Abstract
Elementary algebra refutes Bell’s famous inequality conclusively.

1. Introduction

1.1. The context is John Bell’s famous 1964 essay (freely available, see §4-References). We use \( E \) (not \( P \)) for Bell’s expectation-values, and \( a, b, c \) for Bell’s unit-vectors \( \vec{a}, \vec{b}, \vec{c} \).

1.2. We here refute Bell’s inequality. We thus show that it is not an impediment to our provision of a more complete specification of the Einstein-Podolsky-Rosen-Bohm experiment (EPRB). Nor to our refutation of Bell’s related theorem [see the line below Bell 1964:(3)] and his conclusion:

“In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant,” Bell (1964:199).

2. Analysis

2.1. From Bell 1964:(1)-(2), we have

\[
-1 \leq E(a, b) \leq 1, 
-1 \leq E(a, c) \leq 1, 
-1 \leq E(b, c) \leq 1.
\]

\[
\therefore E(a, b)[1 + E(a, c)] \leq 1 + E(a, c).
\]

\[
\therefore E(a, b) - E(a, c) \leq 1 - E(a, b)E(a, c).
\]

Similarly:

\[
E(a, c) - E(a, b) \leq 1 - E(a, b)E(a, c).
\]

\[
\therefore \pm |E(a, b) - E(a, c)| \leq 1 - E(a, b)E(a, c).
\]

\[
\therefore |E(a, b) - E(a, c)| + E(a, b)E(a, c) \leq 1. \quad \Box
\]

2.2. Then, for comparison with irrefutable (6), here’s Bell’s famous inequality, Bell 1964:(15):

\[
|E(a, b) - E(a, c)| - E(b, c) \leq 1 [\text{sic}].
\]

2.3. So, comparing (7) with (6), Bell 1964:(15) is algebraically false: and seriously false, for

\[
|E(a, b) - E(a, c)| - E(b, c) > 1. \quad \Box
\]

2.4. That is, allowing the expectation values in (1) to range from \(-1\) to \(1\) over \([0, \pi]\) via the proxies

\[
E(a, b) = -\cos(a, b), \ E(a, c) = -\cos(a, c), \ E(b, c) = -\cos(b, c)
\]

[which are consistent with quantum theory] then Bell’s inequality is seriously false whenever

\[
|\cos(a, c) - \cos(a, b)| + \cos(b, c) > 1.
\]

2.5. Or, using (10) with an angular relation commonly found in Bell-studies [eg, Peres (1995:Fig.6.7)],

\[
(b, c) = (a, c) - (a, b) : \text{and, say, with } (a, c) = 3(a, b),
\]

then, in this example, Bell’s inequality is false over 66% of the range \(-\pi < (a, b) < \pi\); to wit,

\[
-\pi < (a, b) < \frac{2\pi}{3}, \quad \frac{\pi}{3} < (a, b) < 0, 0 < (a, b) < \frac{\pi}{3}, \quad \frac{2\pi}{3} < (a, b) < \pi; \text{ etc.}
\]

Corresponding author: eprb@me.com  Subject line: 2018J.v3.
3. Conclusions and the way ahead

3.1. Bell’s inequality [algebraically false; cf (7) with (6)], is seriously false under EPRB; see (12).

3.2. Further, exhausting (1), our inequality (6) becomes

\[ 0 \leq |E(a, b) - E(a, c)| + E(a, b)E(a, c) \leq 1; \]  \hspace{1cm} \text{(13)}

to be compared with Bell’s inequality (7), amended under (11) and the same exhaustion,

\[ -1 \leq |E(a, b) - E(a, c)| - E(b, c) \leq \frac{3}{2}. \]  \hspace{1cm} \text{(14)}

3.3. Thus, in the context of EPRB and Bell 1964, (14) joins our (13) as a truism. And neither presents any impediment to our provision of a more complete specification of EPRB. Nor to our consequent refutation of Bell’s related theorem.

3.4. Nor to our consequent completion—without *spooky-action-at-a-distance*—of Einstein’s argument that EPR correlations can be “made intelligible only by completing the quantum mechanical account in a classical way,” Bell (2004:86).

‘For on one supposition we should absolutely hold fast: the real factual situation of the system \( S_2 \) is independent of what is done with the system \( S_1 \), which is spatially separated from the former,’ after Einstein (1949:85).

3.5. For, based on that supposition, our local hidden-variable theory refutes this:

“If nature follows quantum mechanics in these correlations [which she does], then Einstein’s conception of the world is untenable,” Bell (2004:86).

4. References


