

# A SIMPLE, DIRECT PROOF OF FERMAT'S LAST THEOREM

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ABSTRACT. An open problem is proving FLT *simply* (using Fermat's toolbox) for each  $n \in \mathbb{N}, n > 2$ . Our *direct proof* (not BWOC) of FLT is based on our algebraic identity  $((r + 2q^n)^{\frac{1}{n}})^n - (2^{\frac{2}{n}}q)^n = ((r - 2q^n)^{\frac{1}{n}})^n$  with arbitrary values of  $n \in \mathbb{N}$ , and with  $r \in \mathbb{R}, q \in \mathbb{Q}, n, q, r > 0$ . For convenience, we denote  $(r + 2q^n)^{\frac{1}{n}}$  by  $s$ ; we denote  $2^{\frac{2}{n}}q$  by  $t$ ; and, we denote  $(r - 2q^n)^{\frac{1}{n}}$  by  $u$ . For any given  $n > 2$ : Since the term  $t$  or  $2^{\frac{2}{n}}q$  with  $q \in \mathbb{Q}$  is not rational, this identity allows us to relate null set  $\{(s, t, u) | s, t, u \in \mathbb{Q}, s, t, u > 0, s^n - t^n = u^n\}$  with subsequently proven null set  $\{z, y, x | z, y, x \in \mathbb{Q}, z, y, x > 0, z^n - y^n = x^n\}$ : We show it is true, for  $n > 0$ , that  $\{t | s, t, u \in \mathbb{Q}, s, t, u > 0, s^n - t^n = u^n\} = \{y | z, y, x \in \mathbb{Q}, z, y, x > 0, z^n - y^n = x^n\}$ . Hence, for any given  $n \in \mathbb{N}, n > 2$ , it is a true statement that  $\{(x, y, z) | x, y, z \in \mathbb{N}, x, y, z > 0, x^n + y^n = z^n\} = \emptyset$ .

## 1. INTRODUCTION

FLT states:  $x^n + y^n = z^n$  does not hold for  $n > 2, n, x, y, z \in \mathbb{N}, x, y, z > 0$ .

A *simple* (using Fermat's tools) proof of FLT for each  $n \in \mathbb{N}, n > 2$  is lacking.

For  $n \in \mathbb{N}, n > 2$ : We propose a simple *direct proof* (not the expected BWOC).

(A)  $z^n - y^n = x^n$ , for  $n > 0$ , with  $z, y, x \in \mathbb{Q}, z, y, x > 0$  for which (A) holds.

We want an algebraic identity, with an irrational term for  $n > 2$ , to relate to (A).

(B)  $((r + 2q^n)^{\frac{1}{n}})^n - (2^{\frac{2}{n}}q)^n = ((r - 2q^n)^{\frac{1}{n}})^n$  for  $n \in \mathbb{N}, q \in \mathbb{Q}, r \in \mathbb{R}, n, q, r > 0$  such that  $(r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}}q, (r - 2q^n)^{\frac{1}{n}} \in \mathbb{Q}$  for which (B) holds. From an infinity of identities we choose (B). For  $n = 1, 2$ , but not for  $n > 2$ , equation (B) holds for the elements, above. Yet, even for  $n > 2$ , equation (B) is not inconsistent with (A) since, for  $n > 2$ , no  $z, y, x \in \mathbb{Q}$  is known for which (A) holds. Denoting  $(r + 2q^n)^{\frac{1}{n}}$  by  $s$ ;  $2^{\frac{2}{n}}q$  by  $t$ ;  $(r - 2q^n)^{\frac{1}{n}}$  by  $u$ : We show, below, for  $n > 2$ , with both sets empty, that  $\{(s, t, u) | s, t, u \in \mathbb{N}, s^n - t^n = u^n\} = \{(z, y, x) | z, y, x \in \mathbb{N}, z^n - y^n = x^n\}$

(C)  $(r + q^n)^{\frac{1}{n}})^n - (2^{\frac{1}{n}}q)^n = ((r - q^n)^{\frac{1}{n}})^n$  with  $(r + q^n)^{\frac{1}{n}}, 2^{\frac{1}{n}}q, (r - q^n)^{\frac{1}{n}} \in \mathbb{Q}$ , and  $q \in \mathbb{Q}, r \in \mathbb{R}, n, q, r > 0$  for which (C) holds. Equation (C) has a similar form to (B) but (C) is *very different* from (B) since, for  $n = 2, q \in \mathbb{Q}$ , equation (C) does not hold for  $(r + q^n)^{\frac{1}{n}}, 2^{\frac{1}{n}}q, (r - q^n)^{\frac{1}{n}} \in \mathbb{Q}$ , while (B) does hold for its elements.

So, (C) is not logically consistent with (A) which holds for  $n = 2$ . Thus, (C) is a false premise that would make our argument unsound should (C) be so applied.

