1 w^2; \overrightarrow{p}_4 = -\overrightarrow{p}_4 = \overrightarrow{p}_4 \land \overrightarrow{p}_2 \); \( p_{18}^P = p_{18}^P = 0 \).

Axiom 6. The wavefront of a real preon continues propagating as a G after the reissuue, with spin
\( \overrightarrow{p}_4^G = \overrightarrow{p}_2^G \).

Axiom 7. Let \( C = \{ 3f_H, 01_H, 02_H, 04_H, 20_H, 10_H, 08_H, 3f_H \} \). When a vacuum P is reissuued, then
\( p_{9}^P = C[p_3 \& 07_H] \) and \( p_{9}^P = C[8 - (p_1 \& 07_H)] \).

Axiom 8. UxG interaction

if \( p_{8}^U = +1 \) then

\( p_{15}^U = \text{ON}; \overrightarrow{p}_3^U = -\overrightarrow{p}_2^G \) \( \triangleright \) graviton detection

end if.

Axiom 9. PxG interaction

Move \( p_{10}^U \) value in the direction \( -\left( \frac{1.5 - |\overrightarrow{p}_2^G \bullet \overrightarrow{p}_2^P/|\overrightarrow{p}_2^G||\overrightarrow{p}_2^P||}{|\overrightarrow{p}_2^G||\overrightarrow{p}_2^P||} \right) \overrightarrow{p}_2 \). \( \triangleright \) light bending

Axiom 10. UxU interaction

if \( p_{6}^U \neq p_{6}^U \) then

if \( p_{6}^U = -p_{6}^U \lor p_{6}^U = -p_{6}^U \lor p_{6}^U = p_{6}^U \lor p_{6}^U = p_{6}^U \) then

U1 and U2 merge into a P \( \triangleright \) annihilation

\( \overrightarrow{p}_4^U = -\overrightarrow{p}_4^U = \overrightarrow{p}_4^U \times \overrightarrow{p}_4^U \) \( \triangleright \) spin realignment

end if

else

\( \overrightarrow{p}_4^U = -\overrightarrow{p}_4^U = \overrightarrow{p}_4^U \times \overrightarrow{p}_4^U \) \( \triangleright \) spin realignment

end if.

Axiom 11. UxP interaction

\( p_{18}^P = p_{18}^P + 1 \) \( \triangleright \) update directionality

if \( p_{15}^U = \text{ON} \land P \equiv P_0 \) then \( \triangleright \) recruit vacuum P

kinetic P(\( \overrightarrow{p}_4^U, P \)); \( p_{16}^U = \text{BOUND}; p_{15}^U = \text{OFF} \) \( \triangleright \) grav. acceleration

else if \( \text{pwm}(p_{13}) \land P \equiv P_0 \) then

\( p_{14}^U = p_{14}^U = p_0^P; \overrightarrow{p}_4^U = \overrightarrow{p}_4^U \) \( \triangleright \) static EM interaction

else if \( p_{14}^P = \pm 1 \land \text{pwm}(p_{13}) \land \text{pwm}(p_{13}) \land \text{pol}(8) \) then

\(^{\text{Entanglement must be included in the theory to avoid that it is reduced to the classical theory, cf. [13].}}

5
Axiom 13. The symmetry of $LM$ is broken in one single direction by the value EXCESS.

Remark 3 Implementation notes

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Axiom 12. $PrP$ interaction

if $p_1 \equiv p_2 \equiv P_0$ then
  if $p_{1}^{p_1} = p_{2}^{p_2} \wedge \text{hash}(w^{p_1}) \text{xor} \text{hash}(p_1^{p_1}) = \text{hash}(w^{p_2}) \text{xor} \text{hash}(p_2^{p_2})$ then
    reissue $p_1$ and $p_2$ \>$\triangleright$\> vacuum symmetry breaking
  else if $\text{hash}(w^{p_1}) \text{xor} \text{hash}(p_1^{p_1}) \text{xor} \text{signature}(P_1) = \text{hash}(w^{p_2}) \text{xor} \text{signature}(P_2)$ then
    reissue $P_1$ and $P_2$ \>$\triangleright$\> quantum fluctuation
end if

else if $p_1^{p_1} = (38H,07H) \wedge P_2 \equiv P_0 \wedge \text{pwm}(P_1^{p_1}) \wedge \text{pwm}(P_2^{p_2})$ then
  $p_2^{p_2} = 38H \wedge p_2^{p_2} = 07H$ \>$\triangleright$\> leptonic synthesis
else if $p_2^{p_2} = 0 \wedge p_1^{p_1} = p_2^{p_2}$ then \>$\triangleright$\> kinetic $P \times$ kinetic $P$?

