Autopilot to maintain movement of a drone in a vertical plane at a constant height in the presence of vision-based navigation

Alexander Domoshnitsky[1], Max Kogan[1], Oleg Kupervaser[1,2],Roman Yavich[1]

[1] Department of Mathematics, Ariel University, Israel; [2] TRANSIST VIDEO LLC, Skolkovo, Russia

Abstract

In this report we describe correct operation of autopilot for supply correct drone flight. There exists noticeable delay in getting information about position and orientation of a drone to autopilot in the presence of vision-based navigation. In spite of this fact, we demonstrate that it is possible to provide stable flight at a constant height in a vertical plane. We describe how to form relevant controlling signal for autopilot in the case of the navigation information delay.

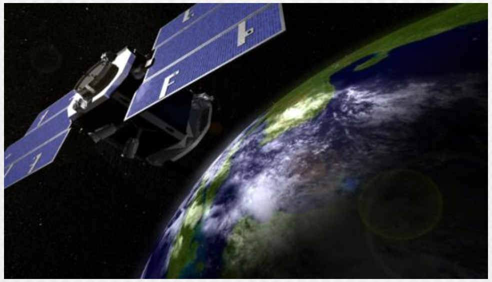
References

[1] Alexander Domoshnitsky, Emilia Fridman "A positivity-based approach to delay-dependent stability of systems with large time-varying delays", Elsevier Journal, Systems & Control Letters 97 (2016)

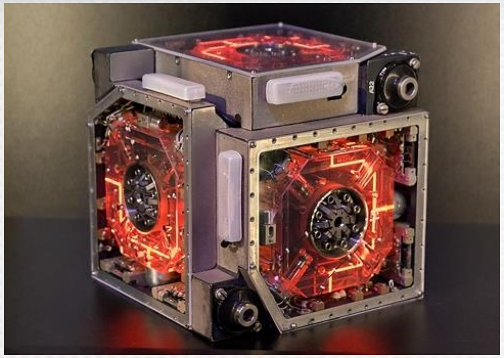
[2] V.A. Bodner, M.S. Kozlov "Stabilization of aerial vehicles and autopilots", Oborongiz, Moscow, 1961 (in Russian)

Unmanned vehicles

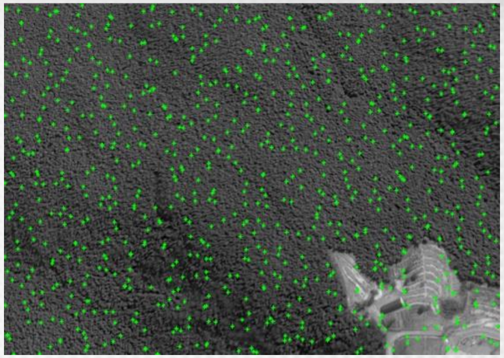
- Unmanned aerial vehicles, satellites, ground robots



