The Standard Model reformulated in terms of a derivative-free analogue of the Lagrangian density

Julian Brown, Gernos Farm, Maesllyn, Wales

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Abstract

This paper introduces a zero dimensional scalar functional ($\mathcal{Q}$). In close analogy with the Lagrangian density ($\mathcal{L}$) of the Standard model, application of a variational principle to $\mathcal{Q}$ appears to yield the correct propagation and interation of physical fields, without recourse to either the gauge freedoms or ghost modes of the conventional theory.

1 Space-time anti-derivatives

Consider a complex spinor $\Lambda = \big(\lambda_1 + i\lambda_2, \quad \lambda_3 + i\lambda_4\big)$ with a $U(1)$ degenerate mapping onto the past null cone with vertex at the origin of x:

$$x_\mu = \Lambda^* \sigma_\mu \Lambda$$

$$d\Lambda = \prod_{i=1}^{4} d\lambda_i = 2\pi\delta(t^2 - r^2) d^4x = 2\pi \frac{d^3\mathbf{r}}{r}$$

It can be shown that, for all $k^2 \neq 0$

$$\int_{-\infty}^{0} e^{-ik_\nu x^\nu} d\Lambda = \frac{1}{k^2} \tag{1}$$

and

$$\int_{-\infty}^{0} x_\mu e^{-ik_\nu x^\nu} d\Lambda = \frac{2ik_\mu}{k^4} \tag{2}$$

The product of two Λ cones maps onto $(x_\mu x^\mu > 0)$ space-time with a $U(1)_L \times SU(2) \times U(1)_R$ degeneracy:

$$x_\mu = a_\mu + b_\mu = (\Lambda_a^*, \Lambda_b^*)[\gamma_\mu \otimes \mathbf{1}](\Lambda_a, \Lambda_b)$$

$$\iint e^{-ik_\nu x^\nu} d\Lambda_a d\Lambda_b = \frac{1}{k^4} \tag{3}$$

$$d\Lambda_a d\Lambda_b = 4\pi^2 d^4x$$

So, for an arbitrary field f:

$$f = \partial_\nu \partial^\nu \int f d\Lambda = \frac{1}{2} \partial^\mu \partial_\nu \partial^\nu \int f x_\mu d\Lambda = (\partial_\nu \partial^\nu)^2 \iint f d\Lambda_a d\Lambda_b \tag{4}$$

2 The Standard Model reconstrued

We will now investigate what happens when we construct dimensionless integrals over cones ($\mathcal{Q}_X$) from scalar products of fields (of definite spin, helicity and charge) and then demand that these integrals ($\mathcal{Q}_X$) be invariant w.r.t. variation in each field.

2.1 Fermions

2.1.1 Massless fermions

Given a massless spinor ν_L we define:

$$\mathcal{Q}_\nu \equiv i \int x^\alpha \nu_L^\dagger \sigma_\alpha \nu_L d\Lambda$$

$$\frac{\partial \mathcal{Q}_\nu}{\partial \nu_L^\dagger} = 0 \implies \int x^\alpha \sigma_\alpha \nu_L d\Lambda = 0 \implies i\sigma_\alpha \partial^\alpha \nu_L = 0$$

.. which describes a freely propagating massless neutrino.

2.1.2 Massive fermions

Given a massive Dirac spinor $\begin{pmatrix} e_L \\ e_R \end{pmatrix}$ we define:

$$\mathcal{Q}_e \equiv i \int x^\alpha \left[e_L^\dagger \sigma_\alpha e_L + e_R^\dagger \tilde{\sigma}_\alpha e_R \right] d\Lambda - m_e \iint \left[e_L^\dagger e_R + e_R^\dagger e_L \right] d\Lambda_a d\Lambda_b$$

$$\frac{\partial \mathcal{Q}_e}{\partial e_L^\dagger} = 0$$

$$\implies \int x^\alpha \sigma_\alpha e_L d\Lambda - m_e \iint e_R d\Lambda_a d\Lambda_b$$

$$\implies i\sigma_\alpha \partial^\alpha e_L - m_e e_R = 0$$

Similiarly,

$$\frac{\partial \mathcal{Q}_e}{\partial e_R^\dagger} = 0 \implies i\tilde{\sigma}_\alpha \partial^\alpha e_R - m_e e_L = 0$$

.. which pair of equations describe a freely propagating electron.

