Beal Conjecture Original Proved

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Using a direct construction approach, the author proved the original Beal conjecture that if $A^x + B^y = C^z$, where $A,B,C,x,y,z$ are positive integers and $x,y,z > 2$, then $A$, $B$ and $C$ have a common prime factor. In the proof, using concrete examples, one begins with $A^x + B^y$ and changes this sum to a product and to a single power, $C^z$. It was determined that if $A^x + B^y = C^z$, then $A$, $B$ and $C$ have a common prime factor. The proof is very simple, and occupies a single page.
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**Process and Requirements Involved in Changing the Sum of Two Powers to a Single Power**

The necessary requirement is that the two powers must have a common power. If this requirement is not satisfied, the sum of the two powers cannot be changed to a product and to a single power.

Step 1: Change the sum of the two powers to a product. If the two powers do not have a common power (and consequently, a common prime factor), you cannot proceed. Any product obtained also has the same common prime factor as the sum of the powers.

Step 2: Change the product to a single power.

**Example 1:** \(2^3 + 2^3 = 2^4\)

Change the sum \(2^3 + 2^3\) to a single power of 2.

Factor out the greatest common factor.

\[
2^3 + 2^3 = 2^3(1 + 1) \quad \text{(G)} \quad \text{------------------------} \\
= 2^3(2) \\
= 2^4
\]

This step requires that \(2^3\) and \(2^3\) have a common prime factor. It is interesting how the "(1+1)" provided the much needed 2.

Note that if \(2^3 + 2^3\) did not have any common factor, one could not factor, and one will not be able to write the sum as a product and subsequently change the product to power form.

The \(2^4\) must have a common factor as \(2^3\) and \(2^3\), from which it was obtained.

From above, the common prime factor is 2.

**Example 2** \(7^6 + 7^7 = 98^3\)

Change the sum \(7^6 + 7^7\) to a single power of 98.

Factor out the greatest common factor.

\[
7^6 + 7^7 = 7^6(1 + 7) \quad \text{(G)} \quad \text{------------------------} \\
= 7^6(8) \\
= 7^6(2^3) \\
= (7^2)^3(2^3) \\
= (49 \cdot 2)^3 \\
= (98)^3 \\
= 98^3
\]

This step requires that \(7^6\) and \(7^7\) have a common prime factor. It is interesting how the "(1+7)" provided the much needed \(2^3\).

Since \(98^3\) was obtained from the sum \(7^6 + 7^7\), which has a common prime factor, \(7\), \(98^3\) has the same common prime factor, \(7\). Therefore \(7^6\), \(7^7\) and \(98^3\) have the common prime factor of \(7\).
### Example 3: $3^3 + 6^3 = 3^5$
Change the sum $3^3 + 6^3$ to a single power of $3$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$3^3 + 6^3$</td>
</tr>
<tr>
<td>2.</td>
<td>$3^3 + (3 \cdot 2)^3$</td>
</tr>
<tr>
<td>3.</td>
<td>$3^3 + 3^3 \cdot 2^3$</td>
</tr>
<tr>
<td>4.</td>
<td>$3^3(1 + 2^3)$  \hspace{1cm} (G) \hspace{1cm} &lt;------------------------</td>
</tr>
<tr>
<td>5.</td>
<td>$3^3(1 + 8)$</td>
</tr>
<tr>
<td>6.</td>
<td>$3^3(9)$</td>
</tr>
<tr>
<td>7.</td>
<td>$3^3 \cdot 3^2$</td>
</tr>
<tr>
<td>8.</td>
<td>$3^5$</td>
</tr>
</tbody>
</table>

This step requires that $3^3$ and $6^3$ have a common prime factor.

It is interesting how the "$(1+ 8 )$" provided the much needed $3^2$.

### Example 4: $2^9 + 8^3 = 4^5$
Change the sum $2^9 + 8^3$ to a single power of $4$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$2^9 + 8^3$</td>
</tr>
<tr>
<td>2.</td>
<td>$2^9 + (2^3)^3$</td>
</tr>
<tr>
<td>3.</td>
<td>$2^9 + 2^9$</td>
</tr>
<tr>
<td>4.</td>
<td>$2^9(1 + 1)^5$  \hspace{1cm} (G) \hspace{1cm} &lt;------------------------</td>
</tr>
<tr>
<td>5.</td>
<td>$2^9 \cdot 2$</td>
</tr>
<tr>
<td>6.</td>
<td>$2^{10}$</td>
</tr>
<tr>
<td>7.</td>
<td>$(2^5)^5$</td>
</tr>
<tr>
<td>8.</td>
<td>$(4)^5$</td>
</tr>
<tr>
<td>9.</td>
<td>$4^5$</td>
</tr>
</tbody>
</table>

This step requires that $2^9$ and $8^3$ have a common prime factor.

