Towards a Proof of the Twin Prime Conjecture

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Abstract: Let P_n be the n_th prime. For twin primes $P_n - P_{n-1} = 2$. We exhibit two formulas for calculating the number of twin primes in the closed interval $[P_n, P_n^2]$. We show there is a lower limit for the number of twin primes in the closed interval $[P_n, P_n^2]$.

Calculating the number of $(6j-1, 6j+1)$ pairs (F_n) with no factor $< P_{n+1}$ in the closed interval $[1, J_n+1]$.

Let the product of the first n primes be $m = 1 \text{ to } n \prod P_m = J_n$. For each $(6j-1, 6j+1)$ pair with no factor less than P_n in $[1, J_{n-1}+1]$ there are pairs $(6j-1+mJ_{n-1}, 6j+1+mJ_{n-1})$ for $m = 0 \text{ to } P_n - 1$ in $[1, J_n+1]$. P_n and J_{n-1} are relatively prime. Thus, P_n divides $6j-1+mJ_{n-1}$ and $6j+1+mJ_{n-1}$ each for exactly one different value of m.
$F_5 = (1/6)(3/5)(J_5)$. $F_7 = (5)(F_5)$. $J_7 = (7)(J_5)$ $F_7 / F_5 = (5/7)(J_7 / J_5)$. $F_7 = (1/6)(3/5)(5/7)(J_7)$.
The number of $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in the interval $[1, J_n+1]$ is exactly $(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n)$. This occurs because J_n is divisible by all primes in the interval $[P_3, P_n]$.
All the $(6j-1, 6j+1)$ pairs with no factor less than P_{n+1} in which $6j < P_{n+1}^2$ are twin primes.

Determining the number of twin primes pairs (TPA_n) in the closed interval $[P_n, P_n^2]$.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. The number of twin prime pairs in $[P_n, P_n^2]$ is $(a_3/5)(a_4/7)(a_5/11)\ldots(a_n/P_n)(X)$. For $3 \leq m \leq n$ $P_m - 4 < a_m < P_m$. **Table 1** illustrates this formula for the interval $[20633, 20633^2]$. For selected P_m it compares the actual number $(6j-1, 6j+1)$ pairs with no factor less than P_{m+1} with the number calculated by setting a_m to $P_m - 4$ and $P_m - 2$.

Table 2 shows the number of twin primes calculated, when a_m is replaced by $P_m - 1$, $P_m - 2$, and $P_m - 3$ for $3 \leq m \leq n$. Comparing this calculation with the actual number of twin prime pairs shows that as P_n gets larger the average value of a_m is slightly less than $P_m - 2$. There are proportionally fewer twin prime pairs in $[P_n, P_n^2]$ than there are $(6j-1, 6j+1)$ pairs with no factors less than P_{n+1} in $[1, J_n+1]$. The number of twin prime pairs in $[P_n, P_n^2]$ is always greater than $(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(X) = 3X/P_n$.

Calculating the number of twin prime pairs (TPC_n) in $[P_n, P_n^2]$ using TPA_{n-1} in $[P_{n-1}, P_{n-1}^2]$. Let $P_n - P_{n-1} = a$
$((F_n)(P_n^2)/J_n) / ((F_{n-1})(P_{n-1})^2/J_{n-1}) = ((F_{n-1})(P_{n-1}+a-2)(P_{n-1}+a)^2/((J_{n-1})(P_{n-1}+a)) / ((F_{n-1})(P_{n-1})^2/J_{n-1}) =$
$(P_{n-1}+a-2)(P_{n-1}+a) / P_{n-1}^2 = 1+(2a-2)/P_{n-1}+(a^2-2a)/P_{n-1}^2$ **table 4** (column D) / (column C)
$(TPA_{n-1})(1+(2a-2)/2P_{n-1}+(a^2-2a)/2P_{n-1}^2)) = TPC_n$ (column B)((column D/column C)+1)/2=(column F)

Analyzing the ratio TPC / TPA.

