Abstract: We evaluate the Hahn-Banach theorem. Without or with the universal quantifiers, the equations are not tautologous. This refutes the Hahn-Banach theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, \( \mathbf{F} \) as contradiction, \( \mathbf{N} \) as truthity (non-contingency), and \( \mathbf{C} \) as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

\[
\text{LET } \sim \text{ Not, } \neg; \quad + \text{ Or, } \lor; \quad - \text{ Not Or; } \& \text{ And, } \land; \quad \backslash \text{ Not And;}
\]
\[
> \text{ Imply, greater than, } \rightarrow; \quad < \text{ Not Imply, less than, } \in
\]
\[
\equiv \text{ Equivalent, } \equiv; \quad @ \text{ Not Equivalent, } \neq;
\]
\[
\% \text{ possibility, for one or some, } \exists, \diamond; \quad # \text{ necessity, for every or all, } \forall, \Box;
\]
\[
\sim(p<q) \quad (p>p), \quad (x\leq y), \quad (x\subseteq y); \quad (p=p) \text{ Tautology.}
\]

From: en.wikipedia.org/wiki/Hahn–Banach_theorem

\[
\text{LET } \quad p, q, r, s, u, v: \quad p, x, \phi, \psi, U, V.
\]

Hahn–Banach theorem (Rudin 1991, Th 3.2). If \( p: V \rightarrow \mathbf{R} \) is a sublinear function, and \( \phi: U \rightarrow \mathbf{R} \) is a linear functional on a linear subspace

\[
U \subseteq V
\]

\[
\sim(v<u) = (p=p); \quad TTTT \ TTTT \ TTTT \ TTTT, \quad FFFF \ FFFF \ FFFF \ FFFF
\]

which is dominated by \( p \) on \( U \), i.e.

\[
\phi(x) \leq p(x) \quad \forall x \in U
\]

Remark 1: We ignore the universal quantification on \( U \) and \( V \) in this test.

\[
\sim((p\&q)<(r\&q)) = (p=p); \quad TTTT \ TTTT \ TTTT \ TTTT
\]

then there exists a linear extension \( \psi: V \rightarrow \mathbf{R} \) of \( \phi \) to the whole space \( V \), i.e., there exists a linear functional \( \psi \) such that

\[
\psi(x) = \phi(x) \quad \forall x \in U,
\]

\[
(r\&q)=(s\&q); \quad TTTT \ TTTT \ FTTT \ TTTT
\]
\[ \psi(x) \leq p(x) \quad \forall x \in V. \quad (3.1) \]

\[ \neg((p \land q) < (s \land q)) = (p = p); \quad \text{TTTF TTTF TTTT TTTT} \quad (3.2) \]

If Eqs 1, then (2 and 3).

\[ \neg((p \land q) < (r \land q)) > (((r \land q) = (s \land q)) \land \neg((p \land q) < (s \land q))); \quad \text{TTTT TTTF TTTT TTTT} \quad (4.2) \]

Eq. 4.2 as rendered is not tautologous, hence refuting the Hahn-Banach theorem.

**Remark 5:** To include the relationship of \( U \) and \( V \) in Eqs. 0 and the universal quantification on \( U \) and \( V \) in 1 and 2 produces this result.

\[ \neg(v < u) > \]

\[ (((q < u) \land \neg((p \land q) < (r \land q))) > \]

\[ (((q < u) \land ((r \land q) = (s \land q))) \land ((q < v) \land \neg((p \land q) < (s \land q)))); \]

\[ \text{TTTT TTCC TTCT TTTT, TTTT TTTT TTTT TTTT} \quad (5.2) \]

Eq. 5.2 is also not tautologous, hence refuting the Hahn-Banach theorem.