Refutation of the modal logic GL_2

Abstract: We evaluate the modal logic GL_2 in two axioms and for satisfiability. One of the two axioms is *not* tautologous, and the five formulas for PSpace complete satisfiability are *not* tautologous. Hence GL_2 is refuted.

We assume the method and apparatus of Meth8/VŁ4 with T autology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆y).

From: Gabelaia, D.; Gogoladzeb, K.; Jibladze, M.; Kuznetsov, E.; Marxa, M. (2018). Modal logic of planar polygons. arxiv.org/pdf/1807.02868.pdf e.kuznetsov@freeuni.edu.ge

LET p, q, r, s: p, q, r, γ.

We also present a slightly more intuitive and concise axiomatization of PL_2 by the following two formulas:

(I) $p \rightarrow \Box[\neg p \rightarrow \Box(p \rightarrow \Box p)]$ (4.2.1)

p>#(~p>#(p>#p)) ; TNTN TNTN TNTN TNTN (4.2.2)

(II) $\Box[(r \wedge q) \rightarrow \gamma] \rightarrow [(r \wedge q) \rightarrow \Diamond(\neg(r \wedge q) \wedge \Diamond\Box p \wedge \Diamond\Box\neg p)]$ (4.3.1)

#((r&q)>s)>((r&q)>%(~(r&q)&(%#p&%#~p))) ;
TTTT TTTT TTTT TTCC (4.3.2)

Where γ is the formula $\Diamond\Box(p \wedge q) \wedge \Diamond\Box(\neg p \wedge q) \wedge \Diamond\Box(p \wedge \neg q)$. (4.4.1)

s=(%#(p&q)&(%#(~p&q)&%#(p&~q))) ;
TTTT TTTT **FFFF FFFF** (4.4.2)

Substituting Eq. 4.4.1 into 4.3.1: (4.5.1)

#((r&q)>(%#(p&q)&(%#(~p&q)&%#(p&~q))))>
((r&q)>%(~(r&q)&(%#p&%#~p))) ; TTTT TTTT TTTT TTTT (4.5.2)

While Eq. 4.5.2 is tautologous as an axiom, Eq. 4.2.2 is *n*ot tautologous as an axiom. This means the slightly more intuitive and concise axiomatization of PL_2 is refuted.

LET p, q, r, s, t: φ, ψ, r, m, e.

Theorem 5.1. The satisfiability problem of our logic is PSpace complete.
Let C be the conjunction of these formulas:

r, m, e are disjoint and one of them holds at each world. (5.0.1.1)

(r+s)+t ;
FFFF TTTT TTTT TTTT, TTTT TTTT TTTT TTTT (5.0.1.2)

$r \rightarrow \Diamond m$ (5.0.2.1)

r>%t ;
TTTT CCCC TTTT CCCC, TTTT TTTT TTTT TTTT (5.0.2.2)

$m \rightarrow \Diamond e$. (5.0.3.1)

s>%t ;
TTTT TTTT CCCC CCCC, TTTT TTTT TTTT TTTT (5.0.3.2)

$C=(r \rightarrow \Diamond m) \& (m \rightarrow \Diamond e)$. (5.0.4.1)

((r+s)+t)&((r>%s)&(s>%t)) ;
FFFF CCCC CCCC CCCC, TTTT CCCC TTTT TTTT (5.0.4.2)

$r \wedge \Box C \wedge \Diamond (m \vee e) \wedge \varphi) \wedge \Box ((m \vee e) \rightarrow \psi)$
is satisfiable in our logic. (5.1.1.1)

((r&#(((r+s)+t)&((r>%s)&(s>%t))))&(%(s>t)&p))&#((s+t)>q) ;
FFFF FFFF FFFF FFFF, **FFFF FFFF FFFF FFF**N (5.1.1.2)

Eq. 5.1.1.2 as rendered is *no*t tautologous and not contradictory, differing from the state of contradiction by one value N. This means PL_2 is *not* satisfiable as PSpace complete.