

Refutation of pure alethic modal logic (PAM)

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Abstract: We evaluate the formula $\Box p \rightarrow p \rightarrow \Diamond p$ as the backbone of pure alethic modal logic (PAM). Two inconsistent results arise from different orders of precedence: a result of *not* tautologous, *not* contradictory; and a tautologous result. That ambiguity refutes PAM.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: \sim Not, \neg ; + Or, \vee, \cup ; - Not Or; & And, \wedge, \cap ; \ Not And;
 > Imply, greater than, \rightarrow, \vdash ; < Not Imply, less than, \Leftarrow
 = Equivalent, \equiv, \vDash , @ Not Equivalent, \neq ;
 % possibility, for one or some, \exists, \diamond ; # necessity, for every or all, \forall, \square ;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); (p=p) Tautology.

See: Béziau, J.-Y. (2012). "Pure alethic modal logic". Coginitio. 13:25-36.

1.1. The backbone of PAM

$$\Box p \rightarrow p \rightarrow \Diamond p \tag{1.1}$$

Remark 1.1: Eq. 1.1 is ambiguous as to order of operation, so we present two interpreted mappings.

$$(\Box p \rightarrow p) \rightarrow \Diamond p \tag{1.1.1}$$

$$(\#p > p) > \%p ; \quad \text{CTCT CTCT CTCT CTCT} \tag{1.1.2}$$

$$\Box p \rightarrow (p \rightarrow \Diamond p) \tag{1.2.1}$$

$$\#p > (p > \%p) ; \quad \text{TTTT TTTT TTTT TTTT} \tag{1.2.2}$$

Because Eq. 1.1.2 as rendered is *not* tautologous, while 1.2.2 is, this ambiguity refutes 1.1 as the backbone of PAM.