(D)  $((r + 2^p q^n)^{\frac{1}{n}})^n - (2^{\frac{p+1}{n}}q)^n = ((r - 2^p q^n)^{\frac{1}{n}})^n$ , for  $n > 0$ , with  $n \in \mathbb{N}$ , and  $p \in \mathbb{I}, p \geq 0$ , and  $r \in \mathbb{R}, q \in \mathbb{Q}, r, q > 0$ , and  $(r + 2^p q^n)^{\frac{1}{n}}, 2^{\frac{p+1}{n}}q, (r - 2^p q^n)^{\frac{1}{n}} \in \mathbb{Q}$  for which the family of identities (D) holds. We have evaluated (D) for usefulness:

We reject (D) with even  $p \geq 0, q \in \mathbb{Q}$  since, for  $n = 2$ , the middle part,  $2^{\frac{p+1}{n}}q$ , is not rational. We reject (D) with odd  $p > 1, q \in \mathbb{Q}$  since for  $2^{\frac{p+1}{n}}q \in \mathbb{Q}$ , equation (B) yields the composite set of all elements contained in every set that (D) yields.

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## 2. OUR DIRECT PROOF

Our argument is a *direct proof*, not deriving a contradiction as is generally expected in proofs. We start in the real realm, ending in the realm of natural numbers.

The algebraic identity we eventually relate to  $z^n - y^n = x^n$ , (A), is (1), below :

$$(1) \quad \left( (r + 2q^n)^{\frac{1}{n}} \right)^n - \left( 2^{\frac{2}{n}} q \right)^n = \left( (r - 2q^n)^{\frac{1}{n}} \right)^n .$$

For all  $n \in \mathbb{N}, n > 0$ , identity (1) holds for all  $r, q \in \mathbb{R}, r, q > 0, r > 2q^n$ .

(2)  $((r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}})$  is the triple for which (1) holds, such that  $(r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}}, r, q \in \mathbb{R}, r, q > 0, r > 2q^n$ .

Throughout this paper,  $n \in \mathbb{N}$ , and,  $n, q, r, x, y, z$  remain positive numbers.

Initially, we need to relate equation (2) with equation (3), below.

(3)  $z^n - y^n = x^n$ . For all values of  $n \in \mathbb{N}, n > 0$ , equation (3) holds for triple  $(z, y, x)$  with  $z, y, x \in \mathbb{R}, z, y, x > 0$ .| The n-th triple for which (3) holds is (4) :

(4)  $\{z^n, y^n, x^n | z, y, x \in \mathbb{R}, z, y, x > 0, z^n - y^n = x^n\}$ .| Expanding (1) yields (5) :

(5)  $(r + 2q^n) - (4q^n) = (r - 2q^n)$ . For some values of  $n > 0$  : (5) holds for  $(r + 2q^n, 4q^n, r - 2q^n)$  such that  $r, q, r + 2q^n, 4q^n, r - 2q^n \in \mathbb{R}, r > 2q^n$ .| So, per (5):

(6)  $\{(r + 2q^n, 4q^n, r - 2q^n) | r, q \in \mathbb{R}, r > 2q^n, (r + 2q^n) - (4q^n) = (r - 2q^n)\}$  is the nth-triple for which identity (5) holds.| We show that, (7), below, is true :

(7)  $\{(r + 2q^n, 4q^n, r - 2q^n) | r, q \in \mathbb{R}, r > 2q^n, (r + 2q^n) - (4q^n) = (r - 2q^n)\} = \{z^n, y^n, x^n | z, y, x \in \mathbb{R}, z, y, x > 0, z^n - y^n = x^n\}$ .

Proof : Equation (7) is true since (4) includes (6), and (6) includes (4).

Set (4) includes set (6), as follows : Terms  $z^n, y^n, x^n$  in (3) have unrestricted positive real values under the conditions imposed by the form of (3).

Thus,  $\{z^n | z^n \in (4)\}$  includes  $\{r + 2q^n | r + 2q^n \in (6)\}$ ;  $\{y^n | y^n \in (4)\}$  includes  $\{4q^n | 4q^n \in (6)\}$ ;  $\{x^n | x^n \in (4)\}$  includes  $\{r - 2q^n | r - 2q^n \in (6)\}$ .

Set (6) includes set (4), per the following argument :

Let  $r + 2q^n = z^n$ . Let  $4q^n = y^n$ . Let  $r - 2q^n = x^n$ . Simultaneous solution of  $r + 2q^n = z^n$  with  $r - 2q^n = x^n$  and  $4q^n = y^n$  yields  $r = \frac{z^n + x^n}{2}$ , and,  $q^n = \frac{z^n - x^n}{4}$ . Since  $r, q$  in identity (5) have unrestricted real values  $> 0$ , we can substitute  $\frac{z^n + x^n}{2}$  for  $r$  in (5), and we can substitute  $\frac{z^n - x^n}{4}$  for  $q^n$  in (5) to transform (5) into (3).