It is interesting how the "$(1+ 1)$" provided the much needed $2$.

### Example 5: $34^5 + 51^4 = 85^4$
Change the sum $34^5 + 51^4$ to a single power of $85$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$34^5 + 51^4$</td>
</tr>
<tr>
<td>2.</td>
<td>$(17 \cdot 2)^5 + (17 \cdot 3)^4$</td>
</tr>
<tr>
<td>3.</td>
<td>$17^5 \cdot 2^5 + 17^4 \cdot 3^4$</td>
</tr>
<tr>
<td>4.</td>
<td>$17^4(17 \cdot 2^5 + 3^4)$  \hspace{1cm} (G) \hspace{1cm} &lt;------------------------</td>
</tr>
<tr>
<td>5.</td>
<td>$17^4(17 \cdot 32 + 81)$</td>
</tr>
<tr>
<td>6.</td>
<td>$17^4(625)$</td>
</tr>
<tr>
<td>7.</td>
<td>$17^4(5^4)$</td>
</tr>
<tr>
<td>8.</td>
<td>$(17 \cdot 5)^4$</td>
</tr>
<tr>
<td>9.</td>
<td>$85^4$</td>
</tr>
</tbody>
</table>

This step requires that $34^5$ and $51^4$ have a common prime factor.

It is interesting how the $17 \cdot 2^5 + 3^4$ provided the much needed magic $625 = 5^4$.

Since $85^4$ was obtained from $34^5$ and $51^4$ which have the common prime factor $17$, $85^4$ has the same common factor, $17$.
Example 6: $3^9 + 54^3 = 3^{11}$

Change the sum $3^9 + 54^3$ to a single power of 3.

Factor out the greatest common factor.

\[
3^9 + 54^3 = 3^9 + (9 \cdot 6)^3 = 3^9 + (3 \cdot 3 \cdot 3 \cdot 2)^3 = 3^9 + (3^3 \cdot 2)^3 = 3^9 + 3^9 \cdot 2^3 = 3^9(1 + 2^3)
\]

\[= (G) 3^9(1 + 8) = 3^9(9) = 3^9 \cdot 3^2 = 3^{11}\]

This step requires that $3^9$ and $54^3$ have a common prime factor.

It is interesting how the $1 + 2^3$ provided the much needed $9$.

Since $3^{11}$ was obtained from $3^9$ and $54^3$ which have the common prime factor, $3$, $3^{11}$ has the common factor $3$.

Example 7: $33^5 + 66^5 = 33^6$

Change the sum $33^5 + 66^5$ to a single power of 33.

Factor out the greatest common factor.

\[
33^5 + 66^5 = (11 \cdot 3)^5 + (11 \cdot 2 \cdot 3)^5 = 11^5 \cdot 3^5 + 11^5 \cdot 2^5 \cdot 3^5 = 11^5 \cdot 3^5(1 + 2^5)
\]

\[= (G) 11^5(1 + 2^5) = 33^5(33) = 33^6\]

This step requires that $33^5$ and $66^5$ have a common prime factor.

It is interesting how the $1 + 2^5$ provided the much needed $33$.

Proof and Conclusion

Based on the above examples, (Examples 1-7), it can be observed that A and B must have a common factor (a prime factor), otherwise, the sum $A^x + B^y$ cannot be changed to a product such that $A, B, C, x, y, z$ are positive integers and $x, y, z > 2$, and subsequently to a single power of $C$.

Step (G) in each example requires that A and B have a common power, since C is produced from $A^x + B^y$, C will have the same common factor as $A^x + B^y$. Therefore, without $A^x + B^y$ with a common factor, there would be no C. Note in the examples that C is produced solely from the sum $A^x + B^y$. Thus to produce C, A and B must have a common prime factor, and if C is produced from $A^x + B^y$ with a common prime factor, C will also have the same common prime factor.

Therefore if $A^x + B^y = C^z$, where $A, B, C, x, y, z$ are positive integers and $x, y, z > 2$, then A, B, and C have a common prime factor.

PS

Other proofs of Beal Conjecture by the author are at viXra:1702.0331; viXra:1609.0383, viXra:1609.0157.

Adonten