There are 3360 primes between 67 and 31333. For the ratio TPC / TPA (**table 4** column H) the first interval of 480 primes has an average value (**table 3** column A) of 0.99357. If this were the first term in an infinite geometric series that summed to one, the average ratio between terms in the geometric series would be (using column A) $A/(1-R) = 1$. $R = 1-A = 0.00643$ (**table 3** column D.)

Table 3 shows the average ratio values for the other six intervals of 480 primes. The TPC / TPA ratios in column C are decreasing faster than ratios for infinite geometric series (column D). $TPC_n / TPA_n < 1$ for all n.
It also appears (column A **table 3** and column H **table 4**) that the $\lim_{n\to\infty} TPC_n / TPA_n = 1$.
The (average number of twin prime pairs) / (average prime value) in **table 3** column G is increasing.

Table 1 - Twin Primes in the interval [*20633, 20633²*]

X_start is the number of *(6j –1, 6j+1)* pairs starting with no factor less than P_m
X_remain is the number of *(6j –1, 6j+1)* pairs remaining with no factor less than P_{m+1}
$(P_m-2)/P_m$ is the number of *(6j –1, 6j+1)* pairs calculated by multiplying $(X\_start)(P_m-2)/P_m$
$(P_m-4)/P_m$ is the number of *(6j –1, 6j+1)* pairs calculated by multiplying $(X\_start)(P_m-4)/P_m$

	A	B	C	D	E	F	G
P_m	X_start	X_remain	A - B	$(P_m-2)/P_m$	B - D	$(P_m-4)/P_m$	B - F
5	67083984	40250390	26833594	40250390	0	13416797	26833593
71	8954354	8702130	252224	8702119	11	8449883	252247
73	8702130	8463717	238413	8463715	2	8225301	238416
1019	3517140	3510080	7060	3510237	-157	3503334	6746
1021	3510080	3502903	7177	3503204	-301	3496328	6575
2087	2824214	2820903	3311	2821508	-605	2818801	2102
2089	2820903	2817631	3272	2818202	-571	2815502	2129
3461	2406424	2404572	1852	2405033	-461	2403643	929
3463	2404572	2402633	1939	2403183	-550	2401795	838
4637	2183932	2182620	1312	2182990	-370	2182048	572
4639	2182620	2181290	1330	2181679	-389	2180738	552
6299	1972174	1971245	929	1971548	-303	1970922	323
6301	1971245	1970348	897	1970619	-271	1969994	354
8009	1828168	1827513	655	1827711	-198	1827255	258
8011	1827513	1826853	660	1827057	-204	1826600	253
9857	1714705	1714258	447	1714357	-99	1714009	249
9859	1714258	1713817	441	1713910	-93	1713562	255
11777	1639317	1638983	334	1639039	-56	1638760	223
11779	1638983	1638694	289	1638705	-11	1638426	268
13931	1576507	1576299	208	1576281	18	1576054	245
13933	1576299	1576093	206	1576073	20	1575846	247
16187	1538268	1538153	115	1538078	75	1537888	265
16189	1538153	1538043	110	1537963	80	1537773	270
18041	1521723	1521665	58	1521554	111	1521386	279
18043	1521665	1521611	54	1521496	115	1521328	283
19991	1515881	1515878	3	1515729	149	1515578	300
19993	1515878	1515875	3	1515726	149	1515575	300
20063	1515866	1515866	0	1515715	151	1515564	302

Table 2 – Twin Primes in the interval $[P_n, P_n^2]$

$(X)(P_m - 1)/P_m$ a_m is replaced by $P_m - 1$ for $m = 3$ to n.

