Abstract: We evaluate the Lyndon interpolation on the logic GL. Each is not tautologous, and the combination is not tautologous, hence rendering both refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)


Definition 2.2. The least normal logic is called K.

\[ K = \Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q) \]  
\[ (#(p>q)>(#p>#q)) ; \quad \text{T T T T T T T T T T T T} \]  

GL = K + {\Box (\Box p \rightarrow p) \rightarrow \Box p}  
\[ (#(p>q)>(#p>#q))&( (#(p>p)>(#p)) ; \quad \text{C T C T C T C T C T C T} \]  

Eq. 2.2.1.2 as GL is not tautologous. This means logic GL is not a logic proved as a theorem.

Definition 2.5. We say a logic L enjoys the Lyndon interpolation property (LIP) if for any formulas \( \phi \) and \( \psi \), if L \( \vdash \)

\[ \phi \rightarrow \psi, \]  
\[ p>q ; \quad \text{T F T T T F T T T F T T} \]  

then there exists a formula \( \theta \) satisfying the following properties:

1. \( v+(\theta) \subseteq v+(\phi) \cap v+(\psi); \)  
\[ \sim((r\&p)\&(r\&q)<(r\&s)) = (p=p) ; \quad \text{T T T T F T T T T T T T} \]
2. \( v-(\theta) \subseteq v-(\phi) \cap v-(\psi) \);  
\[ (2.5.2.1) \]
\[ \neg(((\neg r \& p) \& (\neg r \& q)) \leq (\neg r \& s)) = (p=p); \]
\[ TTTT TTTT TTTT TTTT \]  
\[ (2.5.2.2) \]

3. \( L \vdash \phi \rightarrow \theta; \)  
\[ (2.5.3.1) \]
\[ p>s ; \]
\[ TTTT TTTT TTTT TTTT \]  
\[ (2.5.3.2) \]

4. \( L \vdash 0 \rightarrow \psi. \)  
\[ (2.5.4.1) \]
\[ s>q ; \]
\[ TTTT TTTT FFFT FFFT \]  
\[ (2.5.4.2) \]

Such a formula \( \theta \) is said to be a Lyndon interpolant of \( \phi \rightarrow \psi \) in \( L \).

The argument becomes: \( \phi \rightarrow \psi \) implies that if \((v+(\theta) \subseteq v+(\phi) \cap v+(\psi))\) and \((v-(\theta) \subseteq v-(\phi) \cap v-(\psi))\) and \( \phi \rightarrow \theta \) and \( \theta \rightarrow \psi \), then \( \theta \) as Lyndon interpolant.

\[ (p>q)>((((\neg r \& p) \& (\neg r \& q)) \leq (\neg r \& s)) \]
\[ \& \neg(((r \& p) \& (r \& q)) \leq (r \& s))) \& (p>s) \& (s>q)))>s) ; \]
\[ FFFT FFTT TTTT TTTT \]  
\[ (2.5.5.2) \]

Eq. 2.5.5.2 as rendered is not tautologous. This means the Lyndon interpolation is refuted.

**Remark 5:** To assert that the non-tautologous Lyndon interpolation applies to the non-tautologous logic \( GL \) is a further mistake.

\[ ((p>q)>((((\neg r \& p) \& (\neg r \& q)) \leq (\neg r \& s)) \]
\[ \& \neg(((r \& p) \& (r \& q)) \leq (r \& s))) \& ((p>s) \& (s>q)))>s)) \&
\[ ((\#(p>q)>(\#p>\#q)) \& (\#(p>\#p)) ; \]
\[ FFTT FFTT CTCT CTCT \]  
\[ (5.0.1.2) \]