if align(-1) then
  $P_1 \leftarrow P_0$; $P_2 \leftarrow P_0$ \>$\triangleright$\> cancellation
else if not align(+1) then
  $P_1 = 2 \cdot SIDE - 1$ \>$\triangleright$\> communicated via burst
end if

else if $p_2^{p_2} = \text{LEPT} \wedge p_2^{p_2} = \text{LEPT} \wedge p_2^{p_2} \neq \text{ANTILEPT} \wedge p_2^{p_2} \neq \text{ANTILEPT}$ then
  exchange() \>$\triangleright$\> gluon-gluon interaction
else if $p_2^{p_2} = \text{BOUND} \wedge p_2^{p_2} = \text{BOUND}$ then
  reissue $P_1$ from $p_2^{p_2} - p_2^{p_2} + \lepto_{p_2}^{p_2}$
  reissue $P_2$ from $p_2^{p_2} - p_2^{p_2} + \lepto_{p_2}^{p_2}$ \>$\triangleright$\> neutrino inertia
end if.

Axiom 13. The symmetry of $LM$ is broken in one single direction by the value EXCESS.

Axiom 14. All preons occupy the same 3d address in the initial state of the universe. The charges are evenly distributed between the preons.

3 Implementation notes

Remark 1. Isotropy and spherical wavefront generation are achieved applying the method described in Ref. [14]. The novel feature of that work is that, to obtain the isotropy, is required for each expansion step, executing $n$ steps of the basic algorithm of the automaton, where $n$ is two times the diameter of the universe $D$ (space diagonal). The lattice speed is $s$ and light speed $c$. Then we have the relation
\[ s = 2Dc. \]

In order to synchronize the preons forming a wavefront, each receives the value
\[ t = \lceil 2D|p_2| + 0.5 \rceil. \]

*Remark 2.* A visit-once-tree (see Appendix) is used during preon expansion to avoid cell access conflicts. Burst conflict in the same layer due to multiple detection is solved by a look-ahead algorithm.

*Remark 3.* The time frame is segmented into two steps: one, when the bursts are active, has a duration of \( BURST \) time units. The other, when preons and gravitons are active, has a duration of \( 2D \) time units. The entire frame is termed SYNCH. This is to avoid undesired superposition of a preon wavefront with a burst or gravitons on a common layer (w address).

*Remark 4.* Some properties, e.g. sine wave and polarization, cannot be used directly, but must first be compared against a standard PWM sequence (see Definition 2.2), ruling out the need for an interaction detection mechanism based on an explicit pseudorandom number generator.

*Remark 5.* Additional numerical fields, besides those in Table 1, are necessary to implement the above mechanisms.

4 Discussion

4.1 Conservation laws

Preons are never created or destroyed—their number remains always SIDE. Electric, weak and strong charges are conserved. Spin and helicity are conserved as well. Angular momentum is conserved at the particle level. The other conservation laws are emergent features.

4.2 Conjectures

Based on the axiomatic body presented above, I state now some conjectures related to the expected behavior of the automaton.

**Conjecture 1.** Clusters of Us and associated Ps tend to produce stable or transient patterns of HBAR/2 Us that I call fermions. This quantization effect is supported by Axiom 13 (equivalent to a Dirac monopole [15]) and by the closure of the universe.

**Conjecture 2.** The neutrino is a special fermion made of weakly charged Ps, carrying HBAR/2 units of orbital AM. Also, \( p_5^\nu = -1 \), \( p_5^\nu = +1 \), \( \nu_e \equiv \nu_\mu \equiv \nu_\tau \).

**Conjecture 3.** In a fermion, the spins tend to align either outward or inward, forming a spherically symmetric pattern. These states correspond to either spin up or spin down. This conjecture was inspired by the Hefner electron [16]. Coherence inhibits this tendency.

**Conjecture 4.** The magnetic effects of a still charge over another still charge cancels out due to spherical symmetry. Kinetic Ps can break the symmetry of the cloud, which passes into an oval configuration and consequently induces a magnetic dipole.

**Conjecture 5.** A fermion is in a superposition state when one part of the spins of its Us points inward while the other part points outward. The singlet correlations verified at the ensemble level are byproducts of superposition. Remember that a fermion is formed by a huge number of preons considering the distance between the atomic and Planck scales. ‘Infinite’ Hilbert spaces necessary for contextuality in QM are though supported by the automaton model.

**Conjecture 6.** Gravity is not quantized (adiabatic process).

**Conjecture 7.** Curved spacetime emerges from the combined action of preons.
Conjecture 8. If the alignment predicted in Axiom 11 happens in all Us of a particle, then the Ps merge into a vector boson and escape the influence of the charges, propagating away.

Conjecture 9. Quarks are emergent patterns formed inside hadrons, so are confined. These patterns tend to shrink to a point at higher L.M.

