Bell's inequality refuted via elementary algebra in spacetime

Abstract Bell's inequality is widely regarded as a profound impediment to any intuitive understanding of physical reality. We disagree. So here, via elementary algebra—backed by experiments; and thus with certainty—we refute his famous inequality, correct his key error, resolve his locality-dilemma: all in accord with the antiBellian true-local-realism that we've advanced since 1989. We thus restore commonsense/intuitive ideas to physics—thereby making physical reality more intelligible—by completing the quantum mechanical account of EPR-Bohm correlations via a wholistic/Einsteinian approach in spacetime.

Keywords Bell's inequality, Bell's key error, same-instance-rule, true-local-realism (TLR)

1. Introduction

1.0. Schlosshauer (2011:161) asks a good question (and receives some interesting answers): 'What do the experimentally observed violations of Bell's inequalities tell us about nature?'

1.1. (i) Referring (p161) to Bell's *'steely mathematics'* [sic],[1] he also asks (p165), '[ii] Which of Bell's assumptions will have to go? [iii] If Bell's notion of locality is the sticky issue, can we conclude that nature herself must be nonlocal?' (iv) Here's the gist of some answers from professors in this field:

1.2. (i) Guido Bacciagaluppi (p165): 'It can be safely said that distant correlations in nature cannot [sic] be understood in terms of quite general local models. I am happy to call that nonlocality ... the fact remains that the distant correlations lack any [sic] straightforward explanation in local terms.' (ii) Časlav Brukner (p166): 'There is thus no [sic] way of getting around Bell's theorem.'

1.3. (i) Jeffrey Bub (p168): 'They tell us that we live in a world in which there are nonlocal correlations that are inconsistent [sic] with any explanation in terms of common causes.' (ii) GianCarlo Ghirardi (p172): 'They point to an absolutely fundamental, revolutionary, and unexpected aspect of natural processes, namely that nature is not [sic] locally causal.'

1.4. (i) Shelly Goldstein (p173): 'They tell us that nature is nonlocal [sic] ... that for any theory to predict these violations, it must [sic] be a nonlocal theory.' (ii) Tim Maudlin (p176): '... we can conclude that nature is nonlocal [sic]; ie, in some way certain events at spacelike separation are physically [sic] connected. Einstein's dream of a perfectly local physics ... cannot [sic] be fulfilled.'

1.5. (i) Lee Smolin (p177): 'They imply that there are real physical nonlocal correlations in nature. It seems simplest to suppose these are evidence for nonlocal interactions [sic].' (ii) Antony Valentini (p177): 'They tell us that locality is [sic] violated—if we assume [nb: as we certainly do] that there is no backward causation and that there are not many worlds.'

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[1] This *sic* signals that Bell's mathematics is false: his imagination [as he supposed, see ¶2.3(iii)] too limited. For—as we show via elementary algebra; and as the experimental violations of Bell's inequalities confirm—Bell's analysis is irrelevant to such experiments. Thus, with certainty, each *sic* in this note identifies a Bell-based/Bellian error.

1.6. So, against such answers—see also Watson 2017d and ¶0.1(ix) there—we show that: (i) elementary algebra in spacetime refutes Bell's famous inequality; (ii) Bell errs when *using* [sic]—his word, our emphasis—Bell 1964:(1); (iii) Bellian answers, like those above, are similarly erroneous; (iv) related mathematical claims fall too: eg, Peres (see fn.6 below); 'Bell's theorem is as mathematically robust [sic] as they come,' du Sautoy (2016:170); 'We mathematicians stand in awe of John Bell. That a non-mathematician could derive such a profound [sic] theory from so little,' Harris (2019).

1.7. In brief: Reading Mermin (1988)—2 June 1989, and as discussed with him the next day; consistent with Einstein-locality/separability—we show that: (i) Bell's inequalities are flawed, his conclusions false; (ii) nature is bound by true-local-realism (TLR), see ¶2.4(i);[2] (iii) the intuitive-classicality of our analysis—endorsing local hidden-variables (LHVs)—eliminates any need for 'loop-holes' to rescue such intuitive variables; (iv) against Bell's theorem,[3] paired-tests on correlated particle-pairs produce common-cause-correlated spacelike-separated results (ie, *long-range correlations*) without mystery. Given this background—supported by QM, experiments, Einstein, elementary algebra—let's see.

2. Against Bell: advancing the classicality of wholistic mechanics

> 2.0. 'Bell's theorem stands as an insuperable roadblock in the path to a very desired intuitive solution of the Einstein-Podolsky-Rosen [EPR] paradox and, hence, it lies at the core of the current lack of a clear interpretation of the quantum formalism,' Oaknin (2018).

2.1. Seeking a *more complete specification* of the EPR-Bohm experiment (EPRB), Bell[4] derives an inequality that is false under EPRB. This fact surprises many scientists; eg, Aspect (2004:2). So we use elementary algebra to derive inequalities (5) & (8) that refute Bell's inequality (7) easily/clearly.

2.2. (i) Thus, from first principles, we show that Bell's key error triggers a contagion in need of a warning-sign: BE·1. (ii) For BE·1—the misuse of Bell 1964:(1); by Bell and many others (¶5.4)—limits such analysis to contexts less correlated than EPRB. (iii) They thus miss this fact: the significance of EPRB is that its common-cause correlations can be explained intuitively *and wholistically* via true-local-realism (see ¶2.4): despite the fact that EPRB-based outcomes are said to entail

> 'a kind of correlation of the properties of distant noninteracting systems, which is quite different from previously known kinds of correlation,' Bohm & Aharonov (1957:1070).

