Bell's inequality refuted via elementary algebra

Abstract Bell's inequality is widely regarded as a profound impediment to any intuitive understanding of physical reality. We disagree: for elementary algebra allows us to refute Bell's inequality, identify his errors, dismiss his work generally. We thus begin reiterating the anti-Bellian ideas that we've advanced since 1989: ie, we seek to restore commonsense/intuitive ideas to physics and make physical reality intelligible—like Einstein argued, according to Bell—'by completing the quantum mechanical account in a classical way'.

1. Introduction

> 1.0. 'Bell's theorem stands as an insuperable roadblock in the path to a very desired intuitive solution of the Einstein-Podolsky-Rosen [EPR] paradox and, hence, it lies at the core of the current lack of a clear interpretation of the quantum formalism,' Oaknin (2018).

1.1. Seeking a more complete specification of the EPR-Bohm experiment (EPRB), Bell (1964) famously derives an inequality that is quantum-mechanically false under EPRB. This fact surprises many; eg, see Aspect (2004:2).[1] So in this note we use elementary algebra to identify Bell's first error: his reductive assumption that limits Bellian analyses to contexts less correlated than EPRB.

1.2. We also derive an EPRB-based inequality that identifies Bell's second error and refutes his inequality conclusively. It follows that Bellian analysis is irrelevant to any quantum-mechanical issue. For the significance of EPRB is that its common-cause correlations can be explained classically via true local-realism (see ¶1.4):[2] this despite the fact that EPRB's outcomes are said to entail

> 'a kind of correlation of the properties of distant noninteracting systems, which is quite different from previously known kinds of correlation,' Bohm & Aharonov (1957:1070).

1.3. This allows us to address Bell's false conclusions re *locality* and his many related confusions:

> (i). 'And that is the dilemma. We are led by analysing [the EPRB] situation to admit that in somehow distant things are connected, or at least not disconnected.' (ii) 'Maybe someone will just point out that we were being rather silly .... But anyway, I believe the questions will be resolved.' (iii) 'I think somebody will find a way of saying that [relativity and QM] are compatible. But I haven't seen it yet. For me it's very hard to put them together, but I think somebody will put them together, and we'll just see that my imagination was too limited.' (iv) 'I say only that you cannot get away with locality.' After Bell (1990:7,9,10,13).

1.4. For Watson 2019L resolves Bell's dilemma via true local-realism (TLR): the union of true-locality (no influence propagates superluminally) and true-realism (some existents change interactively). So, akin to Fröhner (1998), we seek to advance *commonsense quantum mechanics*—*Wholistic Mechanics* (WM)—the mathematical unification of inferences to the best explanations of observable facts:— beginning with *a progressively updated classical mechanics* that keeps pace with modern findings.

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[1] Key texts are freely available online, see References. Based on EPR (1935), Bell uses additional variables $\boldsymbol{\lambda}$ with Bohm & Aharonov's (1957) example of EPRB. For a related experiment/false-inequality see Aspect (2004)/CHSH (1969).

[2] We use and defend *true* to distinguish our terms from naive or misleading variants; eg, Bell-locality, naive-realism.

1.5. So, siding with Oaknin's problem-definition and implicit ambitions in ¶1.0 for now, our forthcoming notes will refute Bell's theorem and any analysis that produces false conclusions like these:

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant," Bell (1964:199).

1.6. Progressing via clear-cut elementary facts to irrefutable anti-Bellian conclusions, let's see.

2. Analysis

2.1. As in Bell (1964) and in much Bellian theorizing, the context is EPRB. We use E for expectations (not P, which we reserve for probabilities), and a, b, c for Bell's unit-vectors $\vec{a}, \vec{b}, \vec{c}$. Let (14a)-(14c) identify the unlabelled relations between Bell's (14)-(15); the remainder being (15a), (21a)-(21e), (23).

2.2. Thus, from Bell 1964:(1) and anticipating Bell's further needs (p.198), we have the expectations

$$-1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1. \tag{1}$$

$$\therefore E(a,b)[1+E(a,c)] \leq 1+E(a,c);\ \text{ie, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{2}$$

$$\therefore E(a,b) - E(a,c) \leq 1 - E(a,b)E(a,c). \tag{3}$$

$$\text{Similarly: } E(a,c) - E(a,b) \leq 1 - E(a,b)E(a,c). \tag{4}$$

$$\therefore \pm\Big(E(a,b) - E(a,c)\Big) \leq 1 - E(a,b)E(a,c). \tag{5}$$

$$\therefore |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1. \blacksquare \tag{6}$$

2.3. (For the reduction of (5) to (6) see 5.Appendix.) So—holding unconditionally via (1) and elementary algebra alone—(6) is unconditionally as one with EPRB and Aspect (2004). Further—exhausting (1) unconditionally—our irrefutable (6) becomes pairwise generalized and indisputable (7):

$$0 \leq |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1; \text{etc.} \blacksquare \tag{7}$$

