Evaluation of the Numerical Values of the Normalization Parameter $a$ and the Distance Scale $\alpha$.

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Here we will determine the numerical values of $a$ and $\alpha$ in order to determine with more precisions the impact of dark energy in the vicinity of a black hole.

We know that the metric of a Schwarzschild black in the quintessence field is given by

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

with

$$g(r) = 1 - \frac{2GM}{c^2} \frac{1}{r} - \frac{a}{r^{3\epsilon+1}}.$$  

Then, for the reason of homogeneities, we can think that $a$ is proportional to a power of radius. So, at the event horizon, we have the normalization factor that way

$$a \propto r_h^{3\epsilon+1}.$$  

Then, if the proportional factor is 1, we could have

$$a = r_h^{3\epsilon+1} = \left(\frac{2GM}{c^2}\right)^{3\epsilon+1}.$$  

More over we have written for the distance scale $\alpha$ that

$$\alpha \propto a^{\frac{1}{3\epsilon+1}}.$$  

Now, we have to determine the true range of $a$ and $\alpha$. For that, we know that

$$-1 < \epsilon \leq -0.30$$  

$$G = 6.67408 * 10^{-11} m^3 kg^{-1} s^{-2}$$  

$$C = 299792458 m s^{-1}$$  

For a stellar black hole with the mass $M = 4M_{sun} = 7.9564 * 10^{30} kg$, we have this board

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$a$</th>
<th>$\alpha$ (metre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.99</td>
<td>9.4881 * 10^{-9}</td>
<td>11816.7442</td>
</tr>
<tr>
<td>-0.80</td>
<td>1.9884 * 10^{-6}</td>
<td>11816.7302</td>
</tr>
<tr>
<td>-0.70</td>
<td>3.3132 * 10^{-5}</td>
<td>11816.8612</td>
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<tr>
<td>-0.50</td>
<td>9.1992 * 10^{-3}</td>
<td>11816.7998</td>
</tr>
<tr>
<td>-0.40</td>
<td>1.5328 * 10^{-1}</td>
<td>11818.7771</td>
</tr>
<tr>
<td>-0.30</td>
<td>2.5542</td>
<td>11818.1366</td>
</tr>
</tbody>
</table>

For this board we see that for any value of $\epsilon$ in the range $[-1, -0.30]$, we have some specific values of the normalization parameter $a$ and the distance scale $\alpha$, permitting us to appreciate better the influence of the dark energy in the vicinity of black holes.

Bibliography

Ndongmo et al, Black Hole Mass Decreasing, The power and The Time of Two Black Holes in Coalescence  viXra:1811.0097