Refutation of the Hahn-Banach theorem

Abstract: We evaluate the Hahn-Banach theorem. Without or with the universal quantifiers, the equations are *not* tautologous. This refutes the Hahn-Banach theorem.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨; - Not Or; & And, ∧; \ Not And;
\> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y); (p=p) `T`autology.

From: en.wikipedia.org/wiki/Hahn–Banach_theorem

LET p, q, r, s, u, v: p, x, phi φ, psi ψ, *U*, *V*.

Hahn–Banach theorem (Rudin 1991, Th 3.2). If *p*: *V* → **R** is a sublinear function, and *φ*: *U* → **R** is a linear functional on a linear subspace

$U \subseteq V$ (0.1)

~(v<u) = (p=p) ;
`TTTT TTTT TTTT TTTT,` **`FFFF FFFF FFFF FFFF`** (0.2)

which is dominated by *p* on *U*, i.e.

$\varphi (x) \leq p (x) \ \forall x \in U$ (1.1)

Remark 1: We ignore the universal quantification on *U* and *V* in this test.

~((p&q)<(r&q)) = (p=p) ; `TTT`**`F`**` TTTT TTT`**`F`**` TTTT` (1.2)

then there exists a linear extension *ψ*: *V* → **R** of *φ* to the whole space *V*, i.e., there exists a linear functional *ψ* such that

$\psi (x) = \varphi (x) \ \forall x \in U$, (2.1)

(r&q)=(s&q) ; `TTTT TT`**`FF`**` TT`**`FF`**` TTTT` (2.2)

$\psi (x) \leq p (x) \ \forall x \in V$. (3.1)

~((p&q)<(s&q)) = (p=p) ; TTT**F** TTT**F** TTTT TTTT (3.2)

If Eqs 1, then (2 and 3). (4.1)

~((p&q)<(r&q)) > (((r&q)=(s&q))&~((p&q)<(s&q))) ;
TTTT TT**FF** TT**F**T TTTT (4.2)

Eq. 4.2 as rendered is *not* tautologous, hence refuting the Hahn-Banach theorem.

Remark 5: To include the relationship of *U* and *V* in Eqs. 0 and the universal quantification on *U* and *V* in 1 and 2 produces this result. (5.1)

~(v<u) >
(((#q<u)&~((p&#q)<(r&#q))) >
(((#q<u)&((r&#q)=(s&#q)))&((#q<v)&~((p&#q)<(s&#q)))));
TTTT TTCC TTCT TTTT, TTTT TTTT TTTT TTTT (5.2)

Eq. 5.2 is also *not* tautologous, hence refuting the Hahn-Banach theorem.