Some Neutrosophic Probability Distributions

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Abstract. In this paper, we introduce and study some neutrosophic probability distributions, The study is done through generalization of some classical probability distributions as Poisson distribution, Exponential distribution and Uniform distribution, this study opens the way for dealing with issues that follow the classical distributions and at the same time contain data not specified accurately.

Keywords: Poisson, Exponential & Uniform distributions, Classical Logic, Neutrosophic Logic, Neutrosophic crisp sets.

1 Introduction: Neutrosophy theory introduced by Smarandache in 1995. It is a new branch of philosophy, presented as a generalization for the fuzzy logic [5] and as a generalization for the intuitionistic fuzzy logic [6]. The fundamental concepts of neutrosophic set, introduced by Smarandache in [7, 8, 9, 10], and Salama et al. in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23], provides a new foundation for dealing with issues that have indeterminate data. The indeterminate data may be numbers, and the neutrosophic numbers have been defined in [24, 25, 26, 27]. In this paper, we highlight the use of neutrosophic crisp sets theory [3,4] with the classical probability distributions, particularly Poisson distribution, Exponential distribution and Uniform distribution, which opens the way for dealing with issues that follow the classical distributions and at the same time contain data not specified accurately. The extension of classical distributions according to the neutrosophic logic, means that parameters of classical distribution take undetermined values, which allows dealing with all the situations that one may encounter while working with statistical data and especially when working with vague and inaccurate statistical data, Florentin Smarandache presented the neutrosophic binomial distribution and the neutrosophic natural distribution [1,2] in 2014. In this paper, we will discuss continuous random distributions such as the Exponential distribution and Uniform distribution, and discontinuous random distribution such as Poisson distribution by using neutrosophic logic.

2 TERMINOLOGIES: We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [1, 9, 10], and Salama et al. [22, 23]. We consider the following classic statistical distributions Poisson distribution, Exponential distribution and Uniform distribution.

3 Neutrosophic probability Distributions:

3.1 Neutrosophic Poisson Distribution:

3.1.a Definition: Neutrosophic Poisson distribution of a discrete variable X is a classical Poisson distribution of X, but its parameter is imprecise. For example, λ can be set with two or more elements. The most common such distribution is when λ is interval.

\[ NP(x) = e^{-\lambda_N} \cdot \frac{(\lambda_N)^x}{x!} \quad ; \quad x = 0,1, \ldots \]

\[ \lambda_N: \text{is the distribution parameteer} \]

• \[ \lambda_N: \text{is equal to the expected value and the variance} \]
\[ NE(x) = NV(x) = \lambda_N \]
Where, \( N = d + 1 \) is a neutrosophic statistical number in [2].

### 3.1.b Example for Case study:

In a company, Phone employee receives phone calls, the calls arrive with rate of \([1, 3]\) calls per minute, we will calculate the probability that:

- The employee will not receive any call within a minute:

Assuming \( x \): the number of calls in a minute.

Then:

\[ NP(x = 0) = e^{-\lambda_N} \cdot \frac{(\lambda_N)^0}{0!} = e^{-\lambda_N} = e^{-[1,3]} \]

For \( \lambda = 1 \):
\[ NP(0) = e^{-1} = 0.3679 \]
For \( \lambda = 3 \):
\[ NP(0) = e^{-3} = 0.0498 \]

Thus, the probability that employee won't receive any call, within a minute, ranges between \([0.0498, 0.3697]\).

- the probability that employee won't receive any call, within 5 minutes:

Then:

\[ \lambda_N = 5 \cdot [1,3] = [5, 15] \]

\[ NP(x) = e^{-[5,15]} \cdot \frac{([5,15])^x}{x!} ; \quad x = 0,1, ... \]

\[ NP(x = 0) = e^{-\lambda_N} \cdot \frac{(\lambda_N)^0}{0!} = e^{-\lambda_N} = e^{-[5,15]} \]

For \( \lambda = 5 \):
\[ NP(0) = e^{-5} = 0.0067 \]
For \( \lambda = 15 \):
\[ NP(0) = e^{-15} = 0.000000306 \]