2.3. This *classicality* allows us to resolve Bell's dilemma and refute his false conclusions re *locality*:

> (i). 'And that is the dilemma. We are led by analysing [the EPRB] situation to admit that in somehow distant things are connected, or at least not disconnected.' (ii) 'Maybe someone will just point out that we were being rather silly .... But anyway, I believe the questions will be resolved.' (iii) 'I think somebody will find a way of saying that [relativity and QM] are compatible. But I haven't seen it yet. For me it's very hard to put them together, but I think somebody will put them together, and we'll just see that my imagination was too limited.' (iv) 'I say only that you cannot get away with locality.' After Bell (1990:7,9,10,13).

2.4. (i) For Watson 2017d resolves Bell's dilemma via true-local-realism: the union of *true-locality* (no influence propagates superluminally) and *true-realism* (some existents change interactively). (ii) So, akin to Fröhner (1998), we seek to advance *commonsense mechanics*—*Wholistic Mechanics*—WM: the mathematical unification of inferences to the best explanations of observable facts via *progressive wholistic updating* [¶5.6] that keeps pace with modern findings in spacetime.

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[2] We use and defend *true* to distinguish our terms from misleading or naive variants; eg, Bell-locality, naive-realism.

[3] Bell's theorem is variously defined. Via his 1964 Introduction, the following is adequate here: It is impossible [sic] for any local theory to reproduce the predictions of elementary quantum theory: the physical world is [sic] nonlocal.

[4] Bell (1964:195). Such key texts are freely available online, see References. EPRB is an historic thought-experiment; a related real-experiment/false-inequality is Aspect (2004)/CHSH (1969). Our theory (¶2.4) refutes such inequalities.

2.5. (i) Thus, under a single set of equations—see Appendix A—WM refutes Bell's inequality and theorem in the historic contexts of Bohm's famous thought-experiment (EPRB, spin $s = \frac{1}{2}$) and the related real-experiment (Aspect 2004, $s = 1$). (ii) So WM refutes Bell's work and any analysis that leads to claims—essentially nonlocal/naively-realistic claims—like those in §1 above; and these:

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence [sic] the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously [sic], so that such a theory could not be Lorentz invariant," Bell (1964:199).

2.6. So now, taking math to be the best logic—with probabilities $0 \leq P \leq 1$ being objective degrees of plausibility—we progress via clear-cut elementary facts to irrefutable antiBellian conclusions.

3. Analysis: Bell's inequality refuted

3.1. As in Bell (1964) and in much Bellian theorizing, the context is EPRB. We use E for expectations (not P, which we reserve for probabilities), and a, b, c for Bell's unit-vectors $\vec{a}, \vec{b}, \vec{c}$. Let (14a)-(14c) identify the unlabelled relations between Bell's (14)-(15); the remainder being (15a), (21a)-(21e), (23).

3.2. Thus, from Bell 1964:(1) and anticipating Bell's further needs (p.198), we have the expectations

$$-1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1. \tag{1}$$

$$\therefore E(a,b)[1 + E(a,c)] \leq 1 + E(a,c);\ \text{ie, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{2}$$

$$\therefore E(a,b) - E(a,c) \leq 1 - E(a,b)E(a,c). \tag{3}$$

$$\therefore |E(a,b) - E(a,c)| \leq 1 - E(a,b)E(a,c). \blacksquare \tag{4}$$

3.3. (i) So (3)/(4)[5]—nb: each LHS is from Bell 1964:(14a)/(14c) with an RHS that Bell fails to find—hold unconditionally via (1) and elementary algebra alone: to be as one with QM, EPRB, Aspect (2004). (ii) Further, exhausting (1) unconditionally, (4) becomes indisputable pairwise-generalized (5):

$$0 \leq |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1; \text{etc.}\, \blacksquare \tag{5}$$

3.4. (i) Now whatever form the expectations take, (5) holds for any pair that is consistent with (1). (ii) So (5) holds for EPRB with spin-$\frac{1}{2}$ particles and Aspect's (2004) experiment with photons.[6] (ii) For simplicity, we proceed via (21) under EPRB/Bell (1964) where $s = \frac{1}{2}$. (iv) Henceforth here:

$$E(a,b) = -\cos(a,b),\ E(a,c) = -\cos(a,c),\ E(b,c) = -\cos(b,c). \tag{6}$$

3.5. So here's Bell's famous 1964:(15)—in a form matching our (5)[7]—but note the ***sic*** that follows:

$$-1 \leq |E(a,b) - E(a,c)| - E(b,c) \leq\ 1\ [\textbf{sic}].\, \blacktriangle \tag{7}$$

3.6. ***sic***: for—in the context of EPRB; using (6) with (7) for proof by exhaustion—we find

$$-1 \leq |E(a,b) - E(a,c)| - E(b,c) \leq \tfrac{3}{2}. \blacksquare \tag{8}$$

3.7. Thus, via our also-EPRB-valid (8):[8] (i) For Bell's upper-bound of 1 to be true in his EPRB-false (7), the EPRB expectations in (6) must be reduced by one-third of their value. (ii) So Bell's analysis applies to contexts one-third less-correlated than EPRB. (iii) As we show at §4: Bell's key error, BE·1—¶2.2(i); Bell's use/*misuse* of his 1964:(1) in his/*our* terms; an EPRB-false assumption in general terms—is the source of his serious difficulty at ¶2.3(i). (iv) We therefore resolve Bell's dilemma by refuting the EPRB-false assumption behind BE·1. (v) We find that BE·1's naivety provides no basis for rejecting such commonsense wholistic principles as true-locality and true-realism (¶2.4).