Navigation systems



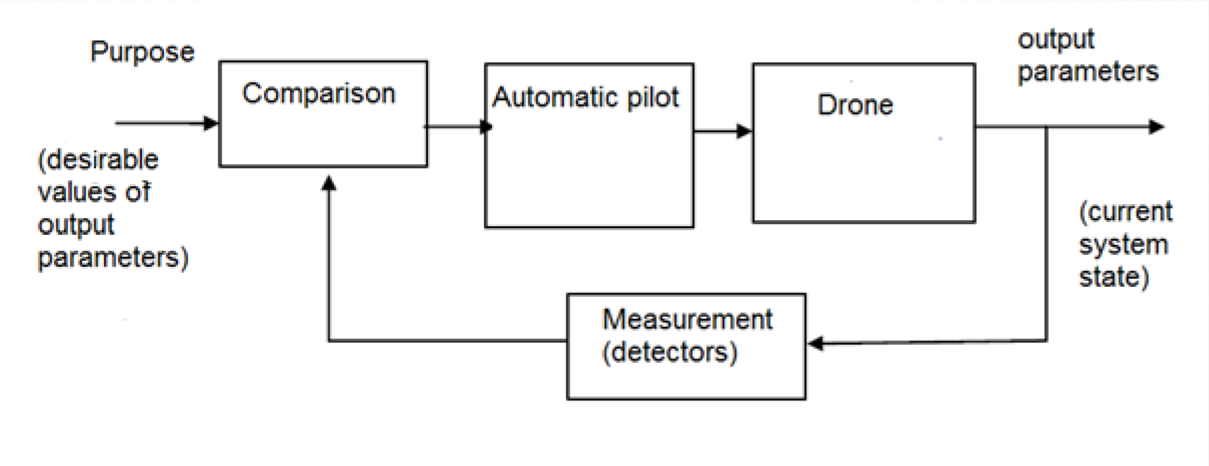
Inertial Navigation System



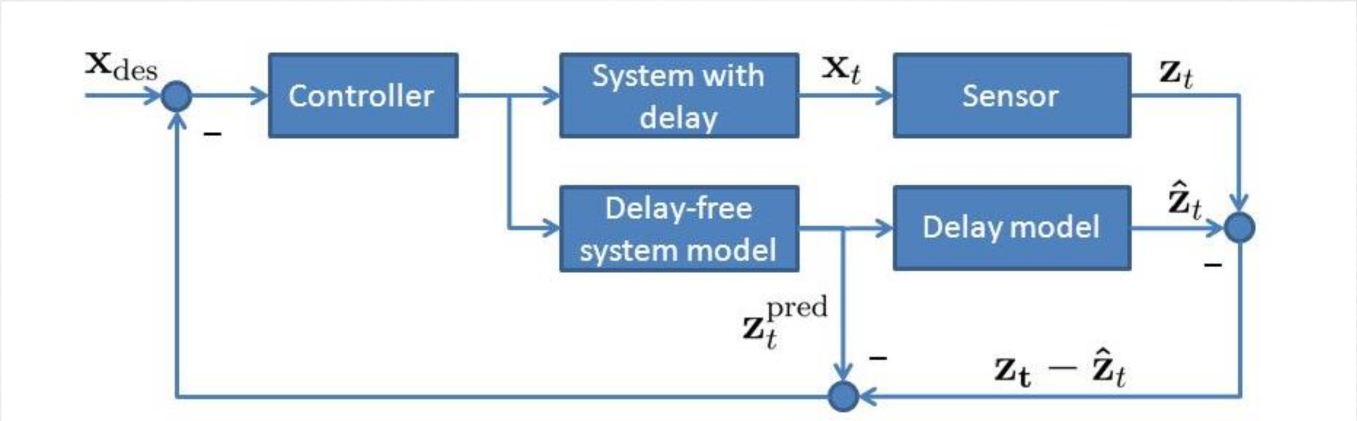
Vision-based Navigation System

GPS Navigation System

Automatic control



Delay of vision-based navigation

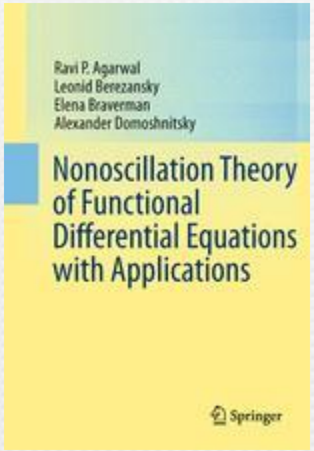


The goal is to maintain **stable movement :**

- Estimate max value of **delay**
- Select **control parameters**

Scientific studies this paper is based on

- A. Domoshnitsky, E. Fridman, “A positivity-based approach to delay-dependent stability of systems with large time-varying delays”
- R.P. Agarwal, L. Berezansky, E.Braverman, A. Domoshnitsky, “Nonoscillation Theory of Functional Differential Equations with Applications”
- V. A. Bodner, M.S. Kozlov “Aircraft Stabilization and Autopilots”



Working process overview

What have we done?