Conjecture 10. Since leptons and hadrons are composite particles, they can possess radial vibration, like a pulsating sphere [17]. The muon is the first excited state of the electromagnetic radial vibrational state of the electron, the bawn is the second, so there is just one stable kind of charged lepton: the electron. For quarks, the charm is the first excited state of the strong radial vibrational state of the up. The top is the second radial vibrational state of the up. The strange is the first radial vibrational state of the down. The bottom is the second radial vibrational state of the down. The down is formed when the up captures a charged lepton. We, therefore, are led to conclude that there is just one kind of stable quark, the up. The W and Z bosons are weak analogous to the single (fundamental) mode. Therefore, the amount of Ps trapped in these resonance modes gives rise to the rest mass of the particles when they emit duo-gravitons in addition to the gravitons emitted by their Us.

Conjecture 11. Weak charged Us are always harvesting vacuum pairs (Axiom 11), causing radial vibration about the weak charges, in the form of virtual weak Ps, therefore, the vibrational patterns do not contribute to the particle’s mass (virtual particle). This harvesting process results in collected Ps (radial vibration) which remain stable up to a threshold around 80 GeV. Unlike in the electromagnetic case, the only observable radial vibration mode is the fundamental one. This process can be hindered by other processes as well, that’s why a neutron in the deutron and in many other nuclei is stable. If there is enough AM available, the newly formed weak boson starts to propagate, escaping the influence of electric/weak charges ($p_{16}^P = p_{16}^{P'} = FREE$) of the Us (inverse or direct beta decay). This boson (whether real or virtual) is inherently unstable, so, a short time afterward, all weak Ps associated with this vibrational mode revert automatically to vacuum Ps.

Conjecture 12. When an interaction occurs, the weak havoc caused by the reissue of preons results in the dissolution of the involved partners, organizing themselves immediately, probably, but not necessarily, in the same particles. This would explain, for example, spin flipping.

Conjecture 13. Preons are reissued at the contact point when the W particle interacts, settling into other combinations of particles. This helps to explain direct/inverse beta decay, for example.

Conjecture 14. Neutrino emission/absorption helps maintain the balance of AM.

Conjecture 15. The Us distribution is in the ratio of 3 quarks to 1 charged lepton. More precisely, 50% up quarks, 25% electrons and 25% down quarks.

4.3 Bridge to quantum and classical mechanics

Can this theory attain classical mechanics in some suitable limit? The answer seems to be affirmative if it satisfies the three axioms presented by Scandolo et al. (see [18]).

Also, identifying the preons distribution within a particle with the phase-waves described by Unnikrishnan in [19], may solve the bridge to QM and CM in one stroke with further simplification of the model, mainly ruling out the enforced nonlocality.

5 Conclusion

The construction of a cellular automaton describing the basic laws of nature is a long-term goal, requiring the synergy of many researchers. In this contribution, I presented a tentative solution to the unification problem using a constructive approach, a framework for further investigation toward a full-fledged unification theory. The scenario is the Planck World. Can it be adjusted to enforce all natural symmetries (see [20]) and relativistic effects? The preliminary results obtained, already suggest a certain resemblance to QM, the SM and experimental data [16, 21–25]. The no-signaling principle is preserved. Conservation laws are mostly emergent characteristics. Since graviton emission is not conditioned to AM transfer, gravity is, in this sense, not quantized. The graviton mechanism adopted implies an arrow of
time, thus preserving the second law from the beginning. The main result is that the mass spectrum can be calculated from first principles (see Conjecture 10).

Note that the term energy has not been used anywhere in the text. Far from being heresy, it simply means that it was not necessary to invoke it at this stage of the model’s development, even though energy is an ill-defined concept in Physics. Clearly, this is a causal theory and therefore, according to Sec. 4.3, SBS states must be sought or enforced, in order to enable it to reach classical theory and account for macroscopic observers.

Except for assisting in the development of the basic mechanisms, the construction of such an automaton for directly solving cosmological problems, or even complex molecules, is inconceivable. Its complete usefulness will come through mathematical analysis in the approximation of large numbers ( [26] being a possible starting point).

6 Compliance with ethical standards

The author declares that he has no conflict of interest.

