Refutation of the Hahn-Banach theorem

Abstract: We evaluate the Hahn-Banach theorem. Without or with the universal quantifiers, the equations are not tautologous. This refutes the Hahn-Banach theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthty (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

FROM: en.wikipedia.org/wiki/Hahn–Banach_theorem

Hahn–Banach theorem (Rudin 1991, Th 3.2). If \( p: V \to \mathbb{R} \) is a sublinear function, and \( \phi: U \to \mathbb{R} \) is a linear functional on a linear subspace

\[
U \subseteq V
\]

\[
\neg(v<u) = (p=p); \quad \begin{array}{ccccccc}
F & F & F & F & F & F & F \\
F & F & F & F & F & F & F \\
F & F & F & F & F & F & F \\
F & F & F & F & F & F & F \\
\end{array} \quad (4), \quad (0.2)
\]

which is dominated by \( p \) on \( U \), i.e.

\[
\phi(x) \leq p(x) \quad \forall x \in U
\]

(1.1)

Remark 1: We ignore the universal quantification on \( U \) and \( V \) in this test.

\[
\neg((p&q)<(r&q)) = (p=p); \quad \begin{array}{ccccccc}
T & T & T & T & F & F & F \\
T & T & T & T & F & F & F \\
T & T & T & T & F & F & F \\
T & T & T & T & F & F & F \\
\end{array} \quad (1.2)
\]

then there exists a linear extension \( \psi: V \to \mathbb{R} \) of \( \phi \) to the whole space \( V \), i.e., there exists a linear functional \( \psi \) such that

\[
\psi(x) = \phi(x) \quad \forall x \in U,
\]

(2.1)

\[
(r&q)=(s&q); \quad \begin{array}{ccccccc}
T & T & F & F & T & F & F \\
T & T & F & F & T & F & F \\
T & T & F & F & T & F & F \\
T & T & F & F & T & F & F \\
\end{array} \quad (2.2)
\]
\[ \psi(x) \leq p(x) \forall x \in V. \] (3.1)

\[ \neg((p \land q) \leq (s \land q)) = (p = p); \quad \text{TTF TTF TTT TTTT} \] (3.2)

If Eqs 1, then (2 and 3).

\[ \neg((p \land q) < (r \land q)) > (((r \land q) = (s \land q)) \& \neg((p \land q) < (s \land q)))); \quad \text{TTTT TFF TTF TTTT} \] (4.1)

Eq. 4.2 as rendered is not tautologous, hence refuting the Hahn-Banach theorem.

**Remark 5:** To include the relationship of \( U \) and \( V \) in Eqs. 0 and the universal quantification on \( U \) and \( V \) in 1 and 2 produces this result. (5.1)

\[ \neg(v < u) > \\
((#q < u) \& \neg((p \& q) < (r \& q))) > \\
((#q < u) \& ((r \& q) = (s \& q))) \& ((#q < v) \& \neg((p \& q) < (s \& q)))); \quad \text{TTTT TCC TCT TTTT} \],
\[ \text{(4), TTTT TTTT TTTT TTTT (12)} \] (5.2)

Eq. 5.2 is also not tautologous, hence refuting the Hahn-Banach theorem.