- Considered a real system of differential equations which describes movement of a flying drone

- Adjusted it to a proper way so that a theory of delay-dependent stability could be applied

- Applied the theory to the concrete case

- Estimated max value of delay at which a stable flight of a drone is possible

- Calculated the control parameters for a stable flight of a drone

Stability of the system. Theorem

We study stability of the following system:

$$x_i'(t) + \sum_{j=1} \sum_{k=1} a_{ij}^k(t) x_j(t - \theta_{ij}^k(t)) = 0, \quad t \in [0, +\infty),$$

$$i = 1, \ldots, n,$$

$$x_i(\xi) = 0, \quad \xi < 0, \ i = 1, \ldots, n,$$

where $a_{ij}^k \in L_\infty$, $\theta_{ij}^k \in L_\infty$ for $k = 1, \ldots, m$.

Stability of the system. Theorem

Theorem. If the following conditions are fulfilled, then the system is exponentially stable.

(1) *for every* $i = 1, \ldots, n$

there exists m_i *such that* $a_{ii}^k(t) \geq 0$ $a_{ii}^j(t) \leq 0$, $\theta_{ii}^k(t) \leq \theta_{ii}^j(t)$ *for* $k = 1, \ldots, m_i, j = m_i + 1, \ldots, m$, $\sum_{k=1}^{m_i} a_{ii}^k(t) \geq \sum_{j=m_i+1}^{m} \left| a_{ii}^j(t) \right|$ *for* $t \in [0, +\infty)$,

$$\int_{t-\theta_{ii}^{+}(t)}^{t} \left\{ \sum_{k=1}^{m_i} a_{ii}^k(s) - \sum_{j=m_i+1}^{m} \left| a_{ii}^j(s) \right| \right\} ds \leq \frac{1}{e},$$

$t \in [0, +\infty)$,

and

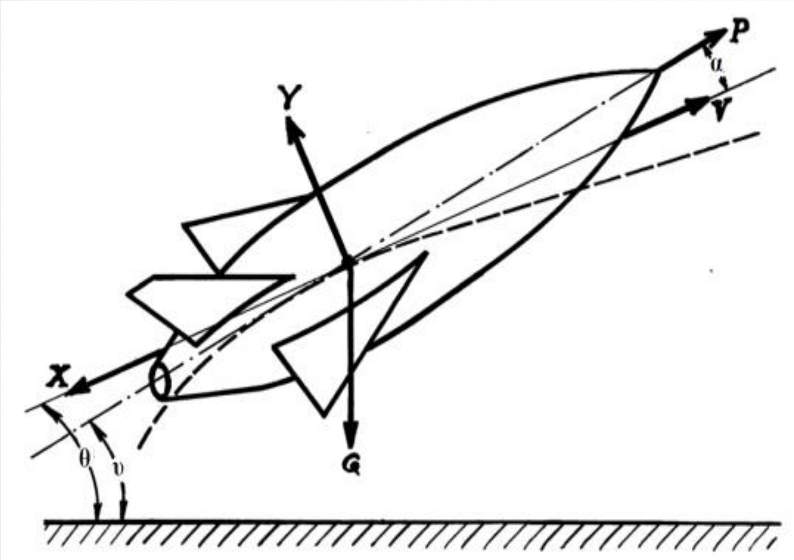
$$\int_{s}^{s+\Delta_i} \sum_{k=1}^{m_i} a_{ii}^k(\xi) d\xi \leq \frac{1}{e} \quad \forall s \geq 0.$$

(2) *There exist positive numbers* $z_1, \ldots, z_n$ *such that*

$$\sum_{k=1}^{m} a_{ii}^k(t) z_i - \sum_{j=1,\, j \neq i}^{n} \sum_{k=1}^{m} \left| a_{ij}^k(t) \right| z_j \geq 1, \quad t \in [0, +\infty),$$

$i = 1, \ldots, n.$

Parameters of drone's motion



V - flight velosity tangent to trajectory
Y - carrying force ortogonal to flight velosity
X - resistance force opposite to V
G - gravitational force
υ - pitch angle, i.e. angle between lengthwise drone axis and horizontal plane
θ - tilting of trajectory about horizontal plane
α - angle of attack, i.e. angle between lengthwise axis and projection of velosity on the symmertry plane of drone
$m = G/g$ - drone mass
P - tractive force directed along lengthwise dron axis

