Beal Conjecture Convincing Proof

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Using a direct construction approach, the author proved the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. By applying numerical examples,, it is shown that one can begin with the sum $A^x + B^y$ and change this sum to a product and then to the single power, C^z. It is concluded that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. It was shown that if $A^x + B^y = C^z$, then A, B and C have a common prime factor.

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Process and Requirements Involved in Changing the Sum of Two Powers to a Single Power

The necessary requirement is that the two powers must have a common power. If this requirement is not satisfied, the sum of the two powers cannot be changed to a product. and to a single power.

Step 1: Change the sum of the two powers to a product. If the two powers do not have a common power. (and consequently, a common prime factor). you cannot proceed. Any product obtained also has the same common prime factor as the sum of the powers

Step 2: Change the product to a single power.

Example 1: $\boxed{2^3 + 2^3 = 2^4}$ $A = 2,\ B = 2,\ C = 2,\ x = 3,\ y = 3,\ z = 4;\ A^x + B^y = C^z$.

Change the sum $2^3 + 2^3$ to a single power of 2.

Factor out the greatest common factor. $2^3 + 2^3$ $= 2^3(1+1)$ (G) <--------------------------------	This step requires that 2^3 and 2^3 have a common prime factor
$= 2^3(2)$ $= 2^4$ Note that if $2^3 + 2^3$ did not have any common factor, one could not factor, and one will not be able write the sum as a product and subsequently change the product to power form.	It is interesting how the "(1+1)" provided the much needed 2.

The 2^4 must have a common factor as 2^3 and 2^3**, f**rom which it was obtained.. From above, the common prime factor is 2,

Example 2 $\boxed{7^6 + 7^7 = 98^3}$ $A = 7,\ B = 7,\ C = 98,\ x = 6,\ y = 7,\ z = 3,\ A^x + B^y = C^z$

Change the sum $7^6 + 7^7$ to a single power of 98.

Factor out the greatest common factor. $7^6 + 7^7$ $= 7^6(1+7)$ (G)<-------------------	This step requires that 7^6 and 7^7 have a common prime factor
$= 7^6(8)$ $= 7^6(2^3)$ $= (7^2)^3(2^3)$ $= (7^2 \bullet 2)^3$ $= (49 \bullet 2)^3$ $= (98)^3$ $= 98^3$	It is interesting how the "(1+ 7)" provided the much needed 2^3.

Since 98^3 was obtained from the sum $7^6 + 7^7$, which has a common prime factor. 7, 98^3 has the same common prime factor, 7, Therefore 7^6, 7^7 and 98^3 have the common prime factor of 7.

Example 3: $\boxed{3^3 + 6^3 = 3^5}$ $A = 3,\ B = 6,\ C = 3,\ x = 3,\ y = 3,\ z = 5,\ A^x + B^y = C^z$

Change the sum $3^3 + 6^3$ to a single power of 3..

Factor out the greatest common factor.	
$3^3 + 6^3$	
$= 3^3 + (3 \bullet 2)^3$	
$= 3^3 + 3^3 \bullet 2^3$	
$= 3^3(1 + 2^3)$ (G)<--------------------	This step requires that 3^3 and 6^3 have a common prime factor
$= 3^3(1 + 8)$	
$= 3^3(9)$	It is interesting how the "(1+ 8)" provided
$= 3^3 \bullet 3^2$	the much needed 3^2.
$= 3^5$	

Since 3^5 was obtained from the sum $3^3 + 6^3$, which has a common prime factor. 3, 3^5 has the same common prime factor, 3,

Example 4 $\boxed{2^9 + 8^3 = 4^5}$ $A = 2,\ B = 8,\ C = 4,\ x = 9,\ y = 3,\ z = 5,\ A^x + B^y = C^z$

Change the sum $2^9 + 8^3$ to a single power of 4.