2.4. Now, whatever their form, (7) holds for any pair of expectations that are consistent with (1). So (7) holds for the EPRB experiment with spin-$\frac{1}{2}$ particles and Aspect's (2004) experiment[3] with photons (spin $s = 1$) where the related expectations take this form:[4]

$$E(a,b) = (-1)^{2s}\cos 2s(a,b),\ E(a,c) = (-1)^{2s}\cos 2s(a,c),\ E(b,c) = (-1)^{2s}\cos 2s(b,c). \tag{8}$$

2.5. However, note the *sic* that follows. For here's Bell's famous 1964:(15) in a form matching our (6):

$$|E(a,b) - E(a,c)| - E(b,c) \leq\ 1\ [\textbf{sic}]. \blacktriangle \tag{9}$$

2.6. [**sic**]: for—in the context of EPRB; using $s = \frac{1}{2}$ in (8) with (9) for proof by exhaustion—we find[5]

$$-1 \leq |E(a,b) - E(a,c)| - E(b,c) \leq \tfrac{3}{2}. \blacksquare \tag{10}$$

2.7. Thus, under EPRB via (10): for Bell's upper-bound of 1 to hold in (9), the EPRB expectations in (8)—and the related correlations—must be reduced to $\frac{2}{3}$ of their value. So, though supposedly bound by EPRB: Bell's errors [see Table 1] move his analysis to contexts less-correlated than EPRB; ie, to contexts consistent with Bell's false/understated upper-bound of 1 in (9).

[3] Peres (1993:163) derives a different Bell-inequality for each experiment. As shown in our 2018K.v4, each is false.

[4] Under QM: or via Bell 1964:(2) and WM under true local-realism; as we show via EPRB in our 2019L.

[5] Eg: the lower-bound when $(a,b) = \frac{\pi}{2}, (a,c) = \frac{3\pi}{2}, (b,c) = \pi$; the upper-bound when $(a,b) = \frac{\pi}{3}, (a,c) = \frac{2\pi}{3}, (b,c) = \frac{\pi}{3}$. nb: including EPRB and Aspect (2004) in one relation, like (8): $-1 \leq |E(a,b) - E(a,c)| + (-1)^{2s}E(b,c) \leq 3s.$ (10a).

Name of relation	Relation to be evaluated under EPRB	T/F	Ref.
Our inequality #1	$\pm[E(a,b)-E(a,c)]+E(a,b)E(a,c)\leq 1$	T	(5)
Our inequality #2	$0\leq\lvert E(a,b)-E(a,c)\rvert+E(a,b)E(a,c)\leq 1$	T	(7)
Our inequality #3	$-1\leq\lvert E(a,b)-E(a,c)\rvert-E(b,c)\leq {}^3\!/\!_2$	T	(10)
Bell's first error	Using Bell 1964:(1) without reference to EPRB instances	F	*
Bell's second error	LHS Bell 1964:(14c) = RHS Bell 1964:(14c)	F	(11)
Bell's inequality	$\lvert E(a,b)-E(a,c)\rvert-E(b,c)\leq 1$	F	(9)

Table 1: Comparison of relations, and whether true or false, under EPRB. We expose Bell's errors as follows: Bell's inequality (9)—aka Bell's third error—is exposed via our (7) or (10). Bell's second error (11) is exposed because it is the source of (9); see Bell 1964:(14c)-(15). ***** Then, included here for completeness (for the details see Watson 2018K.v4), Bell's first error[6]—see line below Bell 1964:(14b)—is exposed because it is the source of Bell's second error, our (11).

2.8. Then, comparing Bellian (9) with our irrefutable (7), Bell's errors under EPRB are clear. Bell's $E(b,c)$[7] breaches Bell's upper-bound of 1 in (9): while our rigorous $E(a,b)E(a,c)$ cannot breach (7)'s upper-bound of 1. Thus, based as it is on Bell's first error, here is Bell's second error (with certainty):

$$\text{LHS Bell 1964:(14c)} = \text{RHS Bell 1964:(14c)}\ \textbf{[sic]}. \blacktriangle \tag{11}$$

2.9. In other words: having shown with certainty that (9) is false, its source—Bell's second error (11)—is also false. Watson (2018K.v4) confirms these facts by correcting Bell's first error. For there we derive (6) in a different way (but again via first-principles): again refuting (11), but in a new way.

