Bell's inequality refuted via elementary algebra

Abstract Using elementary algebra, this note refutes Bell's famous inequality. Later notes will: (i) identify Bell's error; (ii) deliver the more complete local-realistic specification of the Einstein-Podolsky-Rosen-Bohm experiment that Bell sought; (iii) ensure that (ii) is consistent with quantum theory and experiment; (iv) show that Bell's theorem is false and misleading when it comes to understanding the true locality and true realism associated with Einstein-Podolsky-Rosen correlations; (v) resolve Bell's contradictory 1990 dilemma about 'action-at-a-distance' (and his half-expected silliness) in Einstein's favor.

1. Introduction

1.1. Seeking to go beyond quantum mechanics (QM) and provide a more complete specification of the Einstein-Podolsky-Rosen-Bohm experiment (EPRB),[1] Bell (1964) derives an inequality that is—as is well-known—false in that setting. So in this note we use elementary algebra to derive a valid EPRB-based inequality: one that refutes Bell's inequality and allows us to identify Bell's error conclusively.

1.2. Other notes will show that Bell's inequality and his theorem are not impediments to our provision of a more complete specification of EPRB: one that is (of course) consistent with quantum theory and experiment. We therefore reject most EPR-focussed Bellian theorizing, especially this conclusion:

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant," Bell (1964:199).

2. Analysis

2.1. The context here—the basis for much Bellian theorizing—is Bell's famous 1964 essay (freely available, see References). We use E (not P) for expectations, and a, b, c for Bell's unit-vectors $\vec{a}, \vec{b}, \vec{c}$.

2.2. Thus, from Bell 1964:(1) and anticipating his further needs (p.198), we have the expectations

$$-1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1. \tag{1}$$

$$\therefore E(a,b)[1 + E(a,c)] \leq 1 + E(a,c);\ \text{ie, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{2}$$

$$\therefore E(a,b) - E(a,c) \leq 1 - E(a,b)E(a,c). \tag{3}$$

$$\text{Similarly: } E(a,c) - E(a,b) \leq 1 - E(a,b)E(a,c). \tag{4}$$

$$\therefore\ \pm\,[E(a,b) - E(a,c)] \leq 1 - E(a,b)E(a,c). \tag{5}$$

$$\therefore\ |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1;\ \blacksquare \tag{6}$$

2.3. ie, deriving the key term $|E(a,b) - E(a,c)|$ in Bell's famous inequality—Bell 1964:(15)—via (1) alone, we deliver irrefutable inequality (6): which, unlike Bell's inequality, is consistent with EPRB.

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[1] Bell (1964) uses the EPRB example advocated by Bohm & Aharonov (1957).

2.4. That is, exhausting (1) unconditionally—and thus, of course, embracing EPRB; which, see §1, is the centre-piece of Bell (1964)—our irrefutable inequality (6) becomes

$$0 \leq |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1. \blacksquare \tag{7}$$

2.5. Note that (7) holds for any pair of expectations that are consistent with (1), whatever their form: in particular, it holds for Aspect's (2004) experiment with photons and for the EPRB experiment with spin-$\frac{1}{2}$ particles. Further, under EPRB and thus in the context here, expectations are of this form:

$$E(a,b) = -\cos(a,b),\ E(a,c) = -\cos(a,c),\ E(b,c) = -\cos(b,c). \tag{8}$$

2.6. So—as is well-known, and as we now confirm—Bell's inequality is not consistent with EPRB. For, in a format matching our (6), here is—from Bell 1964:(15)—Bell's famous inequality:

$$|E(a,b) - E(a,c)| - E(b,c) \leq\ 1\ [\textbf{sic}]. \tag{9}$$

2.7. Then, when (8) is exhausted, (9) becomes

$$-1 \leq |E(a,b) - E(a,c)| - E(b,c) \leq \tfrac{3}{2}. \tag{10}$$

2.8. Thus, from (10), Bell's upper bound of 1 [sic] in (9) is exceeded by 50% under EPRB: the culprit clearly being Bell's term $-E(b,c)$ when compared with the $+E(a,b)E(a,c)$ in our (7).

2.9. Further, compounding the magnitude of Bell's error, there is also the range of error. By way of example, here's (9) under a typical Bellian angular relation; eg, from Peres (1995:Fig.6.7),

$$(b,c) = (a,c) - (a,b)\text{: and let our example be based on } (a,c) = 3(a,b). \tag{11}$$

2.10. Then, in this example and under EPRB, Bell's inequality is false over $\frac{2}{3}$ of the range $-\pi < (a,b) < \pi$; ie, Bell's inequality is EPRB-false for

$$-\pi < (a,b) < \tfrac{2\pi}{3}, \tfrac{\pi}{3} < (a,b) < 0, 0 < (a,b) < \tfrac{\pi}{3}, \tfrac{2\pi}{3} < (a,b) < \pi;\ \text{etc.} \tag{12}$$

2.11. Thus, under the microscope of elementary algebra, Bell's inequality is—except in so far as it shows what does not work—largely irrelevant to EPRB: and to EPR (1935) studies in general.