7 Summary

This study received no funds.
References

Appendix: Visit-once-tree

To avoid cell access conflict, the path of the expanding preon or burst must be tested using the pseudocode below:

procedure isAllowed(dir, p, d0) begin
    x = p.x + dirs[dir].x
    y = p.y + dirs[dir].y
    z = p.z + dirs[dir].z
    level = abs(x) + abs(y) + abs(z)
    \(\triangleright\) x-axis
    if x > 0 and y = 0 and z = 0 and dir = 0 then
        return true
    else if x < 0 and y = 0 and z = 0 and dir = 1 then
        return true
    end if
    \(\triangleright\) y-axis
    else if x = 0 and y > 0 and z = 0 and dir = 2 then
        return true
    else if x = 0 and y < 0 and z = 0 and dir = 3 then
        return true
    end if
    \(\triangleright\) z-axis
    else if x = 0 and y = 0 and z > 0 and dir = 4 then
        return true
    else if x = 0 and y = 0 and z < 0 and dir = 5 then
        return true
    end if
    \(\triangleright\) xy plane
    else if x > 0 and y > 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 0 and d0 = 2)
        else
            return (dir = 2 and d0 = 0)
        end if
    else if x < 0 and y > 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 1 and d0 = 2)
        else
            return (dir = 2 and d0 = 1)
        end if
    else if x > 0 and y < 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 0 and d0 = 3)
        else
            return (dir = 3 and d0 = 0)
        end if
    else if x < 0 and y < 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 1 and d0 = 3)
        else
            return (dir = 3 and d0 = 1)
        end if
    end if
    \(\triangleright\) yz plane
end if
else if \( x = 0 \) and \( y > 0 \) and \( z > 0 \) then
  if \( \text{level} \mod 2 = 0 \) then
    return (dir = 4 and \( d_0 = 2 \))
  else
    return (dir = 2 and \( d_0 = 4 \))
  end if
else if \( x = 0 \) and \( y < 0 \) and \( z > 0 \) then
  if \( \text{level} \mod 2 = 0 \) then
    return (dir = 4 and \( d_0 = 3 \))
  else
    return (dir = 3 and \( d_0 = 4 \))
  end if
else if \( x = 0 \) and \( y > 0 \) and \( z < 0 \) then
  if \( \text{level} \mod 2 = 0 \) then
    return (dir = 5 and \( d_0 = 2 \))
  else
    return (dir = 2 and \( d_0 = 5 \))
  end if
else if \( x = 0 \) and \( y < 0 \) and \( z < 0 \) then
  if \( \text{level} \mod 2 = 0 \) then
    return (dir = 5 and \( d_0 = 3 \))
  else
    return (dir = 3 and \( d_0 = 5 \))
  end if
end if

\[ \downarrow \] \( \text{zx plane} \)
else if \( x > 0 \) and \( y = 0 \) and \( z > 0 \) then
  if \( \text{level} \mod 2 = 1 \) then
    return (dir = 4 and \( d_0 = 0 \))
  else
    return (dir = 0 and \( d_0 = 4 \))
  end if
else if \( x < 0 \) and \( y = 0 \) and \( z > 0 \) then
  if \( \text{level} \mod 2 = 1 \) then
    return (dir = 4 and \( d_0 = 1 \))
  else
    return (dir = 1 and \( d_0 = 4 \))
  end if
else if \( x > 0 \) and \( y = 0 \) and \( z < 0 \) then
  if \( \text{level} \mod 2 = 1 \) then
    return (dir = 5 and \( d_0 = 0 \))
  else
    return (dir = 0 and \( d_0 = 5 \))
  end if
else if \( x < 0 \) and \( y = 0 \) and \( z < 0 \) then
  if \( \text{level} \mod 2 = 1 \) then
    return (dir = 5 and \( d_0 = 1 \))
  else
    return (dir = 1 and \( d_0 = 5 \))
  end if
else
  \[ \text{spirals} \]
  \[ x_0 = x + \text{SIDE}/2 \]
  \[ y_0 = y + \text{SIDE}/2 \]
  \[ z_0 = z + \text{SIDE}/2 \]
  switch level mod 3 do
case 0
    if \( x_0 \neq \frac{SIDE}{2} \) and \( y_0 \neq \frac{SIDE}{2} \) then
        return \((z_0 > \frac{SIDE}{2} \text{ and } \text{dir} = 4) \text{ or } (z_0 < \frac{SIDE}{2} \text{ and } \text{dir} = 5)\)
    end if
    break
case 1
    if \( y_0 \neq \frac{SIDE}{2} \) and \( z_0 \neq \frac{SIDE}{2} \) then
        return \((x_0 > \frac{SIDE}{2} \text{ and } \text{dir} = 0) \text{ or } (x_0 < \frac{SIDE}{2} \text{ and } \text{dir} = 1)\)
    end if
    break
case 2
    if \( x_0 \neq \frac{SIDE}{2} \) and \( z_0 \neq \frac{SIDE}{2} \) then
        return \((y_0 > \frac{SIDE}{2} \text{ and } \text{dir} = 2) \text{ or } (y_0 < \frac{SIDE}{2} \text{ and } \text{dir} = 3)\)
    end if
    break
end switch
end if
return false
end