$(X)(P_m - 2)/P_m$ a_m is replaced by $P_m - 2$ for $m = 3$ to n

$(X)(P_m - 3)/P_m$ a_m is replaced by $P_m - 3$ for $m = 3$ to n

P_n	TPA_n	$(X)(P_m - 1)/P_m$	$(X)(P_m - 2)/P_m$	$(X)(P_m - 3)/P_m$	$3X/P_n$
71	120	318	108	30	35
73	123	331	111	30	36
1019	8420	41870	8927	1560	509
1021	8450	41994	8944	1562	510
2087	28819	159426	30835	4890	1043
2089	28867	159656	30865	4892	1044
3461	68804	411969	74852	11150	1730
3463	68872	412326	74896	11153	1731
4637	114316	713918	125218	18005	2318
4639	114394	714379	125272	18009	2319
6299	195208	1271159	215116	29844	3149
6301	195319	1271765	215185	29849	3150
8009	297317	2001168	329770	44550	4004
8011	297454	2001918	329852	44556	4005
9857	428957	2961372	476744	62920	4928
9859	429089	2962273	476841	62927	4929
11777	588001	4151976	656480	85094	5888
11779	588163	4153034	656591	85102	5889
13931	791507	5703213	885216	112640	6965
13933	791704	5704441	885343	112648	6966
16187	1033547	7581313	1158580	145151	8093
16189	1033796	7582719	1158723	145160	8094
18041	1254327	9316215	1408395	174550	9020
18043	1254586	9317764	1408551	174560	9021
20147	1527206	11489310	1717636	210513	10073
20149	1527479	11491021	1717806	210524	10074
21839	1763993	13391366	1985849	241423	10919
21841	1764289	13393205	1986031	241434	10920
23741	2047968	15694023	2307974	278252	11870
23743	2048281	15696006	2308168	278264	11871
26861	2555034	19847486	2883531	343443	13430
26863	2555371	19849703	2883746	343456	13431
28619	2861908	22393705	3233701	382811	14309
28621	2862279	22396053	3233927	382824	14310
31319	3365123	26586680	3805992	446665	15659
31321	3365489	26589226	3806235	446679	15660

Table 3

	A	B	C	D	E	F	G
#Primes	Average *F/G* Table 4	Col A from Prev Row	*A/B – 1*	*1 – A*	Average value of Primes	*TPA* Average	Ratio *F/E*
1-480	0.99357			0.00643	1586	22618	14.3
481-960	0.99862	0.99357	0.00508	0.00138	5473	157511	28.8
961-1440	0.99917	0.99862	0.00055	0.00083	9779	428079	43.8
1441-1920	0.99942	0.99917	0.00025	0.00058	14302	834392	58.3
1921-2400	0.99954	0.99942	0.00012	0.00046	19002	1381028	72.7
2401-2880	0.99963	0.99954	0.00009	0.00037	23774	2058288	86.6
2881-3360	0.99968	0.99963	0.00006	0.00032	28702	2882451	100.4

Table 4

A	B	C	D	E	F	G	H	I
prime	TPA_{n-1}	$(F_{n-1})(P_{n-1})^2/J_{n-1}$	$(F_n)(P_n)^2/J_n$	$(D/C+1)/2$	$(B)(E)$	TPA_n	F/G	B/C
71	120	109.0	112.1	1.01422	121.7	123	0.989483	1.10092
73	123	112.1	127.9	1.07047	131.7	138	0.954117	1.09723
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425	0.94233
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872	0.94384
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816	0.93417
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070	0.93482
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302	0.91893
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735	0.91931
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534	0.91274
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023	0.91297
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590	0.90731
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566	0.90754
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664	0.90148
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291	0.90167
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794	0.89967
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986	0.89977
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809	0.89561
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679	0.89571
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823	0.89408
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208	0.89417
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821	0.89203
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691	0.89213
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849	0.89056
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761	0.89064
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871	0.88909
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481	0.88916
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878	0.88824
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602	0.88831
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889	0.88731
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825	0.88737
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905	0.88605
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520	0.88610
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905	0.88499
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793	0.88505
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923	0.88414
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814	0.88418