Nonlinear equations of motion

$$m\frac{dV}{dt} = P\cos\alpha - X - G\sin\theta$$

$$mV\frac{d\theta}{dt} = P\sin\alpha + Y - G\cos\theta$$

$$J_z\frac{d^2\nu}{dt^2} = M_z$$

$$\frac{dH}{dt} = V\sin\theta + U_y$$

$$\frac{dL}{dt} = V\cos\theta + U_x$$

$$P = P(\delta_p, V) \qquad \nu = \theta + \alpha$$

$$X = c_x S\frac{\rho V^2}{2} \qquad Y = c_y S\frac{\rho V^2}{2}$$

$$c_x = c_x(\alpha, \nu, V, H);$$

$$c_y = c_y(\alpha, V, H);$$

$$M_z = m_z b_a S\frac{\rho V^2}{2}$$

$$m_z = m_z(\alpha, \dot{\alpha}, \dot{\nu}, V, \delta_B, \rho)$$

δ_p - position of control knob

δ_B - deviation of elevator

M_z - total moment of aerodynamical forces with respect to transversal axis z

J_z - inertial moment of drone with respect of axis z

ρ - air density

U_x and U_y - wind velosities with respect axes x and y, correspondently

S - area of winds

b_a – length of wind chord

m_z - coefficient of moment

c_x and c_y - coefficients of resistance and carrying forces, correspondently

Linear equations of motion

Desirable steady state trajectory:

$$\upsilon o, \theta o, V_o, \alpha o, H_o$$

Deviations:

$$\Delta\upsilon,\ \Delta\theta,\ \Delta V,\ \Delta\alpha \text{ и } \Delta H,$$

$$\left.\begin{aligned}
(p+n_{11})\nu + n_{12}\alpha + n_{13}\upsilon + n_{14}h &= n_p\delta_p + f_1;\\
-n_{21}\nu + (p+n_{22})\alpha - (p+n_{23})\upsilon + n_{24}h &= f_2;\\
n_{31}\nu + (n_0 p+n_{32})\alpha + (p^2+n_{33}p)\upsilon + n_{34}h &= -n_в\delta_в + f_3;\\
-n_{41}\nu + n_{42}\alpha - n_{42}\upsilon + ph &= \nu_y,
\end{aligned}\right\}$$

n_{ij} - coefficients

f_1, f_2, f_3, ν_y -perturbations

p– operator $\frac{d}{dt}$

Autonomous controls try to decrease deviations to zero.

Linear equations of motion. Processing

$$D(v)(t) = -n_{11}\, v(t) - n_{12}\, \alpha(t) - n_{13}\, \vartheta(t) - n_{14}\, h(t) + n_p\, \delta_p(t-\tau)$$

$$D(\alpha)(t) = \varphi(t) + b_0\, \vartheta(t) + n_{21}\, v(t) - n_{22}\, \alpha(t) + n_{23}\, \vartheta(t) - n_{24}\, h(t)$$

$$D^{(2)}(\vartheta)(t) = -n_0\, D(\alpha)(t) - n_{33}\, D(\vartheta)(t) - n_{31}\, v(t) - n_{32}\, \alpha(t) - n_{34}\, h(t) - n_B\, \delta_B(t-\tau)$$

$$D(h)(t) = n_{41}\, v(t) - n_{42}\, \alpha(t) + n_{42}\, \vartheta(t)$$

$$D(\vartheta)(t) := \varphi(t) + b_0\, \vartheta(t)$$

$$h(t) := \lambda(t) - M v(t)$$

To decrease the order of the system

To get non-zero diagonal coefficients

$$\delta_p(t-\tau) := p_1\, v(t-\tau) + p_2\, \alpha(t-\tau) + p_3\, \vartheta(t-\tau) + p_4\, (\lambda(t-\tau) - M v(t-\tau))$$

$$\delta_B(t-\tau) := b_1\, v(t-\tau) + b_2\, \alpha(t-\tau) + b_3\, \vartheta(t-\tau) + b_4\, (\lambda(t-\tau) - M v(t-\tau))$$

