Confirmation of the Łukasiewicz Square of Opposition via logic VŁ4

Abstract: We evaluate the existential import of the Revised Modern Square of Opposition. We confirm that the Łukasiewicz syllogistic was intended to apply to *all* terms. What follows is that Aristotle was mistaken in his mapping of vertices, which we correct and show fidelity to Aristotle's intentions. We also evaluate the Cube of Opposition of Seuren. Two final claims are not tautologous, hence refuting the Cube, which also contradict criticism of Seuren that was not based on those claims.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y); (p=p) Tautology.

See: Read, S. (2015). Aristotle and Łukasiewicz on Existential Import.
st-andrews.ac.uk/~slr/Existential_import.pdf slr@st-andrews.ac.uk

We map vertices of the first Square of Opposition on page 4 with its words below.

(A)	Every S is P.	#(s= p)=(p=p) ;	NFNF NFNF FNFN FNFN	(0.1.2)
(E)	No S is P.	#(s=~p)=(p=p) ;	FNFN FNFN NFNF NFNF	(0.3.2)
(I)	Some S is P.	%(s= p)=(p=p) ;	TCTC TCTC CTCT CTCT	(0.5.2)
(O)	Not every S is P.	%(~s=p)=(p=p) ;	CTCT CTCT TCTC TCTC	(0.7.2)

Remark 0: The above is from our *revised* Modern Square of Opposition as published.

We map the relations which Aristotle accepts as preserved here.

A- and E-propositions are contrary (cannot both be true) [(A)=T & (E)=T] (1.1.1)

(#(s= p)=(p=p))&(#(s=~p)=(p=p)) ; FFFF FFFF FFFF FFFF (1.1.2)

and I- and O-propositions are subcontrary (cannot both be false) [(I)=**F** & (O)=**F**] (1.2.1)

(%(s= p)=(p@p))&(%(s=~p)=(p@p)) ; FFFF FFFF FFFF FFFF (1.2.2)

A- and O-propositions are contradictories, [(A)&(O)] (2.1.1)

#(s= p)&%(s=~p) ; FFFF FFFF FFFF FFFF (2.1.2) as are I- and E-propositions [(I) & (E)] (2.2.1)

%(s= p)&#(s=~p) ; **FFFF FFFF FFFF FFFF** (2.2.2)

A-propositions imply their subaltern I-proposition, [(A) > (I)] (3.1.1)

#(s= p)>%(s= p) ; TTTT TTTT TTTT TTTT (3.1.2)

and E-propositions their subaltern O-proposition [(E) > (O)] (3.2.1)

#(s=~p)>%(s=~p) ; TTTT TTTT TTTT TTTT (3.2.2)

I- propositions convert simply ‘Some *S* is *P* ’ implies ‘Some *P* is *S*’, (4.1.1)

%(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (4.1.2)

and E-propositions ‘No *S* is *P* ’ implies ‘No *P* is *S*’ (4.2.1)

#(~s=p)>#(~p=s) TTTT TTTT TTTT TTTT (4.2.2)

A-propositions convert accidentally (‘Every *S* is *P* ’ implies ‘Some *P* is *S*’) (5.1.1)

#(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (5.1.2)

and O-propositions don’t convert at all. [Some S is not P implies Every P is not S.] (5.2.1)

%(s=~p)>#(p=~s) ; NNNN NNNN NNNN NNNN (5.2.2)

We present these six equations for the six directed rays in the Square, as previously published.

(A\E) #(s= p) \ #(s=~p) ; TTTT TTTT TTTT TTTT (6.1.2)
(A>I) #(s= p) > %(s= p) ; TTTT TTTT TTTT TTTT (6.2.2)
(A\O) #(s= p) \ %(s=~p) ; TTTT TTTT TTTT TTTT (6.3.2)
(E\I) #(s=~p) \ %(s= p) ; TTTT TTTT TTTT TTTT (6.4.2)
(E>O) #(s=~p) > %(s=~p) ; TTTT TTTT TTTT TTTT (6.5.2)
(I+O) %(s= p) + %(s=~p) ; TTTT TTTT TTTT TTTT (6.6.2)

Remark 6: The new connective distribution is as follows with count. The mappings above allow for replication and confirmation of the 24-syllogisms and with our claim of a minor correction each to Modus Camestros and Modus Cesare.