Thus, the probability that the employee will not receive any call within 5 minutes ranges between \([0.000000306, 0.0067]\).
3.2 The Neutrosophic Exponential Distribution:

3.2.a Definition: Neutrosophic exponential distribution [21] is defined as a generalization of classical exponential distribution, Neutrosophic exponential distribution can deals with all the data even non-specific, we express the density function as:

\[ X_N \sim \exp(\lambda_N) = f_N(x) = \lambda_N e^{-x \lambda_N}; \quad 0 < x < \infty, \]

\[ \exp(\lambda_N) \quad : \text{Neutrosophic Exponential Distribution}. \]

\[ X_N \quad : \text{X neutrosophic random variable [22]}. \]

\[ \lambda_N \quad : \text{distribution parameter}. \]

3.2.b the distribution properties:

1- Expected value:

\[ E(x) = \frac{1}{\lambda_N} \]

Variance:

\[ var(x) = \frac{1}{(\lambda_N)^2} \]

2- Distribution function:

Probability to terminate the client's service in less than a minute:

\[ NF(x) = NP(X \leq x) = \left(1 - e^{-x \lambda_N}\right) \]

Figure 1

3.2.c. Example for Case study:

The time required to terminate client's service in the bank follows an exponential distribution, with an average of one minute, let us write a density function that represents the time required for terminating client's service, and then calculate the probability of terminating client's service in less than a minute.
Solution:
- Assuming $x$: represents the time required for termination of the client's service per minute.
- The average $1/\lambda = 1 \Rightarrow \lambda = 1$

Therefore, the Probability density function:

$$f(x) = e^{-x} ; \quad 0 < x < \infty$$

- The possibility of client's service terminated in less than a minute:

$$p(X \leq 1) = (1 - e^{-x}) = (1 - e^{-1}) = 0.63$$

- The above example is a simple example practically, but if it is changed to the following:

The time required to terminate client's service in the bank follow an exponential distribution, with an average of $[0.67, 2]$ minute. We know that classical exponential distribution only deals with data defined accurately, note that the average here is an interval, how we will deal with this situation.

So, we will turn to the neutrosophic exponential distribution to solve this issue:

For exponential distribution, its average $[0.67, 2]$ minutes, we write:

$$\frac{1}{\lambda_N} = [0.67, 2] \Rightarrow \lambda_N = \frac{1}{[0.67, 2]} = [0.5, 1.5]$$

The probability density function:

$$f_N(x) = \lambda_N e^{-x} \lambda_N ; \quad 0 < x < \infty$$

$$f_N(x) = [0.5, 1.5] e^{-[0.5,1.5]x} ; \quad 0 < x < \infty$$

Probability to terminate the client's service in less than a minute:

$$NP(X \leq 1) = (1 - e^{-[0.5,1.5]}) = (1 - e^{-[0.5,1.5](1)}) = 1 - e^{-[0.5,1.5]}$$

We note:

For $\lambda = 0.5$:

$$NP(X \leq 1) = 1 - e^{-0.5} = 0.39$$

For $\lambda = 1.5$:

$$NP(X \leq 1) = 1 - e^{-1.5} = 0.78$$

That is, the probability of terminating client's service in less than a minute ranges between $[0.39, 0.78]$. 
Note that, the value of the classic probability to terminate client’s service in less than a minute is one of the domain values for the neutrosophic probability:

\[ p(X \leq 1) = 0.63 \in [0.39, 0.78] = NP(X \leq 1) \]

And the solutions are the shaded area in Figure 1.

3.2.d Note: We also mention the relationship of exponential distribution with Poisson distribution. If the occurrence of events follows the Poisson distribution, the duration between the occurrence of two events follows exponential distribution. For example, arrival of customers to a service centre follows the Poisson distribution, the time between the arrival of a customer and the next customer follows the exponential distribution. Thus, when the parameter \( \lambda \) is inaccurately defined, we are dealing with the neutrosophic exponential distribution and the neutrosophic Poisson distribution, and we write:

If an event is repeated in time according to the neutrosophic Poisson distribution:

\[ NP(x) = e^{-\lambda_N} \cdot \frac{(\lambda_N)^x}{x!} \quad ; \quad x = 0,1, .... \]

Then, the time between two events follows the neutrosophic exponential distribution:

\[ f_N(t) = \lambda_N \cdot e^{-\lambda_N} t \quad ; \quad t > 0 \]

3.2.d.i. Example:

Assuming that we have a machine in a factory. The rate of machine breakdowns is \([1, 2]\) per week, let’s calculate the possibility of no breakdowns per week, and calculate the possibility that at least two weeks pass before the appearance of the following breakdowns.