Linear equations of motion. Processing

$$D(v)(t) = -n_{11} v(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}(\lambda(t) - Mv(t)) + n_p(p_1 v(t-\tau) + p_2\alpha(t-\tau) + p_3\vartheta(t-\tau) + p_4(\lambda(t-\tau) - Mv(t-\tau)))$$

$$D(\alpha)(t) = \varphi(t) + b_0\vartheta(t) + n_{21} v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}(\lambda(t) - Mv(t))$$

$$D(\vartheta)(t) = \varphi(t) + b_0\vartheta(t)$$

$$D(\varphi)(t) = -b_0(\varphi(t) + b_0\vartheta(t)) - n_0(\varphi(t) + b_0\vartheta(t) + n_{21} v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}(\lambda(t) - Mv(t))) - n_{33}(\varphi(t) + b_0\vartheta(t)) - n_{31} v(t) - n_{32}\alpha(t) - n_{34}(\lambda(t) - Mv(t)) - n_B(b_1 v(t-\tau) + b_2\alpha(t-\tau) + b_3\vartheta(t-\tau) + b_4(\lambda(t-\tau) - Mv(t-\tau)))$$

$$D(\lambda)(t) = M(p_1 v(t-\tau) + p_2\alpha(t-\tau) + p_3\vartheta(t-\tau) + p_4(\lambda(t-\tau) - Mv(t-\tau)))n_p + (M^2 n_{14} - Mn_{11} + n_{41})v(t) + (-Mn_{13} + n_{42})\vartheta(t) + (-Mn_{12} - n_{42})\alpha(t) - M\lambda(t)n_{14}$$

Stability of the system. Applying

$$1 \le (n_p p_4 M + n_{14} M - n_p p_1 + n_{11}) z_1 - |-n_p p_2 + n_{12}| z_2 - |-n_p p_3 + n_{13}| z_3 - |-n_p p_4 + n_{14}| z_5$$

$$1 \le n_{22} z_2 - |M n_{24} + n_{21}| z_1 - |n_{23} + b_0| z_3 - z_4 - |n_{24}| z_5$$

$$1 \le -b_0 z_3 - z_4$$

$$1 \le (b_0 + n_0 + n_{33}) z_4 - |n_B b_4 M - n_0 n_{24} M + M n_{34} - n_B b_1 - n_0 n_{21} - n_{21}| z_1 - |n_B b_2 - n_0 n_{22} + n_{32}| z_2 - |b_0^2 + n_0 b_0 + n_{33} b_0$$
$$+ n_B b_3 + n_0 n_{23}| z_3 - |n_B b_4 - n_0 n_{24} + n_{34}| z_5$$

$$1 \le (-n_p p_4 M + n_{14} M) z_5 - |-M^2 n_p p_4 + M^2 n_{14} + M n_p p_1 - M n_{11} + n_{41}| z_1 - |-n_p p_2 M + M n_{12} + n_{42}| z_2 - |-n_p p_3 M$$
$$+ M n_{13} - n_{42}| z_3$$

$$\tau \le \frac{1}{e\left(n_{14} M + n_{11} - n_p \cdot p_1 - |n_p \cdot p_4 \cdot M|\right)}$$

Stability of the system. Calculation

- Get from the table

n_{11}	0,024
n_{12}	-0,11
n_{13}	0,2
n_{14}	$-4{,}3\cdot10^{-4}$
n_{21}	-0,4
n_{22}	2,4
n_{23}	0
n_{24}	$-1{,}22\cdot10^{-2}$
n_{31}	0
n_0	0,4
n_{32}	38
n_{33}	2,45
n_{34}	-0,053
n_B	49
n_p	0.022
n_{41}	0
n_{42}	1

