Proof of the Twin Prime Conjecture
Jim Rock

Abstract: Let P_n be the n_th prime. For twin primes $P_n - P_{n-1} = 2$. We show that in the interval (P_n, P_n^2) as P_n gets larger, there is an increasing number of twin primes.

Determining the twin primes pairs (TPA_n) in the interval (P_n, P_n^2).
Let the product of the first n primes be $_{m=1 \text{ to } n}\prod P_m = J_n$. We group the positive integers of the form $(6j-1, 6j+1)$ in the interval $(1, J_n+1)$ into pairs. We eliminate all pairs, which contain a prime factor $P \le P_n$. All $(6j-1, 6j+1)$ left for which $6j < P_n^2$ are twin primes.

Calculating the number of $(6j-1, 6j+1)$ pairs (F_n) with no factor $< P_{n+1}$ in the interval $(1, J_n+1)$.
$(1/6)(3/5)(5/7)\ldots((P_n-2)/P_n)(J_n) = F_n$ is the number of pairs $(6j-1, 6j+1)$, in $(1, J_n+1)$ with no factor less than P_{n+1}. For each $(6j-1, 6j+1)$ with all prime factors greater than P_{n-1} in $(1, J_{n-1}+1)$ there are pairs $(6j-1+mJ_{n-1}, 6j+1+mJ_{n-1})$ for $m = 0$ to $P_n - 1$ in $(1, J_n+1)$. P_n and J_{n-1} are relatively prime.
Thus, P_n divides $6j-1+mJ_{n-1}$ and $6j+1+mJ_{n-1}$ each for exactly one different value of m.

The number of twin prime pairs (TPA) in (P_{n+1}, P_{n+1}^2) is greater than the number in (P_n, P_n^2)
Let $P_{n+1} - P_n = a$. (P_{n+1}, P_{n+1}^2) is $(P_n+a, (P_n+a)^2) - (P_n, P_n^2) = 2aP_n+a^2-a$. The number of $(6j-1, 6j+1)$ pairs in $(P_{n+1}, P_{n+1}^2) - (P_n, P_n^2)$ is the integer part of $(2aP_n+a^2-a)/6 = x$. The number of pairs with $6j \ge P_{n+1}$ is approximately $(3/5)(5/7)\ldots((P_n-2)/P_n)(x)$ and is greater than
$(3/5)(5/7)(7/9)\ldots(P_n-2)/P_n)(x)=(3)(2aP_n+a^2-a)/6P_n = a$. $TPA_{n+1} - TPA_n > P_{n+1} - P_n$.

Calculating the number of twin prime pairs (TPC_{n+1}) in (P_{n+1}, P_{n+1}^2) using TPA_n in (P_n, P_n^2).
In table 2 (column D) / (column C) equals $((F_{n+1})(P_{n+1}^2)/J_{n+1}) / ((F_n)(P_n)^2/J_n) =$
$((F_n)(P_n+a-2)(P_n+a)^2/((J_n)(P_n+a)) / ((F_n)(P_n)^2/J_n) = (P_n+a-2)(P_n+a) / P_n^2 = 1+(2a-2)/P_n+(a^2-2a)/P_n^2$
$(TPA_n)(1+(2a-2)/2P_n+(a^2-2a)/2P_n^2))=TPC_{n+1}$ is (column B)((column D/column C)+1)/2=(column F)
$(1+(2a-2)/2P_n+(a^2-2a)/2P_n^2)) \sim 1+a/P_n \sim 1+(\ln(P_n)/P_n$. TPC_{n+1} (column F) $< TPA_{n+1}$ (column G).

Analyzing the ratio TPC / TPA.
There are ***3360*** primes between ***67*** and ***31333***. For the ratio TPC / TPA (column H in table 2) the first interval of 480 primes has an average value (column A in table 1) of ***0.99357***. If this were the first term in an infinite geometric series that summed to one, the average ratio between terms in the geometric series would be (using column A) $A/(1-R) = 1$. $R = 1-A = 0.00643$ (column D in table 1.)

Table 1 shows the average ratio values for the other six intervals of 480 primes. The ratios in column C are decreasing faster than ratios for an infinite geometric series shown in column D. $TPC_n / TPA_n < 1$ for all n. It also appears (column A table 1 and column H table 2) that the $\lim_{n\to\infty} TPC_n / TPA_n = 1$.
The (average number of twin prime pairs) / (average prime value) in column G table 1 increasing.

Table 1

	A	B	C	D	E	F	G
#Primes	Average F/G Table 2	A Prev	A/B – 1	1 – A	Average value of Primes	TPA Average	Ratio F/E
1-480	0.99357			0.00643	1586	22618	14.3
481-960	0.99862	0.99357	0.00508	0.00138	5473	157511	28.8
961-1440	0.99917	0.99862	0.00055	0.00083	9779	428079	43.8
1441-1920	0.99942	0.99917	0.00025	0.00058	14302	834392	58.3
1921-2400	0.99954	0.99942	0.00012	0.00046	19002	1381028	72.7
2401-2880	0.99963	0.99954	0.00009	0.00037	23774	2058288	86.6
2881-3360	0.99968	0.99963	0.00006	0.00032	28702	2882451	100.4

Table 2

A	B	C	D	E	F	G	H	I
prime	TPA	$((F_n)(P_n)^2/J_n)$	C_Next	(D/C+1)/2	BxE	B_Next	F/G	B/C
71	120	109.0	112.1	1.01422	121.7	123	0.989483	1.10092
73	123	112.1	127.9	1.07047	131.7	138	0.954117	1.09723
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425	0.94233
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872	0.94384
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816	0.93417
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070	0.93482
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302	0.91893
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735	0.91931
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534	0.91274
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023	0.91297
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590	0.90731
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566	0.90754
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664	0.90148
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291	0.90167
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794	0.89967
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986	0.89977
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809	0.89561
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679	0.89571
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823	0.89408
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208	0.89417
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821	0.89203
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691	0.89213
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849	0.89056
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761	0.89064
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871	0.88909
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481	0.88916
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878	0.88824
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602	0.88831
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889	0.88731
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825	0.88737
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905	0.88605
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520	0.88610
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905	0.88499
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793	0.88505
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923	0.88414
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814	0.88418