2.2 Charged massive fermions

Given additionally a 4-component field $\mathcal{A}$ that couples to e_L and e_R equally, we define:

$$\begin{aligned} \mathcal{Q}_{ae} \equiv \mathcal{A}_j^2 - m_e \iint \left[\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L \right] d\Lambda_a d\Lambda_b \\ + \int x^\alpha \left[\psi_L^\dagger e^{q\mathcal{A}_j\sigma_j} \sigma_\alpha e^{q\mathcal{A}_j\sigma_j} \psi_L + \psi_R^\dagger e^{-q\mathcal{A}_j\sigma_j} \tilde{\sigma}_\alpha e^{-q\mathcal{A}_j\sigma_j} \psi_R \right] d\Lambda \end{aligned} \tag{5}$$

..where $\psi_L \equiv e^{-q\mathcal{A}_j\sigma_j} e_L$ and $\psi_R \equiv e^{q\mathcal{A}_j\sigma_j} e_R$ replace e_L and e_R as the independent fermionic fields.

$$\frac{\partial \mathcal{Q}_{ae}}{\partial \psi_L^\dagger} = 0$$

$$\implies \int x^\alpha e^{q\mathcal{A}_j\sigma_j} \sigma_\alpha e_L d\Lambda - m_e \iint e^{q\mathcal{A}_j\sigma_j} e_R d\Lambda_a d\Lambda_b = 0$$

$$\implies e^{q\mathcal{A}_j\sigma_j} \left[i\sigma_\alpha \overleftrightarrow{\partial}^\alpha e_L - m_e e_R \right] = 0$$

$$\implies i\sigma_\alpha (\partial^\alpha - qA^\alpha) e_L - m_e e_R = 0$$

.. which describes an electron interacting with an electromagnetic field having (fixed gauge) 4-vector A_μ components that are related to the $j = \{1, 2, 3\}$ components of $\mathcal{A}$ by:

$$A^0 = i\partial_j \mathcal{A}_j \qquad A^j = i\epsilon^{jkl}\partial_k \mathcal{A}_l - i\partial_0 \mathcal{A}_j \tag{6}$$

The requirement that

$$\frac{\partial \mathcal{Q}_{ae}}{\partial \mathcal{A}_j^*} = 0$$

yields the source equation

$$\implies \mathcal{A}_j = -q \int x^\alpha \left[e_L^\dagger \sigma_j \sigma_\alpha e_L + e_R^\dagger \sigma_j \tilde{\sigma}_\alpha e_R \right] d\Lambda$$

with conserved charge:

$$\mathcal{A}_0 = -q \int x^\alpha \left[e_L^\dagger \sigma_\alpha e_L + e_R^\dagger \sigma_j e_R \right] d\Lambda = \int x^\alpha j_\alpha d\Lambda$$

2.3 Scalar fields

Given a scalar field $\phi = \frac{1}{\sqrt{2}}(v + h)$ where v is a v.e.v. and h is a dynamic perturbation, we define:

$$\mathcal{Q}_\phi \equiv \int \phi^* \phi d\Lambda - \lambda \iint (\phi^* \phi)^2 d\Lambda_a d\Lambda_b$$

The requirement that:

$$\frac{\partial \mathcal{Q}_H}{\partial \phi^*} = 0$$

yields:

$$\implies \int \phi d\Lambda + 2\lambda \iint (\phi^* \phi) \phi d\Lambda_a d\Lambda_b = \iint \left[\partial_\nu \partial^\nu - 2\lambda(\phi^* \phi) \right] \phi d\Lambda_a d\Lambda_b = 0$$

$$\implies [\partial_\nu \partial^\nu - \lambda h^2 - 2\lambda v h - \lambda v^2] h = 0$$

..which describes a scalar field of mass $M_H = \sqrt{\lambda} v$, with (3- and 4-vertex) self-couplings.

2.4 Massive bosons

In the $\begin{pmatrix}\nu_L\\ e_L\\ e_R\end{pmatrix}$ space, the free Higgs field responsible for lepton mass has operator form:

$$\Phi_0 = \frac{1}{\sqrt{2}}(v+h)\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$\langle 0|\,\Phi_0\,|0\rangle = 0$ but $\langle 0|\,\Phi_0^2\,|0\rangle = \frac{1}{2}v^2$
In the presence of $SU(2)\times U(1)_Y$ field bosons, the fermions transform as

$$\psi \equiv \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix} \equiv e^{\Theta}\begin{pmatrix}\nu_L\\ e_L\\ e_R\end{pmatrix} \quad , \quad \psi^\dagger \equiv \begin{pmatrix}\nu_L^\dagger & e_L^\dagger & e_R^\dagger\end{pmatrix} e^{\Theta} \tag{7}$$

and the Higgs operator transforms as:

$$\Phi \equiv \frac{1}{\sqrt{2}}(v+h)e^{-\Theta}\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}e^{-\Theta} \tag{8}$$