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[5] See Appendix B for the move (3)-(4); the absolute brackets are to match those in Bell's famous EPRB-false inequality.

[6] Bellians require a different Bell-inequality for each experiment; eg, Peres (1993:163). WM shows that each is false.

[7] The valid lower-bound is included for completeness. nb: Bell's inequality is false under EPRB, not in general (¶5.3).

[8] Eg: the lower-bound when $(a,b) = \frac{\pi}{2}, (a,c) = \frac{3\pi}{2}, (b,c) = \pi$; the upper-bound when $(a,b) = (b,c) = \frac{\pi}{3}, (a,c) = \frac{2\pi}{3}$.

Name of relation	Relation to be evaluated under EPRB	T/F	Eq.
Our key result	$0 \leq \lvert E(a,b) - E(a,c)\rvert + E(a,b)E(a,c) \leq 1$; etc.	T	(5)
Bell's error BE·1	Bell's use of 1964:(1) to derive false (14b) from true (14a).	F	(9)
Our inequality	Under EPRB: $-1 \leq \lvert E(a,b) - E(a,c)\rvert - E(b,c) \leq 3/2$.	T	(8)
Bell's inequality	Under EPRB: $-1 \leq \lvert E(a,b) - E(a,c)\rvert - E(b,c) \leq 1$. [sic]	F	(7)

Table 1: Comparison of relations, and whether true or false, under EPRB. (i) We first expose BE·1 logically: Bell's inequality Bell 1964:(15) [see our (7)] is EPRB-false via our EPRB-true (5) and (8). (ii) So the related 1964:(14b) & (14c) are false. (iii) So it is Bell's use of his 1964:(1)—see note below Bell 1964:(14b)—that is false: for it downgrades EPRB-valid (14a) to EPRB-false (14b). (iv) In short: Bell's misuse of his 1964:(1) under EPRB creates (7); to thus require our remedial (8). (v) nb: we expose and correct BE·1 mathematically at §4 below. (vi) For BE·1's influence on Bell's results, see Appendix C.

3.8. (i) So, comparing Bellian (7) with our irrefutable **(5)**: we see that $E(b,c)$—a consequence of BE·1—breaches Bell's upper-bound of 1 in (7). (ii) Yet our rigorous $E(a,b)E(a,c)$ satisfies the same bound in (5). (iii) Thus, wrt Bell's claim below his 1964:(14b) and with certainty, here is BE·1:

Under EPRB, BE·1 is Bell's misuse of his 1964:(1) to derive false (14b) from true (14a). (9)

3.9. Thus: (i) (5) & (8) show that (7) is false under EPRB. (ii) It follows that (7)'s key source—BE·1, (9)—requires correction under EPRB. To that end: (iii) We use Bell 1964:(1) correctly in (11)-(14), consistent with irrefutable (5). (iv) *We then follow Bell's example* and misuse Bell 1964:(1) in (15)-(19). (v) We thus confirm (9) and define Bell's misuse more precisely—in (19)—on Bell's terms.

3.10. So, with certainty: Bellian analysis and Bell's famous (7) are irrelevant to EPRB and QM.[9]

4. Analysis: Bell's key error—BE·1—corrected

4.0. [i] 'Too bad for Einstein ... you cannot get away with locality,' after Bell (1990:6,13).

[ii] 'Since the initial quantum mechanical wavefunction [for EPRB] does not determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the [EPRB] state.

[iii] Let this more complete specification be effected by means of parameters $\boldsymbol{\lambda}$. It is a matter of indifference in the following whether $\boldsymbol{\lambda}$ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if $\boldsymbol{\lambda}$ were a single continuous parameter. [iv] The result A of measuring $\sigma_1 \cdot a$ is then determined by a and $\boldsymbol{\lambda}$, and the result B of measuring $\sigma_2 \cdot b$ *in the same-instance* [i] is determined by b and $\boldsymbol{\lambda}$, ...' after Bell (1964:196): with our emphasis and additions [.].

4.1. (i) Thus i, a mnemonic *instance-identifier*—reminds us that EPRB correlations and expectations arise from the union of paired-results obtained in *the same-instance*. (ii) So, via that *same-instance-rule* (SIR): instance-identifiers help us avoid BE·1 and apply Bell 1964:(1) correctly. (iii) We therefore incorporate SIR into our (10)—thereby clarifying Bell 1964:(1)—as follows.

4.2. (i) Under EPRB as in Bell 1964—and still taking math to be the best logic—we first consider 'the governing physical principles before heading to the math ... when exploring quantum phenomena,' after Aharonov et al (2019:1). (ii) Thus, for us: under EPRB, the governing principle is the pairwise conservation of total angular momentum *in the same-instance*. (iii) Then, backtracking as above from (7)—Bell's famous (known, now shown to be false) inequality—we identified BE·1 in (9).

