Abstract: We evaluate a modal logic for partial awareness from a published example. The definitions and conjectures are not tautologous. We show how to exclude a priori logical clauses to promote a perhaps unintended tautology for the example. However, our evaluation does not rely on modal operators, suggesting that the system as proffered should be renamed to a logic for awareness, without the word modal.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

Remark 4.3: Since Ex. 4.1 and 4.2 are related with total verbiage greater than Ex 4.3, we select Ex. 4.3 to evaluate.


Remark 4.3: We evaluate Example 4.3 because its verbiage is less than that for the related Examples 4.1 and 4.2.

\[
\begin{align*}
P_{w_1}^i &= d_1 \\
(p \& x) &= u; & \begin{array}{ccccccc}
T & T & T & T & T & T & T \\
F & F & F & F & F & F & F \\
T & F & T & F & T & F & T \\
F & T & F & T & F & T & F \\
\end{array} & \quad (4.3.1.1) \\
Q_{w_3}^i &= d_2 \\
(q \& z) &= v; & \begin{array}{ccccccc}
T & T & T & T & T & T & T \\
F & F & F & F & F & F & F \\
T & F & F & F & F & F & F \\
F & F & F & F & F & F & F \\
\end{array} & \quad (4.3.2.1) \\
P_{w_2}^i &= P_{w_3}^i = Q_{w_2}^i = \text{zero} & \quad (4.3.3.1) \\
(p \& y) &= ((p \& z) = (q \& y) = (p \& p)); & \begin{array}{ccccccc}
T & T & T & T & T & T & T \\
F & F & F & F & F & F & F \\
T & F & F & F & F & F & F \\
F & F & F & F & F & F & F \\
\end{array} & \quad (4.3.3.2) \\
R_w^i &= d_1 \\
(r \& w) &= u; & \begin{array}{ccccccc}
T & T & T & T & T & T & T \\
F & F & F & F & F & F & F \\
T & T & T & T & T & T & T \\
F & F & F & F & F & F & F \\
\end{array} & \quad (4.3.4.1)
\end{align*}
\]
\( A_1w = \text{null [or] (P or Q) [or] zero} \) \hspace{1em} (4.3.5.1)

\[
(s&w)=((p@p)+((p+q)+(p@p))) ;
\]
\[
\text{FFF TFF TFFF TFFF, TFFFF TFFTT FTTT} \hspace{1em} (4.3.5.2)
\]

\( A_2w = \text{null [or] (Q or R) [or] zero} \) \hspace{1em} (4.3.6.1)

\[
(t&w)=((p@p)+((q+r)+(p@p))) ;
\]
\[
\text{TTFF FFFFF TTTT TTTT, FFFFF TTTT} \hspace{1em} (4.3.6.2)
\]

"agent_1 wants \( d_1 \) only when it has property \( P \) (to trade in states \( w_2 \) [or] \( w_3 \)), and agent_2 wants \( d_2 \) only when it has property \( Q \) (to trade in states \( w_1 \) and \( w_2 \))"

"for agent_1, \( w_2 \) and \( w_3 \) are equivalent, and for agent_2, \( w_1 \) and \( w_2 \) are equivalent."

\[
((p&(y+z))>(s>u))>(y=z) ;
\]
\[
\text{TTTT TTTT TTTT TTTT, FFFFF FFFFF FFFFF FFFFF, FFFFF FFFFF FTFT FTTT} \hspace{1em} (4.3.7.2)
\]

\[
((q&(w&x))>(t>v))>(x=y) ;
\]
\[
\text{TTTT TTTT TTTT TTTT, FFFFF FFFFF FFFFF FFFFF} \hspace{1em} (4.3.8.2)
\]

"However, neither agent can propose an acceptable contract."

**Remark 9.1:** To evaluate Eq. 4.3.9/10 we process Eqs. 4.3.7.2 or 4.3.8.2 respectively as the consequent of the definitions in Eqs. 4.3.1.2/6.2.

\[
((((p&x)=u)&((q&z)=v))&(((p&y)=((p&q)=(p@p))))&((r&w)=u))&(((s&w)=((p@p)+((p+q)+(p@p)))&((t&w)=((p@p)+((q+r)+(p@p)))))) > (((p&(y+z))>(s>u))+(q&(w&x))>(t>v))>(x=y)) ;
\]
\[
\text{TTTT TTTT TTTT TTTT, FTTTT TTTT FTTTT TTTT, FTTTT TTTT TTTT TTTT, TTTTT TTTT TTTT TTTT} \hspace{1em} (4.3.9.2)
\]

Eqs. 4.3.9.2 as rendered is *not* tautologous, and hence as presented "neither agent can propose an acceptable contract."

**Remark 4.3.10:** To rehabilitate Eq. 4.3.9.2, we exclude the agent clauses from Eqs. 4.3.7/8 for \( w \)-equivalences as potential a priori commentary.

\[
((((p&x)=u)&((q&z)=v))&(((p&y)=((p&q)=(p@p))))&((r&w)=u))&(((s&w)=((p@p)+((p+q)+(p@p)))&((t&w)=((p@p)+((q+r)+(p@p)))))) > (((p&(y+z))>(s>u))+(q&(w&x))>(t>v)) ;
\]
\[
\text{TTTT TTTT TTTT TTTT} \hspace{1em} (4.3.10.2)
\]
Eq. 4.3.10.2 is tautologous, hence without the injected agent w-equivalences, the agents can propose an acceptable contract. We do not guess what that contract is.

Excepting Eq. 4.3.10, the others are *not* tautologous. This means the example does not support a modal logic for partial awareness. We note that modal operators were not used by us here at all.

Our conclusion is not to refute the notion of a partial awareness in semantics. This can be construed as a newly coined academic term for $\mathcal{VL}4$, where the four-valued logic purposely codifies falsity and truthity based on exact truth table results in the range from contradiction to tautology. Because of that, $\mathcal{VL}4$ is better suited for the *exact* analysis of partial awareness with or without modal operators.