where

$$\Theta \equiv \begin{pmatrix}\Theta_L & 0\\ 0 & \Theta_R\end{pmatrix} \equiv \frac{1}{2}\begin{pmatrix}g\mathcal{W}_j^3 - g'\mathcal{B}_j & g\mathcal{W}_j^+ & 0\\ g\mathcal{W}_j^- & -g\mathcal{W}_j^3 - g'\mathcal{B}_j & 0\\ 0 & 0 & -2g'\mathcal{B}\end{pmatrix}\sigma_j$$

$$\begin{aligned}\langle 0|\,\Phi^2\,|0\rangle &= \frac{1}{8}(v+h)^2\left[4 + g^2\mathcal{W}_j^-\mathcal{W}_j^+ + g^2\mathcal{W}_j^3\mathcal{W}_j^3 - 2gg'\mathcal{B}_j\mathcal{W}_j^3 + g'^2\mathcal{B}_j\mathcal{B}_j\right]\\ &= \frac{1}{8}(v+h)^2\left[4 + g^2\mathcal{W}_j^-\mathcal{W}_j^+ + (g^2+g'^2)\mathcal{Z}_j\mathcal{Z}_j\right]\end{aligned} \tag{9}$$

where we have replaced $\mathcal{W}^3, \mathcal{B}$ by mass eigenstates:

$$\mathcal{Z}_j \equiv \frac{1}{\sqrt{g^2+g'^2}}(g\mathcal{W}_j^3 - g'\mathcal{B}_j) \qquad \mathcal{A}_j \equiv \frac{1}{\sqrt{g^2+g'^2}}(g'\mathcal{W}_j^3 + g\mathcal{B}_j)$$

we define:

$$\begin{aligned}\mathcal{Q}_H &\equiv \mathcal{W}_j^{1*}\mathcal{W}_j^1 + \mathcal{W}_j^{2*}\mathcal{W}_j^2 + \mathcal{W}_j^{3*}\mathcal{W}_j^3 + \mathcal{B}_j^*\mathcal{B}_j\\ &\qquad + \int \langle 0|\,\Phi^2\,|0\rangle\, d\Lambda - \lambda \iint \langle 0|\,\Phi^4\,|0\rangle\, d\Lambda_a d\Lambda_b\\ &= \mathcal{W}_j^+\mathcal{W}_j^- + \mathcal{Z}_j^*\mathcal{Z}_j + \mathcal{A}_j^*\mathcal{A}_j - \frac{\lambda}{4}\iint (v+h)^4 d\Lambda_a d\Lambda_b\\ &\qquad + \int \frac{1}{8}(v+h)^2\left[4 + [g^2\mathcal{W}_j^+\mathcal{W}_j^- + (g^2+g'^2)\mathcal{Z}_j\mathcal{Z}_j]\right]d\Lambda\end{aligned} \tag{10}$$

where

$$\begin{aligned}\frac{\partial \mathcal{Q}_H}{\partial \mathcal{W}_j^+} = 0 &\implies [\partial_\mu\partial^\mu - \frac{1}{4}g^2(v+h)^2]\mathcal{W}_j^- = 0\\ \frac{\partial \mathcal{Q}_H}{\partial \mathcal{Z}_j} = 0 &\implies [\partial_\mu\partial^\mu - \frac{1}{4}(g^2+g'^2)(v+h)^2]\mathcal{Z}_j = 0\\ &\implies M_W = \frac{1}{2}gv \quad M_{\mathcal{Z}} = \frac{1}{2}\sqrt{g^2+g'^2}\ v \quad M_{\mathcal{A}} = 0\end{aligned}$$

2.5 Electroweak charged fermions

Using the definitions (7) and (8) and

$$\Theta \equiv \begin{pmatrix} \Theta_L & 0 \\ 0 & \Theta_R \end{pmatrix} \equiv \frac{g}{2} \begin{pmatrix} c_\theta^{-1} \mathcal{Z}_j & \mathcal{W}_j^+ & 0 \\ \mathcal{W}_j^- & (s_\theta t_\theta - c_\theta)\mathcal{Z}_j - 2 s_\theta \mathcal{A}_j & 0 \\ 0 & 0 & 2(s_\theta \mathcal{A}_j - t_\theta \mathcal{Z}_j) \end{pmatrix} \sigma_j$$

$$\Longrightarrow \ e^{\Theta} = \begin{pmatrix} e^{\Theta_L} & 0 \\ 0 & e^{\Theta_R} \end{pmatrix}$$