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[9] Agreeing with Peres 1995:162, 'Bell's theorem is *not* a property of QM.' However, we reject the related Peres/Bellian claim: 'It applies to any [sic] physical system with dichotomic variables, whose values are arbitrarily called 1 and -1.' For Bell's famous inequality 1964:(15)—(7); influencing his theorem, see fn.3—does not satisfy that claim: our (5) does.

4.3. (i) So we now reveal BE·1's origin by using instance-identifiers to avoid it. (ii) We can then—see Appendix D—independently derive our indisputable (4) and (5) in a fresh irrefutable way: to thereby show that, in contravening SIR (¶4.1)—against his own words, ¶4.0[iv]—Bell voids his own conclusions. (iii) Thus, using instance-identifier i with Bell's 1964:(1) & 1964:(13); allowing $\boldsymbol{\lambda}_i^- \equiv -\boldsymbol{\lambda}_i$ when clarity requires (eg, in text):

$$A(a,\boldsymbol{\lambda})_i \equiv A^{\pm} = \pm 1.\; B(b,\boldsymbol{\lambda})_i \equiv B^{\mp} = \mp 1 = -A(b,\boldsymbol{\lambda})_i = A(b,-\boldsymbol{\lambda})_i \equiv A(b,\boldsymbol{\lambda}^-)_i. \tag{10}$$

4.4. (i) The first and last terms in (10) show that—under SIR—the correlations between spacelike separated events are explicable via *a common function* A of *local* variables: ie, in Alice's locale (space `A`), detector $\Delta_a^{\pm}$[10] responds to the local physical input $\boldsymbol{\lambda}$; in Bob's locale (space `B`), detector $\Delta_b^{\pm}$ responds to the local physical input $\boldsymbol{\lambda}^-$. (ii) We thus tackle Bell's polemics—eg, ¶4.0[i]—directly.

4.5. (i) For, consistent with Bell's *vital assumption re locality*—see the line below Bell 1964:(1)—all that Bell *implies* in Bell 1964:(1) is now *explicit* in (10). (ii) Thus, via WM/true-local-realism (¶2.4):

> 'Whatever exists (ie, is real) is localized in time and space: the real in space `A` exists independently of the real in space `B`; and vice-versa. So that which exists in `B` [allowing that it may soon be disturbed; but only via a local interaction] does not depend on what kind of measurement—if any—is carried out in space `A`. So we can hardly consider the quantum-theoretical description [nor Bell's *analysis* of EPRB] to be a complete representation of the physically real,' based on Einstein in Born (1971:164).

4.6. (i) Further—re BE·1—i serves as a single/silent mnemonic in (10): reminding us that SIR (¶4.1) applies. (ii) So, since Bell's opening inequality-formalism—Bell 1964:(14a)—*includes two instances*: we now use two identifiers, i & j. Thus, noting the irrefutable rigor of our (13) versus Bell's (14b):

$$\begin{aligned}
\text{Bell 1964:(14a)} \;&\equiv\; E(a,b) - E(a,c) && (11)\\
&\equiv\; -\int d\lambda\, \rho(\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i + \int d\lambda\, \rho(\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda})_j A(c,\boldsymbol{\lambda})_j && (12)\\
&=\; \int d\lambda\, \rho(\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i [A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i A(a,\boldsymbol{\lambda})_j A(c,\boldsymbol{\lambda})_j - 1] && (13)\\
&\neq\; \text{Bell 1964:(14b).} \blacksquare \text{ QED.} && (14)
\end{aligned}$$

4.7. (i) nb: (13) differs from any equation in Bell (1964). (ii) For we track instances rigorously under SIR—beginning with Bell's (14a); our (11)-(12)—to show the utility of i and j. (iii) At the same time (of course) we show that our analysis is EPRB-valid in each instance. To be clear: (iv) (13) is true under EPRB and (10), and thus—please note—under Bell 1964:(1). (v) Crucially: (13) reduces [see Appendix D] to irrefutable EPRB-valid (4) and (5). (vi) Further, and next: (13) readily exposes the breaches of SIR that Bell requires to (falsely) equate his (14b) with his (14a) under EPRB.

4.8. Thus, with ▲ doubly denoting absurdity and the beginning and end of an absurd Bellian sandwich:

$$\begin{aligned}
\blacktriangle\ \text{Neglect SIR (¶4.1)} \;&:\; \text{ie, delete } i,j \text{ from (13); the remnant is then equal [with} && (15)\\
\text{care] to Bell 1964:(14a)} \;&=\; \int d\lambda\, \rho(\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda}) A(b,\boldsymbol{\lambda}) [A(a,\boldsymbol{\lambda}) A(b,\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda}) A(c,\boldsymbol{\lambda}) - 1] && (16)\\
&=\; \int d\lambda\, \rho(\boldsymbol{\lambda}) A(a,\boldsymbol{\lambda}) A(b,\boldsymbol{\lambda}) [A(b,\boldsymbol{\lambda}) A(c,\boldsymbol{\lambda}) - 1];\ \text{ie, carelessly} && (17)\\
\text{(or, identically, via BE·1):} \;&=\; \text{Bell 1964:(14b)} \blacktriangle\text{: ie, under EPRB,} && (18)
\end{aligned}$$

BE·1 (¶2.2) is Bell's violation of SIR (¶4.1) to derive absurd (17) from true (13)/(16). (19)

4.9. So, re ¶3.3(i) and the logical analysis below Table 1: (i) Beginning with (11) under EPRB, Bell derives EPRB-false (17). (ii) We note that i,j in (12) are helpful sentinels against silliness. (iii) For—comparing valid (13) & (16) with absurd (17)—Bell requires $A(a,\boldsymbol{\lambda})_j = A(a,\boldsymbol{\lambda})_i$ and $A(c,\boldsymbol{\lambda})_j = A(c,\boldsymbol{\lambda})_i$: though instance i might occur in India, with instance j in Japan. (v) Thus, all our truly-local and truly-realistic results (¶2.4) remain valid under elementary algebra, EPRB, QM, etc. QED.