Factor out the greatest common factor.	
$2^9 + 8^3$	
$= 2^9 + (2^3)^3$	
$= 2^9 + 2^9$	This step requires that 2^9 and 8^3
$= 2^9(1 + 1)$ (G)<----------------------	have a common prime factor
$= 2^9 \bullet 2$	
$= 2^{10}$	It is interesting how the "(1+ 1)" provided
$= (2^2)^5$	the much needed 2.
$= (4)^5$	
$= 4^5$	

Since 4^5 was obtained from the sum $\boxed{2^9 + 8^3}$, which has a common prime factor. 2, 4^5 has the same common prime factor, 2,

Example 5 $\boxed{34^5 + 51^4 = 85^4}$ $A = 34,\ B = 51,\ C = 85,\ x = 5,\ y = 4,\ z = 4,\ A^x + B^y = C^z$

Change the sum $34^5 + 51^4$ to a single power of 85.

Factor out the greatest common factor.	
$34^5 + 51^4$	
$= (17 \bullet 2)^5 + (17 \bullet 3)^4$	
$= 17^5 \bullet 2^5 + 17^4 \bullet 3^4$	
$= 17^4(17 \bullet 2^5 + 3^4)$ (G) <--------------	This step requires that 34^5 and 51^4 have a common prime factor
$= 17^4(17 \bullet 32 + 81)$	
$= 17^4(625)$	It is interesting how the
$= 17^4(5^4)$	$\underbrace{17 \bullet 2^5 + 3^4}_{\text{magic}}$ provided the much needed
$= (17 \bullet 5)^4$	
$= 85^4$	$625 = 5^4$

Since 85^4 was obtained from 34^5 and 51^4 which have the common prime factor , 17, 85^4 has the same common factor, 17.

Example 6: $\boxed{3^9 + 54^3 = 3^{11}}$ $A = 3,\ B = 54,\ C = 3,\ x = 9,\ y = 3,\ z = 11$, $A^x + B^y = C^z$

Change the sum $3^9 + 54^3$ to a single power of 3.

Factor out the greatest common factor.	
$3^9 + 54^3$	
$= 3^9 + (9 \bullet 6)^3$	
$= 3^9 + (3 \bullet 3 \bullet 3 \bullet 2)^3$	
$= 3^9 + (3^3 \bullet 2)^3$	
$= 3^9 + 3^9 \bullet 2^3$	This step requires that 3^9 and 54^3 have a common prime factor
$= 3^9(1 + 2^3)$ (G) <----------------------	
$= 3^9(1 + 8)$	It is interesting how the $1 + 2^3$ provided the much needed 9.
$= 3^9(9)$	.
$= 3^9 \bullet 3^2$	
$= 3^{11}$	

Since 3^{11} was obtained from 3^9 and 54^3 which have the common prime factor , 3, 3^{11} has the common factor 3.

Example 7: $\boxed{33^5 + 66^5 = 33^6}$ $A = 33,\ B = 66,\ C = 33,\ x = 5,\ y = 5,\ z = 6$, $A^x + B^y = C^z$

Change the sum $33^5 + 66^5$ to a single power of 33..

Factor out the greatest common factor.	
$33^5 + 66^5$	
$= (11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5$	
$= 11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5$	This step requires that 33^5 and 66^5 have a common prime factor
$= 11^5 \bullet 3^5(1 + 2^5)$ (G) <---------------------	
$= (11 \bullet 3)^5(1 + 2^5)$	
$= 33^5(33)$	It is interesting how the $1 + 2^5$ provided the much needed 33
$= 33^6$	

Similary, as from above, 33^6 has the common prime factors 3 and 11

Proof and Conclusion

Based on the above examples, (Examples 1-7). it can be observed that A and B must have a common factor (a prime factor), otherwise, the sum $A^x + B^y$ cannot be changed to a product such that A, B, x, y are positive integers and $x, y, > 2$, and subsequently to a single power of C.
Step (G) in each example requires that A and B have a common power. Since C is derived from $A^x + B^y$, C will have the same common factor as $A^x + B^y$, Therefore, without $A^x + B^y$ with a common factor, there would be no C. Note in the examples that C is derived solely from the sum $A^x + B^y$. Thus to derive C, A and B must have a common prime factor, and if C is derived from $A^x + B^y$ with a common prime factor, C will also have the same common prime factor.

Therefore if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

PS

Other proofs of Beal Conjecture by the author are at viXra:1702.0331; viXra:1609.0383, viXra:1609.0157.

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