Refutation of the lonely runner conjecture with three runners

Abstract: We evaluate the conjecture of the lonely runner with three runners. We do not assume a runner may be stationary as a no-go contestant. The result is that the conjecture diverges from tautology by one logical value and hence is refuted. We also assume a runner can be stationary; those non tautologous results also refute the conjecture.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET   p, q, r, s:   runner-1, runner-2, runner-3, number of runners;
~ Not;   +  Or ;   &  And;   \ Not And;   >  Imply;   =  Equivalent;   @  Not Equivalent;
(p=p) tautology;   (p@p) contradiction, zero 0.

From: en.wikipedia.org/wiki/Lonely_runner_conjecture

Remark 0: Other implementations of the conjecture assume a runner may not run but remain stationary, and name that the lonely runner. However this initial implementation makes no such assumption because a non-runner is not a runner and hence removed from consideration.

No runner is stationary.   (1.1.1)

(((p+q)+r)@(p@p)) = (p=p) ;   FTTT TTTT FTTT TTTT   (1.1.2)

No runner as equivalent to another runner implies the number of runners.   (1.2.1)

((((p@q)&(q@r))&(p@r))>s) = (p=p) ;   TTTT TTTT TTTT TTTT   (1.2.2)

No runner is stationary, and no runner as equivalent to another runner implies the number of runners.   (1.3.1)

((((p+q)+r)@(p@p))&((((p@q)&(q@r))&(p@r))>s)) = (p=p) ;
FTTT  TTTT  FTTT  TTTT

Remark 1.1/2/3: While the truth table results for Eqs. 1.1.2 and 1.3.2 as rendered are equivalent, Eq. 1.2.2 is needed to establish that the unique runners establish the number of runners. Eqs. 1 as cast with model operators weaken the result.

One runner implies the fraction of a runner divided by the number of runners.   (2.1.1)

(((p>((q+r)\s))+(q>((p+r)\s)))+(r>((p+q)\s))) = (p=p) ;
TTTT  TTTT  TTTT  TTTT  F
One runner is equivalent to the fraction of a runner divided by the number of runners. \( (p=((q+r)s)+(q=((p+r)s))+(r=((p+q)s))) = (p=p) ; \)

\[ (p=p) \]

Remark 2: Eqs. 2.1 and 2.2 show respectively one runner implying a fraction is closer to tautology than is one runner equivalent to a fraction.

We evaluate two arguments with the antecedent of Eqs. 1 and consequent of 2.1 / 2.2.

No runner is stationary, and no runner as equivalent to another runner implies the number of runners which implies one runner implies the state of 1 divided by the number of runners.

\[ (((p+q)+r)@(p@p))&(((p@q)&(q@r))&(p@r))>s) >
(((p>((q+r)s))+(q>((p+r)s)))+(r>((p+q)s))) ; \]

\[ TTTT TTTT TTTT TTTF \]

Remark 3: Eqs. 3.1 and 3.2 produce the same truth table result as close to tautology but divergent by one \( F \) value. This is due to \( T>F=F \).

If we ignore Eq. 1.1 to establish that a runner can be permitted as stationary, to adopt the common assumption, the truth table result analogs for Eqs. 3 become:

\[ (((p@q)&(q@r))&(p@r))>s) >
(((p>((q+r)s))+(q>((p+r)s)))+(r>((p+q)s))) ; \]

\[ FTTT TTTT FTTT TTTF \]

Remark 4: By admitting a stationary runner, the conjecture is weakened for the implication of a runner as a fraction, but remains the same result as Eq. 3.2 for the equivalence of a runner as a fraction.

Excepting Eq. 1.2, the other Eqs. are not tautological. This means that with or without assuming a runner can be stationary as a no-go, the lonely runner conjecture is refuted.