- Express

$$p_2 := -\frac{\frac{\left|\frac{n_{42}}{M}\right|}{2} + |n_{12}|}{n_p}$$

$$p_3 := \frac{\frac{\left|\frac{n_{42}}{M}\right|}{2} + |n_{13}|}{n_p}$$

$$b_4 := \frac{n_0\, n_{24} - n_{34}}{n_B}$$

$$b_2 := \frac{n_0\, n_{22} - n_{32}}{n_B}$$

$$b_3 := -\frac{b_0^2 + n_0\, b_0 + n_{33}\, b_0 + n_0\, n_{23}}{n_B}$$

$$b_1 := -\frac{n_{21}\left(n_0 + 1\right)}{n_B}$$

- Handle analytically

$$b_0 := -1.001$$

$$z_1 := 1$$

$$z_2 := 1$$

$$z_3 := \frac{1}{10000000000}$$

$$z_4 := \frac{1}{10000000000}$$

$$z_5 := 1$$

$$M := -63$$

- Calculate the rest in MatLab

Stability of the system. Results

$$\delta_p(t-\tau) := p_1 v(t-\tau) + p_2 \alpha(t-\tau) + p_3 \vartheta(t-\tau) + p_4 h(t-\tau)$$

$$\delta_B(t-\tau) := b_1 v(t-\tau) + b_2 \alpha(t-\tau) + b_3 \vartheta(t-\tau) + b_4 h(t-\tau)$$

$b_1 := 0.01142857143$

$b_2 := -0.7559183673$

$b_3 := 0.03777242857$

$b_4 := 0.0009820408163$

$p_1 := -35$

$p_2 := -5.360750359$

$p_3 := 9.451659450$

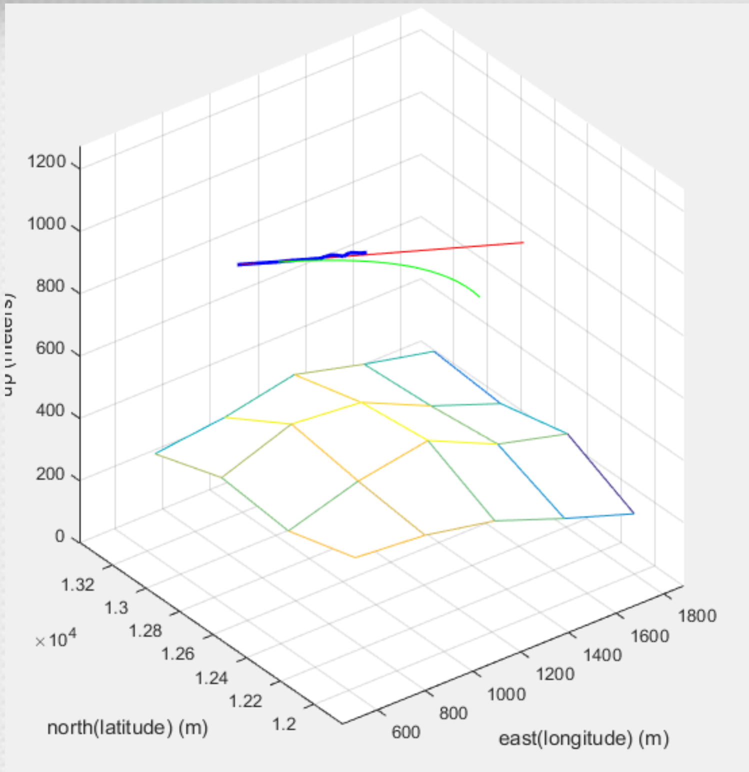
$p_4 := 0.5512345678$

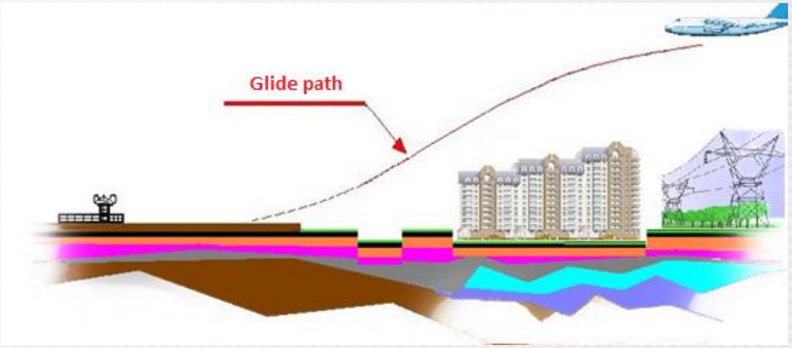
Control parameters

$\tau < 24.5\ s$

Delay estimation

Future plans





Actual path

Inertial system

Vision-based system