

## Refutation of the definition of Sacchetti's modal logics of provability

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**Abstract:** We evaluate the definition of Sacchetti's modal logics of provability. It is *not* tautologous. Therefore it is a mistake to use it as a basis for constructing fixed point procedures.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨; - Not Or; & And, ∧; \ Not And;  
 > Imply, greater than, →; < Not Imply, less than, ∈  
 = Equivalent, ≡; @ Not Equivalent, ≠;  
 % possibility, for one or some, ∃, ∂; # necessity, for every or all, ∀, □;  
 ~( y < x) ( x ≤ y), ( x ∈ y).

From: Kurahashi, T.; Okaw, Y. (2018). Effectively constructible fixed points in Sacchetti's modal logics of provability. [arxiv.org/pdf/1811.12827.pdf](https://arxiv.org/pdf/1811.12827.pdf) kurahashi@n.kisarazu.ac.jp

"We give a purely syntactical proof of the fixed point theorem for Sacchetti's modal logics  $\mathbf{K} + \Box(\Box^n p \rightarrow p) \rightarrow \Box p (n \geq 2)$  of provability. (1.0)  
 From our proof, an effective procedure for constructing fixed points in these logics is obtained."

**Remark 1.0:** At Eq. 1.0 we ignore **K** for  $\Box(p > q) > (\Box p > \Box q)$ , as we previously show as tautologous, and rewrite using the power series  $\Box^n$  as  $\Box^*n$ .

[LET p, q, r, n: p, q, r, s.]

$$\Box(\Box^n p \rightarrow p) \rightarrow \Box p (n \geq 2) \tag{1.1}$$

$$\#((\#p \& s) > p) > \#(p \& \sim((\%r \< \#r) > s)); \text{ cccc cccc cccc cccc} \tag{1.2}$$

**Remark 1.2:** The table result value of c for contingency as falsity is the closest pure state to **F** as contradiction. This means Eq. 1.2 is not a contradiction or a tautology, but rather an intermediate state of falsity.

Eqs. 1 as rendered are *not* tautologous, hence refuting Sacchetti's modal logics of provability. What follows to obtain a procedure for constructing fixed points in these logics is mistaken.