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[10] In our terms, from Watson 2017d:(3): $\Delta_a^{\pm}$ is Alice's polarizer-analyzer Δ—a disturber/detector in space `A`—its principal-axis oriented a, its output-channels a^+ and a^-. Likewise, $\Delta_b^{\pm}$ is Bob's polarizer-analyzer in space `B`; etc.

5. Conclusions

> 5.0. 'The purpose of this first part is to convince the reader that the *formalism leading to Bell's Inequalities is very general and reasonable.* What is *surprising* is that such a reasonable formalism [sic] *conflicts with Quantum Mechanics*,' Aspect (2004:2); his emphasis.

5.1. (i) Avoiding Bell's errors, we conclude that Bell's *very general and reasonable* formalism holds. (ii) For it's Bell's inadequate analysis—due to his key error; BE·1, (9), (19)—that creates his conflict with QM and nature. (iii) Thus, beginning with Bell 1964:(14a); ie, LHS of our (3): we derive irrefutable (4) & (5), and Bell derives false (7). (iv) Then, in seeking a more complete specification of EPRB: we derive irrefutable (8) while Bell again offers false (7). (v) See Appendix E for further discussion.

5.2. Thus, via EPRB and true-local-realism (¶2.4)—with every relevant existent included in our math for completeness; against Bell and similar others ¶¶1.1-1.5, etc—we confirm the following certainties:

(i) (5) Our EPRB-true inequality, irrefutable under algebra, refutes Bell's 1964:(15).
(ii) (7) Bell's famous inequality 1964:(15)[11] is EPRB-false under (5) and (8).
(iii) (8) Our EPRB-true inequality refutes and replaces Bell's famous 1964:(15).
(iv) (9) BE·1 (¶2.2) is Bell's misuse of his 1964:(1) to derive false (14b) from true (14a).
(v) (10) Equates to Bell's 1964:(1); facilitates instance-tracking to avoid/correct BE·1.
(vi) (14) Bell's 1964:(14b) $\neq$ Bell 1964:(14a).
(vii) (18) Bell's violation of SIR (¶4.1)—see (15)—equates his false (14b) with true (14a).
(viii) (19) BE·1 (¶2.2) violates SIR (¶4.1) to derive absurd (17) from true (13)/(16).
(ix) (33) Bell 1964:(14a) equates to our irrefutable (5) under EPRB and SIR (¶4.1).
(x) ¶4.5 Via the local-causality in (10), we show that each ***sic*** in this note is valid.

5.3. (i) So Bell's inequality (7) and Bellian analysis are irrelevant to EPRB and QM. (ii) For the experimentally observed violations of Bell's inequalities have to do with neglecting SIR (¶4.1). (iii) They have nothing to do with the proffered answers in §1: all of which, as foreshadowed in fn.1, are false. (iv) Claims re Bell's *steely mathematics*—eg, ¶¶1.1 & 1.6(iv)]—are also refuted: it being understood that BE·1 is compatible with experiments less correlated than EPRB and Aspect (2004).

5.4. Then, re ¶2.2(i)/BE·1 and the associated contagion: communicators include CHSH [1969: first math expression; unnumbered], Bell 1971:(8)-(11), Clauser & Shimony [1978:(3.6a)-(3.6c); our numbering], Peres 1995:(6.25) & (6.29), Griffiths (1995:378; 2005:425), Ballentine (1998:589), Aspect 2004:(17), Lubos Motl (2007:8),[12] Mikko Levanto (2016).

5.5. Thus: (i) Oaknin's 2018 'insuperable roadblock' ¶2.0 is a mirage on the road just travelled. (ii) Elementary algebra and EPRB facts resolve Bell's dilemma ¶2.3 and antiEinstein polemic ¶4.0[i] in Einstein's favor. (iii) Against Bell's supposed *generality and reasonableness* (¶5.0), a new generality and factualness applies. (iv) For WM respects that crucial EPRB boundary-condition—in our terms, the *same-instance-rule* (SIR) at ¶4.1—spelt out by Bell in the line before Bell 1964:(1).

5.6. Then, wrt ¶2.4 and *wholistic* updating in spacetime: (i) As Einstein's relativistics update classical mechanics, so we seek to update the ensuing wholism via QM and GA (Watson 2017d:§5). (ii) Thus, under WM and via true-realism—¶2.4(i)—we allow that polarizer-analyzer/particle-field interactions may here change/disturb each particle's total angular momentum: to thus reveal the (likely-new) orientation of its spin (its intrinsic angular momentum). (iii) Born's rule—the Riesz-Fejér theorem (RFT) of 1915; Fröhner (1998)—follows *wholistically* via Fourier analysis.