Taking a rational subset of each side of (7) :  $(r + 2q^n, 4q^n, r - 2q^n) \in \mathbb{Q}$  implies  $q^n, r \in \mathbb{Q}$ , resulting in (8) with both subsets empty, or both subsets nonempty :

(8)  $\{(r + 2q^n, 4q^n, r - 2q^n) | r, q^n \in \mathbb{Q}, q \in \mathbb{R}, r > 2q^n, (r + 2q^n) - (4q^n) = (r - 2q^n)\} = \{(z^n, y^n, x^n) | z, y, x \in \mathbb{R}, z^n, y^n, x^n \in \mathbb{Q}, z^n - y^n = x^n\}$ , yielding (9):

(9)  $(r + 2q^n, 4q^n, r - 2q^n) \in (8) = (z^n, y^n, x^n) \in (8)$ .

Taking the n-th root on each side of (9) yields (10) :

(10)  $((r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}} \in (9)) = (z, y, x) \in (9)$ .

So, we get (11), for  $n > 0$ , with both sets empty, or both nonempty, as follows :

(11)  $\{((r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}}) | q \in \mathbb{R}, (r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}}, r, q^n \in \mathbb{Q},$   
with  $r > 2q^n, ((r + 2q^n)^{\frac{1}{n}})^n - (2^{\frac{2}{n}} q)^n = ((r - 2q^n)^{\frac{1}{n}})^n = \{(z, y, x) | z, y, x \in \mathbb{R}, z^n, y^n, x^n \in \mathbb{Q}, z^n - y^n = x^n\}$ .

Taking a more restricted rational subset of each side of (11) yields (12), below, with both subsets empty, or both subsets nonempty, for  $n > 0$ , with  $r > 2q^n$  :

(12)  $\{((r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}}) | q, r, (r + 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}} q, (r - 2q^n)^{\frac{1}{n}} \in \mathbb{Q}$  with  $((r + 2q^n)^{\frac{1}{n}})^n - (2^{\frac{2}{n}} q)^n = ((r - 2q^n)^{\frac{1}{n}})^n = \{(z, y, x) | z, y, x \in \mathbb{Q}, z^n - y^n = x^n\}$ .

## 3. RESULTS AND CONCLUSION

Hence, per (12), values of  $q$  that are solely  $q \in \mathbb{Q}$  are sufficient for our proof.

In this section, as with section (1), above, for convenience only :

(13) Let  $(r + 2q^n)^{\frac{1}{n}} \in (12)$  be  $s$ .

(14) Let  $2^{\frac{2}{n}}q \in (12)$  be  $t$ .

(15) Let  $(r - 2q^n)^{\frac{1}{n}} \in (12)$  be  $u$ .

(16)  $\{(s, t, u) | s, t, u \in (12)\} = \{(z, y, x) | z, y, x \in (12)\}$  per (12),(11),(12),(13).

Taking the integral subset of each side of (16) results in (17), below :

(17)  $\{(s, t, u) | s, t, u \in \mathbb{N}, s^n - t^n = u^n\} = \{(z, y, x) | z, y, x \in \mathbb{N}, z^n - y^n = x^n\}$ .

Some concrete examples of (17) : For  $n = 2$ , with  $z = 5, y = 4, x = 3$ , there is a corresponding  $s = 5, t = 4, u = 3$  resulting from  $r$  in (B) = 17 and  $q$  in (B) = 2. For  $n = 1$ , with  $z = 13, y = 12, x = 1$  in (A), there is a corresponding  $s = 13, t = 12, u = 1$  resulting from  $r$  in (B) = 7 and  $q$  in (B) = 3.

(18)  $\{t | t \in \mathbb{Q}, s, u \in \mathbb{R}, s, t, u > 0, s^n - t^n = u^n\} = \emptyset$  for  $n > 2$ , which is true since  $t$  is  $2^{\frac{2}{n}}q$ , per (14), so,  $2^{\frac{2}{n}}q$  is irrational with  $q \in \mathbb{Q}$ .| Hence, per (17),(18) :

(19)  $\{y | z, y, x \in \mathbb{N}, z, y, x > 0, z^n - y^n = x^n\} = \emptyset$  for  $n > 2$ .| Thus, per (A),(19):

(20)  $\{(z, y, x) | z, y, x \in \mathbb{N}, z, y, x > 0, z^n - y^n = x^n\} = \emptyset$  for  $n > 2$ .| So, per (20):

(21)  $x^n + y^n = z^n$ , for  $n \in \mathbb{N}, n > 2$ , does not hold for  $x, y, z \in \mathbb{N}, x, y, z > 0$ .

QED.