Solution:

- The possibility of no breakdowns in the week:

Assume \( x \): variable represents occurrence of breakdowns in the week.

We note that, \( x \) is a variable that is subject to the neutrosophic Poisson distribution, the distribution parameter is

\[ \lambda_N = [1,2] \], thus:

\[ NP(x = 0) = e^{-\lambda_N} \cdot \frac{(\lambda_N)^x}{x!} = e^{-\lambda_N} \cdot \frac{(\lambda_N)^0}{0!} = e^{-\lambda_N} = e^{-[1,2]} \]

Then, the possibility of no breakdowns in the week ranges between \([0.135, 0.368]\).

- Assuming \( y \): is represent the time before the appearance of the following breakdowns, we note that \( y \) is a variable following the neutrosophic exponential distribution, then:

\[ NF(x) = NP(X \leq x) = (1 - e^{-x \lambda_N}) \]

\[ NP(y > 2) = 1 - NP(y \leq 2) = 1 - NF(2) = 1 - (1 - e^{-2 \lambda_N}) \]

\[ = e^{-2 \lambda_N} = e^{-2[1,2]} = e^{-[4,2]} \]
Thus, the possibility that at least two weeks pass before the appearance of the following breakdowns, ranges between [0.018, 0.135].

3.3 Neutrosophic Uniform Distribution:
3.3.a Definition: Neutrosophic Uniform distribution of a continuous variable $X$, is a classical Uniform distribution , but distribution parameters $a$ or $b$ or both are imprecise. For example, $a$ or $b$ or both are sets with two or more elements (may $a$ or $b$ or both are intervals) with $a < b$.

3.3.b Example for Case study:

Assuming $x$ is a variable represents a person's waiting time to passengers' bus (in minutes), bus's arrival time is not specified, the station official said:

1- the bus arrival time is: either from now to 5 minutes $[0,5]$ or will arrive after 15 to 20 minutes$[15,20]$, then:

$a=[0,5]$ , $b=[15,20]$

Then, the density function:

$$f_N(x) = \frac{1}{b-a} = \frac{1}{[15,20]-[0,5]} = \frac{1}{[10,20]} = [0.05, 0.1]$$

The solution in the Graph is the shaded area, with the probability to moving (a) between $[0, 5]$ and (b) between $[15, 20]$.

2- The bus arrives after five minutes or will arrive after 15 to 20 minutes $[15, 20]$, then:

$a=5$  $b=[15, 20]$

Then, the density function:

$$f_N(x) = \frac{1}{b-a} = \frac{1}{[15,20]-5} = \frac{1}{[10,15]} = [0.067, 0.1]$$

The solution is the shaded area, with the probability to moving (b) between $[15, 20]$. 
There are many non-specific situations that we encounter about the values a, b such as a, b or both are intervals (a, b or both are sets with two or more elements), we deal with these situations as the cases studied above.

4 The research gap

The classical probability distributions only deal with the specified data. The classical distribution parameters are always given with a specified value. This paper contributes to the study of classical distributions with undetermined values, and distribution parameters such as periods. We call these distributions neutrosophic probability distributions.

Conclusion:

We conclude from this paper that the neutrosophic probability distributions gives us a more general and clarity study of the studied issue, So that the classical probability is one solution among the solutions resulting from the study, of course, this is produced by giving the distribution parameters several options possible and does not remain linked to a single value. This paper is to present some the neutrosophic probability distributions, and we present various solved for the problems that classic logic is not deal with it. We look forward in the future to study other types of probability distributions according to the neutrosophic logic, especially the gamma distribution and student distribution and other distributions that have not yet been studied.

References


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