Affirmation of the Craig interpolation theorem

Abstract: We evaluate the Craig interpolation theorem, find a mistake in a Craig lemma as rendered by Feferman, and affirm the theorem.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y).

From: en.wikipedia.org/wiki/Craig_interpolation;

> **Remark 0:** We use only four variables to minimize table results to 4-rows or 16-values (instead of 512-rows or 2048-values) . Hence we avoid direct assignment of φ, ψ as separate variables.
>
> LET p, q, r, s: P, Q, R, T.

In propositional logic, let

φ = ~(P ∧Q) → (~R ∧Q) (1.1.1)

~(p&q)>(~r&q) ; **FF**TT **FFF**T **FF**TT **FFF**T (1.1.2)

ψ = (T → P) ∨(T → ~R). (1.2.1)

(s>p)+(s>~r) ; TTTT TTTT TTTT **F**T**F**T (1.2.2)

Then φ tautologically implies ψ. (1.3.1)

(~(p&q)>(~r&q))>((s>p)+(s>~r)) ; TTTT TTTT TTTT TTTT (1.3.2)

Eq. 1.3.2 affirms the Craig interpolation theorem.

> **Remark 1:** Eq. 1.3 is a tautology via contradiction implying contradiction (**F**>**F**=T). This form of proof is not constructive in an affirmative or positive sense. A much longer constructive proof exists, but it can be minimized by its use of *induction*.

From: Feferman, S. (2008). Harmonious Logic: Craig's Interpolation Theorem and its Descendants
math.stanford.edu/~feferman/papers/Harmonious%20Logic.pdf

[Here $\vdash$ is validity in classical first order logic with equality (FOL), ϕ, ψ, θ are sentences, and R, S, and T are sequences of relation symbols for which the sequence S is non-empty.]

A common statement of Craig's theorem (initially referred to by him as a lemma) goes as follows:

[LET r, s, t, w, x, y: R, S, T, ϕ, ψ, θ.]

Suppose $\vdash \phi(R, S) \to \psi(S, T)$. Then there is a $\theta(S)$
such that $\vdash \phi(R, S) \to \theta(S)$ and $\vdash \theta(S) \to \psi(S, T)$. (2.1)

Remark 1.1: For "and" above, read "plus".

(((w&(r&s))>(x&(s&t)))=(p=p)) >
((y&s)>((((w&(r&s))>(y&s))=(p=p))+(((y&s)>(x&(s&t)))=(p=p)))) ;
TTTT TTTT TTTT TTTT (2.2)

Accounting for the unclear writing of Craig (and Feferman), the lemma of Eq. 2.2 as rendered affirms the Craig interpolation as a theorem.