..we define:

$$\mathcal{Q}_{bf} \equiv \mathcal{W}_j^+ \mathcal{W}_j^- + \mathcal{Z}_j^2 + \mathcal{A}_j^2 - y_e \iint \psi^\dagger \Phi \psi d\Lambda_a d\Lambda_b \\ + \int x^\alpha \big[\psi_L^\dagger e^{-\Theta_L} \sigma_\alpha e^{-\Theta_L} \psi_L + \psi_R^\dagger e^{-\Theta_R} \tilde{\sigma}_\alpha e^{-\Theta_R} \psi_R \big] d\Lambda \quad (11)$$

$$\frac{\partial \mathcal{Q}_{bf}}{\partial \psi^\dagger} = 0 \ , \qquad h = 0 \ , \qquad m_e = \frac{1}{\sqrt{2}} y_e v$$

$$\Longrightarrow \ \int x^\alpha e^{-\Theta} \begin{pmatrix} \sigma_\alpha & 0 & 0 \\ 0 & \sigma_\alpha & 0 \\ 0 & 0 & \tilde{\sigma}_\alpha \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \\ e_R \end{pmatrix} d\Lambda - m_e \iint e^{-\Theta} \begin{pmatrix} 0 \\ e_R \\ e_L \end{pmatrix} d\Lambda_a d\Lambda_b = 0$$

$$\Longrightarrow \ e^{-\Theta} \begin{pmatrix} i\sigma_\alpha \overleftrightarrow{\partial_\alpha} & 0 & 0 \\ 0 & i\sigma_\alpha \overleftrightarrow{\partial_\alpha} & m_e \\ 0 & m_e & i\tilde{\sigma}_\alpha \overleftrightarrow{\partial_\alpha} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \\ e_R \end{pmatrix} = 0$$

$$\Longrightarrow \begin{cases} i\sigma_\alpha [\partial_\alpha - \partial_\alpha \Theta_L] \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = m_e \begin{pmatrix} 0 \\ e_R \end{pmatrix} \\ i\tilde{\sigma}_\alpha [\partial_\alpha - \partial_\alpha \Theta_R] \begin{pmatrix} 0 \\ e_R \end{pmatrix} = m_e \begin{pmatrix} 0 \\ e_L \end{pmatrix} \end{cases}$$

...which given (6) together with:

$$W^0 = i\partial_j \mathcal{W}_j \qquad W^j = i\epsilon^{jkl} \partial_k \mathcal{W}_l - i\partial_0 \mathcal{W}_j \quad (12)$$

$$Z^0 = i\partial_j \mathcal{Z}_j \qquad Z^j = i\epsilon^{jkl} \partial_k \mathcal{Z}_l - i\partial_0 \mathcal{Z}_j \quad (13)$$

...yields the Dirac equation.
The complete scalar functional describing the propagation and interaction of leptons and electroweak bosons is:

$$\mathcal{Q} = \mathcal{W}_j^+ \mathcal{W}_j^- + \mathcal{Z}_j^* \mathcal{Z}_j + \mathcal{A}_j^* \mathcal{A}_j - \frac{\lambda}{4} \iint (v+h)^4 d\Lambda_a d\Lambda_b \\ + \int (v+h)^2 \big[\frac{1}{2} + \frac{1}{8}[g^2 \mathcal{W}_j^+ \mathcal{W}_j^- + (g^2 + g'^2) \mathcal{Z}_j \mathcal{Z}_j]\big] d\Lambda - y_e \iint \psi^\dagger \Phi \psi d\Lambda_a d\Lambda_b \\ + \int x^\alpha \big[\psi_L^\dagger e^{-\Theta_L} \sigma_\alpha e^{-\Theta_L} \psi_L + \psi_R^\dagger e^{-\Theta_R} \tilde{\sigma}_\alpha e^{-\Theta_R} \psi_R \big] d\Lambda \quad (14)$$

From (6), (12) and (13) it follows that for bosons with 4-momentum $k = (E, 0, 0, p)$, the 3-component fields corresponding to gauge vector fields for helicity eigenstates (normalized to E particles per unit volume) are:

$$
\begin{aligned}
\{\hat{\mathcal{W}}, \hat{\mathcal{Z}}\}_\pm &= \frac{\sigma_1 \pm i\sigma_2}{\sqrt{2}(E+p)} e^{-ik\cdot x} \iff \{\hat{A}, \hat{W}, \hat{Z}\}_\pm = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} e^{-ik\cdot x} \\
\{\hat{\mathcal{W}}, \hat{\mathcal{Z}}\}_L &= \frac{\sigma_3}{M_{\{W,Z\}}} e^{-ik\cdot x} \iff \{\hat{W}, \hat{Z}\}_L = \frac{1}{M_{\{W,Z\}}} \begin{pmatrix} p \\ 0 \\ 0 \\ E \end{pmatrix} e^{-ik\cdot x}
\end{aligned}
\tag{15}
$$

where $M^2 = E^2 - p^2$, so the zero helicity (longitudinal) mode of the massless $\mathcal{A}$ does not exist.