Refutation of graded modal logic

Abstract: We evaluate definitions of five frame classes and a main theorem of satisfiability. Three of the five frame classes are *not* tautologous and the designated example for satisfiability is *not* tautologous. This refutes graded modal logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y); (p=p) Tautology.

From: Kazakov, Y.; Pratt-Hartmann, I. (ca. 2009). A note on the complexity of the satisfiability problem for graded modal logics. yevgeny.kazakov@comlab.ox.ac.uk , ipratt@cs.man.ac.uk uni-ulm.de/fileadmin/website_uni_ulm/iui.inst.090/Publikationen/2008/KazPH08Graded_TR.pdf

LET p, q, r, s: x, y, r, z

Table I lists these frame classes together with their respective defining first-order sentences.

Reflexive frames: ∀x.r(x,x) (1.1.1)

r&#p ; **FFFF F**N**F**N **FFFF F**N**F**N (1.1.2)

Serial frames: ∀x∃y.r(x,y)(1.2.1)

r&(#p&%q) ; **FFFF FFF**N **FFFF FF**FN (1.2.2)

Symmetric frames: ∀x∀y.(r(x,y) → r(y,x)) (1.3.1)

(r&(#p&#q))>(r&(#q&#p)) ; TTTT TTTT TTTT TTTT (1.3.2)

Transitive frames: ∀x∀y∀z.(r(x,y) ∧r(y,z) → r(x,z)) (1.4.1)

(r&(#p&#q))&((r&(#q&#s))>(r&(#p&#s))) ;
FFFF FFFN **FFFF FFF**N (1.4.2)

Euclidean frames: ∀x∀y∀z.(r(x,y) ∧r(x,z) → r(y,z)) (1.5.1)

((r&(#p&#q))&(r&(#p&#s)))>(r&(#q&#s)) ;
TTTT TTTT TTTT TTTT (1.5.2)

Because three of the five frame class definitions are *not* tautologous, the system as refuted.

Theorem 4: ... consider the formula ϕ given by

$$\phi := q0 \wedge \Diamond_{\geq 2}(\neg q0 \wedge q1 \wedge \Diamond_{\geq 1}(\neg q0 \wedge \neg q1)) \wedge \Diamond_{\leq 1} \neg q1 \tag{4.1}$$

The formula ϕ is certainly satisfiable over transitive frames; however, it is not satisfiable over tree-shaped transitive frames.

Remark 4.1: We map $\Diamond_{\geq 2}$ as 2*◊ and $\Diamond_{\geq 1}$ or $\Diamond_{\leq 1}$ as 1*◊ to mean ◊, that is the iteration to be the designated ordinal in the sub-scripted relation.

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(p&((%s<#s)&%((~p&q)&%(~p&~q))))&%~q ;
                                        FCFC FCFC FCFC FCFC      (4.2)
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Because Eq. 4.2 is *not* tautologous as the example for Thm. 4, we say that Thm. 4 is also *not* tautologous, and hence do not proceed further through Thm. 6 and Lem. 3 as the proof path for Thm 4.