Confirmation of the Łukasiewicz Square of Opposition via logic VŁ4

Abstract: We evaluate the existential import of the revised, modern square of opposition. We confirm that the Łukasiewicz syllogistic was intended to apply to *all* terms. What follows is that Aristotle was mistaken in his mapping of vertices, which we correct and show fidelity to Aristotle's intentions.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y); (p=p) Tautology.

See: Read, S. (2015). Aristotle and Łukasiewicz on Existential Import.
st-andrews.ac.uk/~slr/Existential_import.pdf slr@st-andrews.ac.uk

We map vertices of the first Square of Opposition on page 4 with its words below.

(A)	Every S is P.	#(s= p)=(p=p) ;	N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N	(0.1.2)
(E)	No S is P.	#(s=~p)=(p=p) ;	**F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F**	(0.2.2)
(I)	Some S is P.	%(s= p)=(p=p) ;	TCTC TCTC CTCT CTCT	(0.3.2)
(O)	Not every S is P.	%(~s=p)=(p=p) ;	CTCT CTCT TCTC TCTC	(0.4.2)

Remark 0: The above is from our *revised* Modern Square of Opposition as published.

We map the relations which Aristotle accepts as preserved here.

A- and E-propositions are contrary (cannot both be true) [(A)=T & (E)=T] (1.1.1)

(#(s= p)=(p=p))&(#(s=~p)=(p=p)) ; **FFFF FFFF FFFF FFFF** (1.1.2)

and I- and O-propositions are subcontrary (cannot both be false) [(I)=**F** & (O)=**F**] (1.2.1)

(%(s= p)=(p@p))&(%(s=~p)=(p@p)) ; **FFFF FFFF FFFF FFFF** (1.2.2)

A- and O-propositions are contradictories, [(A)&(O)] (2.1.1)

#(s= p)&%(s=~p) ; **FFFF FFFF FFFF FFFF** (2.1.2)

as are I- and E-propositions [(I) & (E)] (2.2.1)

%(s= p)&#(s=~p) ; **FFFF FFFF FFFF FFFF** (2.2.2)

A-propositions imply their subaltern I-proposition, [(A) > (I)] (3.1.1)

#(s= p)>%(s= p) ; TTTT TTTT TTTT TTTT (3.1.2)

and E-propositions their subaltern O-proposition [(E) > (O)] (3.2.1)

#(s=~p)>%(s=~p) ; TTTT TTTT TTTT TTTT (3.2.2)

I- propositions convert simply 'Some *S* is *P* ' implies 'Some *P* is *S*', (4.1.1)

%(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (4.1.2)

and E-propositions 'No *S* is *P* ' implies 'No *P* is *S*' (4.2.1)

#(~s=p)>#(~p=s) TTTT TTTT TTTT TTTT (4.2.2)

A-propositions convert accidentally ('Every *S* is *P* ' implies 'Some *P* is *S*') (5.1.1)

#(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (5.1.2)

and O-propositions don't convert at all. [Some S is not P implies Every P is not S.] (5.2.1)

%(s=~p)>#(p=~s) ; NNNN NNNN NNNN NNNN (5.2.2)

Remark 5: We avoid evaluation of the Cube of Opposition because the basis for vertices is not in Eqs. 0.

We present these six equations for the six directed rays in the Square, as previously published.

(A\E)	#(s= p) \ #(s=~p) ;	TTTT TTTT TTTT TTTT	(6.1.2)
(A>I)	#(s= p) > %(s= p) ;	TTTT TTTT TTTT TTTT	(6.2.2)
(A\O)	#(s= p) \ %(s=~p) ;	TTTT TTTT TTTT TTTT	(6.3.2)
(E\I)	#(s=~p) \ %(s= p) ;	TTTT TTTT TTTT TTTT	(6.4.2)
(E>O)	#(s=~p) > %(s=~p) ;	TTTT TTTT TTTT TTTT	(6.5.2)
(I+O)	%(s= p) + %(s=~p) ;	TTTT TTTT TTTT TTTT	(6.6.2)

Remark 6: The new connective distribution is as follows with count. The mappings above allow for replication and confirmation of the 24-syllogisms and with our claim of a minor correction each to Modus Camestros and Modus Cesare.

(1)	Contraries	Not And (\);
(1)	Subcontraries	Or (+);
(2)	Subalterns	Imply (>); and
(2)	Contradictories	Not And (\)

We conclude that Łukasiewicz was not mistaken in his rendition of the Square of Opposition.