The new bivalent, three-valued logic VŁ3

Abstract: We build and test a new bivalent three-valued logic named VŁ3. Recent advances are support of the classical tautologies, modal definitions, the law of excluded fourth, and extended contradiction principle.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q : A, B;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ¢ ; < Not Imply, less than, ∈;
= Equivalent, ≡, ¨ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(%p<#p) C as contingency; (p=p) T as tautology; (p@p) **F** as contradiction;
~(y < x) (x ≤ y), (x ⊆ y).

From: en.wikipedia.org/wiki/Three-valued_logic

We build bivalent truth tables for the connectives Or, And, Imply, and Equivalent using the 2-tuple in bits as: 00 False (contradiction); 01,10 (unknown); and 11 Tautology (designated truth value).

+	**00**	**01,10**	**11**	
00	00	01,10	11	
01,10	01,10	01,10,11	11	
11	11	11	11	(1.1)

&	**00**	**01,10**	**11**	
00	00	00,00	00	
01,10	00,00	00,01,10	01,10	
11	00	01,10	11	(1.2)

>	**00**	**01,10**	**11**	
00	11	11,11	11	
01,10	10,01	11,10,01	11	
11	00	01,10	11	(1.3)

=	**00**	**01,10**	**11**	
00	11	00,00	00	
01,10	00,00	00,11	00,00	
11	00	00,00	11	(1.4)

We rewrite Eqs. 1 removing: 01,10 for U.

+	F	U	T	
F	F	U	T	
U	U	U,T	T	
T	T	T	T	(2.1)

&	F	U	T	
F	F	F,F	F	
U	F,F	F,U	U	
T	F	U	T	(2.2)

>	F	U	T	
F	T	T,T	T	
U	U	T,U	T	
T	F	U	T	(2.3)

=	F	U	T	
F	T	F,F	F	
U	F,F	F,T	F,F	
T	F	F,F	T	(2.4)

We rewrite Eqs. 2 by removing: x, U for U; T,T for T; F,F for F; F,T for U.

+	F	U	T	
F	F	U	T	
U	U	U	T	
T	T	T	T	(3.1)

&	F	U	T	
F	F	F	F	
U	F	U	U	
T	F	U	T	(3.2)

>	F	U	T	
F	T	T	T	
U	U	U	T	
T	F	U	T	(3.3)

=	F	U	T	
F	T	F	F	
U	F	U	F	
T	F	F	T	(3.4)

	~	
F	T	
U	U	
T	F	(3.5)

We evaluate two classical tautologies usually falsified by common three-valued logic systems (L3):

A+~A (4.1)

p+~p ; TTTT TTTT TTTT TTTT (4.2)

~(A&~A) (5.1)

~(p&~p) = (p=p) ; TTTT TTTT TTTT TTTT (5.2)

We evaluate three classical tautologies:

$A \vee B = (A \rightarrow B) \rightarrow B$ (6.1)

(p+q)=((p>q)>q) ; TTTT TTTT TTTT TTTT (6.2)

$A \wedge B = \neg(\neg A \vee \neg B)$ (7.1)

(p&q)=~(~p+~q) ; TTTT TTTT TTTT TTTT (7.2)

$A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$ (8.1)

(p=q)=((p>q)&(q>p)) ; TTTT TTTT TTTT TTTT (8.2)

We evaluate three modal definitions:

$\mathbf{M}A = \neg A \rightarrow A$, corrected as: (9.1)

%p=(#~p>#p) ; TTTT TTTT TTTT TTTT (9.2)

$\mathbf{L}A = \neg\mathbf{M}\neg A$ (10.1)

#p=~%~p ; TTTT TTTT TTTT TTTT (10.2)

$\mathbf{I}A = \mathbf{M}A \wedge \neg\mathbf{L}A$,
with **I**A meaning "it is contingent that" (11.1)

(%p<#p)=(%p&~#p) ; TTTT TTTT TTTT TTTT (11.2)

We also evaluate:

$A \vee \mathbf{I}A \vee \neg A$ (law of excluded fourth) (12.1)

(p+((%p>#p)=(#p&~%p)))+~p ; TTTT TTTT TTTT TTTT (12.2)

$\neg(A \wedge \neg\mathbf{I}A \wedge \neg A)$ (extended contradiction principle) (13.1)

~((p+((%p>#p)=(#p&~%p)))+~p) = (p@p) ;
TTTT TTTT TTTT TTTT (13.2)

Eqs. 1-13 are tautologous, confirming VŁ3 as a bivalent, three-valued logic in support of five classical tautologies, three modal definitions, the law of excluded fourth, and extended contradiction principle.