(1) Contraries Not And (\);
(1) Subcontraries Or (+);
(2) Subalterns Imply (>); and
(2) Contradictories Not And (\)

We conclude that Łukasiewicz was not mistaken in his rendition of the Square of Opposition.

We now turn to the criticism of the Cube of Opposition of Seuren to map and interleave the additional vertices from the diagram on page 8. While * marks predicate negation with the term "-P", we use $ to mark copula negation with the term "not P", and mark the negation of $ using !.

(A) Every S is P. #(s= p) =(p=p) ; N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.1.1)
(A*) Every S is not-P. ~(#(s=p)=(p=p))=(p=p) ;
as Not (Every S is P.) CTCT CTCT TCTC TCTC (7.1.2)
(A$) Every S is not P. #(s=~p)=(p=p) ; **F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.1.3)
(A!) Not (Every S is not P.) ~(#(s=~p)=(p=p))=(p=p) ;
TCTC TCTC CTCT CTCT (7.1.4)

(E) No S is P. #(s=~p)=(p=p) ; **F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.1)
(E*) No S is not-P. ~(#(s=~p)=(p=p))=(p=p) ;
as Not (No S is P.) TCTC TCTC CTCT CTCT (7.2.2)
(E$) No S is not P. #(~s=~p)=(p=p) ; N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.3)
(E!) Not (No S is not P.) ~(#(~s=~p)=(p=p))=(p=p) ;
CTCT CTCT TCTC TCTC (7.2.4)

(I) Some S is P. %(s= p)=(p=p) ; TCTC TCTC CTCT CTCT (7.3.1)
(I*) Some S is not-P. ~(%(s= p)=(p=p))=(p=p) ;
as Not (Some S is P). **F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.3.2)
(I$) Some S is not P. %(s=~p)=(p=p) ;
CTCT CTCT TCTC TCTC (7.3.3)
(I!) Not (Some S is not P.) ~(%(s=~p)=(p=p))=(p=p) ;
N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.3.4)

(O) Not every S is P. %(~s=p)=(p=p) ; CTCT CTCT TCTC TCTC (7.4.1)
(O*) Not every S is not-P. ~(%(~s=p)=(p=p))=(p=p) ;
as Not(Not every S is P.) N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.4.2)
(O$) Not every S is not P. %(~s=~p)=(p=p) ; TCTC TCTC CTCT CTCT (7.4.3)
(O!) Not (Not every S is not P.) ~(%(~s=~p)=(p=p))=(p=p) ;
FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.4.4)

The following are supposed to hold:

~I* = *E: ~(~(%(s= p)=(p=p))=(p=p)) = (~(#(s=~p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (8.1.1)

~A* = O*: ~(~(#(s=p)=(p=p))=(p=p)) = (~(%(~s=p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (8.1.2)

A* > E: (~(#(s=p)=(p=p))=(p=p)) > (#(s=~p)=(p=p)) ;
NNNN NNNN NNNN NNNN (9.1.1)

A > E*: (#(s= p) =(p=p)) > (~(#(s=~p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (9.1.2)

I > O*: (%(s= p)=(p=p)) > (~(%(~s=p)=(p=p))=(p=p)) ;
NNNN NNNN NNNN NNNN (9.1.3)

I* > O: (~(%(s= p)=(p=p))=(p=p)) > (%(~s=p)=(p=p)) ;
TTTT TTTT TTTT TTTT (9.1.4)

Eqs. 9.1.1 (A* > E) and 9.1.3 (I > O*) are *not* tautologous, albeit truthities. This means that the final claims of Seuren's Cube of Opposition are mistaken, but also that the criticism of Seuren as based not on those claims is also mistaken.