2.10. So, also with certainty: Bellian analysis and Bell's famous (9) are irrelevant to EPRB and QM.[8]

2.11. Finally, to close this analysis, but compounding the extent of Bell's shortfall,[9] there is also the consequential range of error in Bell's inequality (the latter aka Bell's third error). Thus—under a typical Bellian/planar angular-relation; eg, from Peres (1995:Fig.6.7)—let

$$(b,c)=(a,c)-(a,b)\colon \text{and let } (a,c)=3(a,b);\ \text{so } (b,c)=2(a,b). \tag{12}$$

2.12. Then, in this antecedent/consequent example: if Bell's inequality is (9) under EPRB, then it is false over $\frac{2}{3}$ of the range $-\pi<(a,b)<\pi$; ie, in this example, Bell's inequality is EPRB-false for

$$-\pi<(a,b)<\tfrac{2\pi}{3},\tfrac{\pi}{3}<(a,b)<0, 0<(a,b)<\tfrac{\pi}{3},\tfrac{2\pi}{3}<(a,b)<\pi;\ \text{etc.} \tag{13}$$

3. Conclusions and the way ahead

3.0. 'The purpose of this first part is to convince the reader that the *formalism leading to Bell's Inequalities is very general and reasonable.* What is *surprising* is that such a reasonable formalism *conflicts with Quantum Mechanics*,' Aspect (2004:2); his emphasis.

3.1. En route to debunking the supposed roadblock (¶1.0) and resolving Bell's dilemma (¶1.3)—and against Aspect's *generality and reasonableness* (¶3.0)—we confirm the certainty expressed in ¶2.10:

Bell's inequality (9) and Bell-based analyses are irrelevant to EPRB and QM.

[6] Limiting Bell's analysis to settings less correlated than EPRB, this error is discussed and corrected in 2018K.v4.

[7] Introduced via Bell's first error; see comment accompanying Table 1.

[8] Agreeing with Peres 1995:162, 'Bell's theorem is *not* a property of QM.' However, as above, we refute the related Bellian/Peres' claim that 'Bell's theorem applies to any physical system with dichotomic variables, whose values are arbitrarily called 1 and -1.' For it is our (7) that satisfies that claim: not (9), Bell's famous but false 1964:(15) inequality.

[9] From (10): Bell's false upper-bound of 1 in (9) needs correction to $\frac{3}{2}$ to be consistent with EPRB.

3.2. By observation, Bell's first error is identified by the fact that it leads to two further errors: (i) Bell's second error (11); (ii) Bell's inequality (9); see ¶2.8-2.9. For Bell's first error (in a string that leads to his theorem) breaches the crucial EPRB boundary-condition—in our terms, the *same-instance rule* (SIR)—spelt out by Bell in the line before Bell 1964:(1). But then neglected by Bell.

3.3. Thus, in our 2018K.v4, we show that SIR leads to our irrefutable (7): not Bell's inequality (9).

3.4. Then, as future notes will confirm: Bell's logically-false[10]—and thus experimentally-false—results (and CHSH 1969 similarly) have everything to do with neglecting SIR; and nothing at all to do with entanglement, hidden-variables, locality, realism, separability, *spooky* mechanisms (¶1.5).

3.5. Further, supporting Einstein, Watson 2019L: (i) refutes Bell's theorem; (ii) delivers the missing-links—the functions A and B—in Bell 1964:(1); (iii) counters Bell's erroneous (2004:86) conclusions:

> "Einstein argued that the EPR correlations could be made intelligible only by completing the quantum mechanical account in a classical way. But detailed analysis [sic] shows that any classical account of these correlations has to contain just such a 'spooky action at a distance' as Einstein could not believe in ... Einstein's conception of the world is untenable."

3.6. Thus, consistent with Watson 2017D and against Bellian conclusions as in ¶1.5/3.5, we show that SIR and additional variables deliver a more complete truly-local-and-realistic specification of EPRB:

> One like EPR advanced, and Bell sought; one that Bell's erroneous inequality precludes.

3.7. For, in line with Aharonov et al. (2019:1), Watson 2019L first considers 'the governing physical principles before heading to the math ... when exploring [supposed] puzzling quantum phenomena'.

3.8. In sum: (i) Under EPR (1935) we have EPRB and Aspect (2004). (ii) Relatedly, against Bell and under Wholistic Mechanics/true-local-realism (WM/TLR), we have two inequalities (7)/(10) and one set of expectations (8). (iii) So Bell's inequality (9) is certainly false under EPRB: an error in a string of errors—that lead to his false theorem—as we'll show.

4. Acknowledgments It's a pleasure to again thank Roger Mc Murtrie and Diane Jean Fitton for continuing beneficial correspondence.

5. Appendix: Steps in the reduction of (5) to (6).

$$\text{Let } -1 \leq X \leq 1, -1 \leq Y \leq 1, \text{then (5) may be written} \pm (X - Y) \leq 1 - XY. \tag{14}$$

$$\therefore \text{(i)}: X - Y \leq 1 - XY. \text{ And (ii)}: Y - X \leq 1 - XY \therefore X - Y \geq -(1 - XY). \tag{15}$$

$$\text{Thus, from (15), (i)-(ii)}: -(1 - XY) \leq X - Y \leq 1 - XY. \tag{16}$$

$$\therefore |X - Y| \leq 1 - XY :- \text{as in (6)}. \tag{17}$$

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[10] *Logically-false* because Bell's first error limits his *supposed* EPRB analysis to settings less-correlated than EPRB.

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