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[11] (7) is the first in a family of Bellian inequalities—see Clauser & Shimony (1978:1889)—all of which are false.

[12] Lubos Motl is not a Bellian. This example is from his Lecture 36.

5.7. Thus this note and Watson (2017d) advance our theory—wholistic mechanics—WM. (i) For here and at 2017d:§6, WM refutes Bell's famous inequality as well as his famous theorem.[13] (iii) There too—at 2017d:(75)—WM delivers the function A in our (10) and spacetime. (iv) So too, WM counters Bell's erroneous (2004:86) conclusions:

> 'Einstein argued that the EPR correlations could be made intelligible by completing the quantum mechanical account in a classical way. Detailed analysis [sic] shows that any classical account of these correlations has to contain just such a 'spooky action at a distance' as Einstein could not believe in ... Einstein's conception of the world is untenable [sic].'

5.8. Thus, against each sic-identified error here, we show that SIR and additional ['hidden'] variables deliver a more complete truly-local-and-realistic Lorentz-invariant specification of EPRB:

> One like EPR advanced, and Bell sought; one that Bell's work no longer precludes.

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7. Appendix A: WM's generalized relations for EPRB and Aspect (2004)

7.1. (i) Given (8), true-local-realism (¶2.4) leads to a generalization. (ii) Combining EPRB and Aspect (2004) under spin s and the unified condition ω (which includes appropriate spin-s polarizer-analyzers), WM derives MLx—Malus' Law extended; a 'fuller [more complete] specification of the law of Malus', Watson (1989d; 2017d)—from first principles:

$$P(A^+B^+|\omega) = P(A^+|\omega)P(B^+|\omega, A^+) = P(B^+|\omega)P(A^+|\omega, B^+) = \tfrac{1}{2}\cos^2 s(a, -b); \text{etc.} \tag{20}$$

7.2. Thus, under EPRB and Aspect (2004), expectations (6) and our EPRB-true (8) generalize to:

$$E(a,b|\omega) = P(A^+B^+|\omega) - P(A^+B^-|\omega) - P(A^-B^+|\omega) + P(A^+B^+|\omega) = (-1)^{2s}\cos 2s(a,b). \tag{21}$$

$$-1 \leq |E(a,b|\omega) - E(a,c|\omega)| + (-1)^{2s}E(b,c|\omega) \leq 3s. \tag{22}$$

8. Appendix B: Steps in the reduction of (3) to (4)

$$\text{Let } -1 \leq X \leq 1, -1 \leq Y \leq 1, \text{then (3) may be written} \pm (X - Y) \leq 1 - XY. \tag{23}$$

$$\therefore \text{(i)}: X - Y \leq 1 - XY. \text{ And (ii)}: Y - X \leq 1 - XY \therefore X - Y \geq -(1 - XY). \tag{24}$$

$$\text{Thus, from (24), (i)-(ii)}: -(1 - XY) \leq X - Y \leq 1 - XY. \tag{25}$$

$$\therefore |X - Y| \leq 1 - XY: \text{ as in (4)}. \tag{26}$$

9. Appendix C: The influence of Bell's key error BE·1 on Bell's results

9.1. Bell's false upper-bound of 1 in (7) needs correction to $\frac{3}{2}$ to be consistent with EPRB; see (8). There is also the consequential range of error in Bell's inequality. Thus—under a typical Bellian/coplanar angular-relation; eg, similar to that in Peres (1995:Fig.6.7)—let

$$(b, c) = (a, c) - (a, b): \text{and let } (a, c) = 3(a, b); \text{ so } (b, c) = 2(a, b). \tag{27}$$

9.2. Then, in this antecedent/consequent example: if Bell's inequality is (7) under EPRB, then it is false over $\frac{2}{3}$ of the range $-\pi < (a, b) < \pi$; ie, in this example, Bell's inequality is EPRB-false (fn.7) for

$$-\pi < (a, b) < \tfrac{2\pi}{3}, \tfrac{\pi}{3} < (a, b) < 0, 0 < (a, b) < \tfrac{\pi}{3}, \tfrac{2\pi}{3} < (a, b) < \pi; \text{ etc.} \tag{28}$$

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[13] In our terms, Bell's theorem is the impossibility claim stated in the line below his 1964:(3).

10. Appendix D: From Bell 1964:(14a) via irrefutable (13) to irrefutable (4)

$$
\begin{align}
\text{Bell 1964:(14a)} &\equiv E(a,b)-E(a,c) \qquad \text{[From (11), with (13) reproduced next.]} \tag{29}\\
&= \textstyle\int d\lambda\rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i[A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i A(a,\boldsymbol{\lambda})_j A(c,\boldsymbol{\lambda})_j - 1] \tag{30}\\
\text{Then} &\ : \ \text{taking absolute values, with } A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i \leq 1. \tag{31}\\
\therefore |E(a,b)-E(a,c)| &\leq \textstyle\int d\lambda\rho(\boldsymbol{\lambda})[1 - A(a,\boldsymbol{\lambda})_i A(b,\boldsymbol{\lambda})_i A(a,\boldsymbol{\lambda})_j A(c,\boldsymbol{\lambda})_j] \tag{32}\\
&\leq 1 - E(a,b)E(a,c)\text{: equates to irrefutable (4), hence (5). QED.} \tag{33}
\end{align}
$$

10.1. (i) We thus confirm a well-known relation: the expectation over the product of two independent and uncorrelated random variables is the product of their individual expectations. (ii) Hence the utility of instance-identifiers—see (10), (13), (30)—in reproducing irrefutable (4). (iii) And thus—correcting Bell's EPRB-false inequality (7)—the need for our EPRB-true inequality (8).