3. Conclusions and the way ahead

3.1. Bell's inequality—algebraically false; cf (9) with (6)—is seriously false under EPRB; see ¶2.8 & 2.10. We thus refute claims[2] like: (i) Bell inequalities are "valid for any set of dichotomic variables;" (ii) Bell inequalities are "an *upper limit to the correlation of distant events* if the principle of local causes is valid." (iii) We will, in another note, refute Bell's *so-called 'principle of local causes'*.

3.2. Further, in the context of EPRB and Bell 1964, (10) joins our (7) as a truism. And neither presents any impediment to our provision of a more complete specification of EPRB:[3] which was—as we noted, and as is too often overlooked—Bell's goal.

3.3. Thus, looking ahead, Watson 2018K derives (6) anew and refutes (9): thereby identifying the contagious error that voids Bell (1964) and much of Bell's EPRB-based theorizing.

[2] See Peres 1995:165 (with his emphasis), which also refers to CHSH (1969).

[3] Drafted in Watson (2017D); to be revised. See Watson 2018L for a more complete specification of Aspect (2004).

3.4. And Watson 2018L refutes Bell's famous theorem in the context of a real experiment, Aspect (2004): thereby delivering a photonic model in 3-space, contrary to Aspect's (2004:5) difficulties. Moreover, in that context—a real experiment—we again refute the Bellian claim [eg, Peres 1995:165; his emphasis] that Bell's inequality is "an *upper limit to the correlation of distant events*, if the principle of local causes is valid."

3.5. We will thus show: (i) Bell's inequality [algebraically false] is experimentally false. (ii) Bell's subsequent theorem is theoretically and experimentally false. (iii) Further—dismissing *spooky-action-at-a-distance* and citing Bell (2004:86)—we complete Einstein's argument that EPR correlations can be "made intelligible only by completing the quantum mechanical account in a classical way."

> 'For on one supposition—*the separability principle*—we absolutely hold fast: the real factual situation [the real physical state] of the system S_2 is independent of what is done with the system S_1, which is spatially separated from S_2,' after Einstein (1949:85).

3.6. Relatedly [as is well-known] and *truly separably* [as is less well-known]: under the correlation associated with EPRB — according to the type of interaction that we have with S_1— we learn an indirectly [ie, a non-causatively] correlated fact about S_2 under a similar interaction.

3.7. Thus, based on the supposition of true separability, our truly local and truly realistic hidden-variable theory refutes the Bellian conclusion cited in ¶1.2 and these further claims:

> Here's Bell, confused about the physical significance of his theorem: (i) '... I cannot say that action at a distance [AAD] is required in physics. I can say that you cannot get away with no AAD. You cannot separate off what happens in one place and what happens in another.' (ii) 'And that is the dilemma. We are led by analysing this situation to admit that in somehow distant things are connected, or at least not disconnected.' (iii) '... the Einstein program fails [sic]. ... it might be that we have to learn to accept not so much AAD, but [the] inadequacy of no AAD.' (iv) 'And that is the dilemma. ... I step back from asserting that there is AAD, I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood,' after Bell (1990:5,6,7,13).
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> "If nature follows quantum mechanics in these correlations [which she does; as we show, independent[4] of QM], then Einstein's conception of the world is untenable," Bell (2004:86).

3.8. For, without question (as we'll show), special relativity holds—undiminished—in our theory, under EPRB and throughout QM: with resolutions of the 'collapse' and 'measurement' problems being fringe-benefits of our approach.

3.9. Thus, as introduced in §1, we show that much Bellian theorizing is irrelevant to any understanding of the true locality and true realism associated with EPR-correlated outcomes. For, after Watson 2017D:

> (i) Our *true locality* insists that no influence propagates superluminally. (ii) Our *true realism* allows that some existents may change interactively; to thus specifically amend and avoid the *naive-realism* that might be associated with this: "Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone," Clauser & Shimony (1978:1883).

[4] Though our ideas and formalisms differ markedly from QM, we nevertheless show that QM is better founded than many in the 'quantum-foundations' community believe: QM being essentially the proper probability formalism for systems that obey the classical (Hamiltonian) canonical equations; see Fröhner (1998).

3.10. We thus resolve another Bellian dilemma; a source of much confusion. For, under the above conditions and via probability theory: *indirectly correlated outcomes* (as in EPRB and Aspect 2004) — ie, paired outcomes indirectly correlated via Bell's 1964:(1) paired $\boldsymbol{\lambda}$s, *consistent with true locality, true realism and true separability* — are those in which the general product rule

$$P(XY\,|\,\psi) \;=\; P(X\,|\,\psi)P(Y\,|\,\psi X) \text{ does not reduce} \tag{13}$$

$$\text{to the special product rule } P(XY\,|\,\phi) \;=\; P(X\,|\,\phi)P(Y\,|\,\phi): \tag{14}$$

where the conditions ψ and ϕ are included to show that (in our work) all probability assignments are conditional.

3.11. In sum, we deliver the answers that Bell half-expected:

> 'Now, its my feeling that all this action-at-a-distance and no action-at-a-distance business will go the same way [as the ether]. That someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we [Bellians] were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved,' after Bell (1990:9).

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