11. Appendix E: Bell's inadequate analysis; ¶5.1 continued

11.0. We say God plays dice, stochastically and deterministically. Einstein agrees.[14]

11.1. nb: (i) Our arguments are not general objections to Bell's interest in researching alternative physical systems: eg, Bell's inequality is certainly compatible with classical locally-causal experiments that Malus could have conducted with paired light-beams—correlated via a common linear-polarization—c.1810. (ii) However—given the equal certainty of BE·1 under EPRB-related experiments—such certainties move us to identify Bell's conceptual-error;[15] say, for convenience here, BCE·1. (iii) So, given that Bell (c.1964) 'became obsessed' with EPR—see Jammer (1974:306)—we now study the extent to which BCE·1 might arise from Bell's interpretation of EPR's *elements of physical reality*:

> 'If, without in any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality *corresponding to* this physical quantity,' EPR (1935:777); our emphasis.

11.2. (i) Now, for us, *'corresponding to'*—which licenses naive-realism here—is too loose for the above *sufficient condition* to be a satisfactory *definition* of an existent (an element of physical reality). (ii) So—based on Watson 2017d:(3)-(7) in spacetime—here's our rephrasing:

> If, without disturbing a particle $\tilde{q}(\boldsymbol{\lambda})$, we can predict with certainty the result A^+ of a test—ie, the value $+1$ when $\tilde{q}(\boldsymbol{\lambda})$ interacts[16] with Alice's polarizer-analyzer $\Delta_a^\pm$—then existents $\boldsymbol{\lambda}$ and $\Delta_a^\pm$ in Alice's space $\mathtt{A}$ *determine* this result; ie, $A^+ = A(a,\boldsymbol{\lambda})_i$ as in (10).

11.3. (i) And here's Bell's (2004:147) related conceptualization: 'To explain this dénouement [of his work] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a).'

> (ii) So here's d'Espagnat (1979:166), recast for EPRB and 11.2(ii), *with our emphasis*: A physicist 'can infer that in every pair, one particle has the property A^+ [a positive spin-component along axis a] and the other has the property A^-. Similarly, he can conclude that in every pair one particle has the property B^+ and one B^-, and one has property C^+ and one C^-. *These conclusions require a subtle but important extension of the meaning assigned to our notation* A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis a would give the definite result A^+, then that particle is

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[14] Re Einstein (c1945), see Weisberger (2019): "God tirelessly plays dice under laws which he has himself prescribed."

[15] ie, BE·1 flows from Bell's use of his 1964:(1): so we now study how such use might flow from EPR and Bell's logic.

[16] Which, under the true-realism of ¶2.4—against the naive d'Espagnat/Bell realism in ¶11.3—may be a disturbance.

> said to have the property A^+. *In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times.* ... This view is contrary to the conventional interpretation of QM, but it is not contradicted by any fact that has yet been introduced.' [nb: *definite spin components at all times* = preexisting.]

11.4. However, to the contrary under true-realism: (i) d'Espagnat's inferences are false; (ii) weaker, more-general, inferences are available; (iii) there's no need to contravene known facts re QM; (iv) and no need to negate Bohr's insight: which—supported by Bell hereunder—bolsters our case against d'Espagnat's 'Bell-endorsed' inferences. [(v) See also Kochen (2015:5): in QM, physicists 'do not believe that the value of the spin component (S_z)—our A^+—exists' prior to the (polarizer) interaction.]

> (vi) Here's Bell (2004: xi-xii): It's "Bohr's insight that the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems [to Bell] that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [like Bell's *impossibility* theorem; see fn.13 above], and most of 'quantum logic'."

11.5. (i) We agree: under true-realism at ¶2.4—*some existents change interactively*—we do not assume that all 'measured' properties already exist prior to 'measurement' interactions. (ii) Thus, under true-realism, we allow that Malus' classical experiments may also involve disturbing interactions between polarizers and light-beams.

11.6. (i) So to be clear, and consistent with Bohr's insight, true-realism goes beyond d'Espagnat/Bell inferences (¶11.3) wherein the 'measured' property is *equated* to a pristine property. (ii) That is—going beyond d'Espagnat's *subtle extension* cited in ¶11.3—we instead infer here to *equivalence under a 'polarizing' operator*; say $\partial_x^{\pm}$. (iii) For *equivalence*—a relation without which science would hardly be possible; a weaker, more general relation than *equality*—is here compatible with QM, Bohr's view, and the consequent need to recognize the effect of 'the means of observation' on EPRB inputs:

> "... the unavoidable interaction between the objects and the measuring instruments sets an absolute limit to the possibility of speaking of a behaviour of atomic objects which is independent of the means of observation," Bohr (1958:25).

11.7. Finally: (i) In stochastic Malusian experiments—with polarization-correlated light-beams, ¶11.1; and appropriate re-alignment/modification of our (EPRB-based) detectors, $\Delta_x^{\pm}$—Bell's inequality (7) would hold: for the related correlations would be one-half those in Aspect (2004).[17] (ii) Further, in settings that allow d'Espagnat/Bell inferences to be valid, Bell's inequality (7) would hold deterministically. (iii) However, more interestingly, and acknowledging David Bohm's genius: in EPRB/Aspect-based settings—dependent on the information that God makes available—as in Watson 2017d:(52)-54):

God plays dice—stochastically and deterministically—in spacetime.

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[17] And thus consistent with our finding that Bell's inequality is OK in such low-correlation settings; eg, see ¶5.3(iv).

12. References

1. Aharonov, Y., E. Cohen and D. Oaknin (2019). "Why physical understanding should precede the mathematical formalism - conditional quantum probabilities as a case-study."
https://arxiv.org/pdf/1902.08798.pdf

2. Aspect, A. (2004). "Bell's theorem: The naive view of an experimentalist."
http://arxiv.org/pdf/quant-ph/0402001v1.pdf

3. Ballentine, L. E. (1998). Quantum Mechanics: A Modern Development. Singapore, World Scientific.

4. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200.
http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

5. Bell, J. S. (1971). "Introduction to the hidden-variable question." *Foundations of Quantum Mechanics*. Proceedings of the International School of Physics 'Enrico Fermi', course IL, New York, Academic: 171-181. Also in Bell (2004:29-39).

6. Bell, J. S. (1990). "Indeterminism and nonlocality." Transcript of 22 January 1990, CERN Geneva. Bell, J. S. (1997). Indeterminism and nonlocality. Mathematical Undecidability, Quantum Nonlocality & the Question of the Existence of God. A. Driessen and A. Suarez. 83-100.
http://www.quantumphil.org./Bell-indeterminism-and-nonlocality.pdf

7. Bell, J. S. (2004). Speakable and Unspeakable in Quantum Mechanics. Cambridge, Cambridge University.

8. Bohm, D. and Y. Aharonov (1957). "Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky." Physical Review 108(4): 1070-1076.

9. Bohr, N. (1958). Atomic Physics and Human Knowledge. New York, John Wiley.

10. Born, M. (1971). The Born-Einstein Letters. Macmillan Press, London.
https://archive.org/details/TheBornEinsteinLetters

11. CHSH (1969). "Proposed experiment to test local hidden-variable theories." Physical Review Letters 23(15): 880-884.

12. Clauser, J. F. & A. Shimony (1978). "Bell's theorem: experimental tests and implications." Reports on Progress in Physics **41**: 1881-1927.

13. d'Espagnat, B. (1979). "The quantum theory and reality." Scientific American 241(5): 158-181.
https://static.scientificamerican.com/sciam/assets/media/pdf/197911_0158.pdf

14. d'Espagnat, B. (1979a). A la Recherche du Réel. Paris, Gauthier-Villars.

15. du Sautoy, M. (2016). What We Cannot Know. London, 4th Estate.

16. EPR (1935). "Can quantum-mechanical description of physical reality be considered complete?" Physical Review 47(15 May): 777-780. http://journals.aps.org/pr/pdf/10.1103/PhysRev.47.777

17. Fröhner, F. H. (1998). "Missing link between probability theory and quantum mechanics: the Riesz-Fejér theorem." Z. Naturforsch. 53a, 637-654.
http://zfn.mpdl.mpg.de/data/Reihe_A/53/ZNA-1998-53a-0637.pdf

18. Griffiths, D. J. (1995). Introduction to Quantum Mechanics. New Jersey, Prentice Hall.

19. Griffiths, D. J. (2005). Introduction to Quantum Mechanics: 2nd ed. New Jersey, Pearson Prentice Hall.

20. Harris, A. (2019). Pers. comm. 2 January. UNE (Armidale NSW Australia).

21. Jammer, M. (1974). The Philosophy of Quantum Mechanics: The interpretations of quantum mechanics in historical perspective. New York, John Wiley & Sons.

22. Kochen, S. (2015). "A reconstruction of quantum mechanics." Foundations of Physics 45(5): 557-590.
https://arxiv.org/pdf/1306.3951.pdf [DA20170328]

23. Levanto, M. J. (2016). "On how Bell 1964:(14b) = Bell 1964:(14a)." http://vixra.org/abs/1403.0089 [201610270728]

24. Mermin, N. D. (1988). "Spooky actions at a distance: mysteries of the quantum theory." The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

25. Motl, L. (2007). "Lecture 36: Entanglement at a distance." https://www.scribd.com/document/80665127/Entanglement-at-a-Distance-L-Motl

26. Oaknin, D. H. (2018). "The Bell's theorem revisited: a subtle, though crucial, assumption has gone unnoticed, and it might not be justified!" www.researchgate.net/publication/328722979

27. Peres, A. (1995). Quantum Theory: Concepts & Methods. Dordrecht, Kluwer Academic.

28. Schlosshauer, M., Ed. (2011). Elegance and Enigma: The Quantum Interviews. Berlin, Springer.

29. Watson, G. S. (1989d). Pers. comm. to N D Mermin, 15 September. "There are no spooky actions at a distance: particles carry instruction sets."

30. Watson, G. S. (2017d.v3). "Bell's dilemma resolved, nonlocality negated, QM demystified, etc." http://vixra.org/abs/1707.0322?ref=10704673 (nb: the Abstract that displays is from v2.)

31. Weisberger. M. (2019). ""God plays dice ..." Einstein writes." Live Science, 12 June. https://www.livescience.com/65697-einstein-letters-quantum-physics.html