Refutation of the Frauchiger-Renner thought paradox as a quantum model

Abstract: We use modal logic to evaluate a quantum model of the Frauchiger-Renner thought experiment as not a contradiction (paradox) *and* not a tautology (theorem). The example misapplies the Born rule which we refute elsewhere.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p , q , r, s, t , u , v , w , x , y , z :
Alice, |1>, R, S, memory, Ursula, Bob, Wigner, √1, √2, √3,
~q |0>;
~ Not; + Or ; & And; \ Not And; > Imply; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p=p) ok, tautology, ordinal three 3; (p@p) fail, contradiction, zero 0;
(%p>#r) truthity, ordinal one 1; (%p>#r) falsity, ordinal two 1;
a=0 ((p&t)=(~q&p)); a=1 ((p&t)=(q&p);
b=0 ((v&t)=(~q&v)); b=1 ((v&t)=(q&v).

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Initial settings:

R = ((√(1/3)|0>$_R$)+((√(2/3)|1>$_R$) (1.0.1.1)

r=(((x\y)&(~q&r))+((y\z)&(q&r))) ;
TTTT TTTT TTTT TTTT, TTTT **FF**TT TTTT **FF**TT
TTTT TT**FF** TTTT TT**FF**, TTTT **FFFF** TTTT **FFFF** (1.0.1.2)

S = |0>$_S$ (1.0.2.1)

s=(~q&s) ;
TTTT TT**FF** TTTT TT**FF** (1.0.2.2)

Alice memory = |0>$_A$ (1.0.3.1)

(p&t)=(~q&p) ;
T**F**TT T**F**TT T**F**TT T**F**TT, TTT**F** TTT**F** TTT**F** TTT**F** (1.0.3.2)

Bob memory = |0>$_B$ (1.0.4.1)

(v&t)=(~q&v) ;
TTTT TTTT TTTT TTTT(4), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.4.2)

Ursula memory = $|0>_{U}$ (1.0.5.1)

(u&t)=(~q&u) ;
TTTT TTTT TTTT TTTT(2), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.5.2)

Wigner memory = $|0>_{W}$ (1.0.6.1)

(w&t)=(~q&w) ;
TTTT TTTT TTTT TTTT(8), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.6.2)

T 1: **Remark 1:** We map agent *T* = <1, 2, 3, 4, 5> using instructions in the text as best as we can follow.

R = $|0>_{R}$, $|1>_{R}$ (1.1.1.1)

r=((~q&r)+(q&r)) ;
TTTT TTTT TTTT TTTT(128) (1.1.1.2)

Alice records the result in her memory A. (1.1.2.1)

(p&t)=((~q&p)+(q&p));
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**, TTTT TTTT TTTT TTTT (1.1.2.2)

Alice prepares S :
if outcome a = 0 then her memory is $|0>_{A}$ and S is $|0>_{S}$; or
if outcome a = 1 then her memory is $|1>_{A}$ and S is
$(1/\sqrt{2})((|0>_{S})+|1>_{S}))$ (1.1.3.1)

(((p&t)=(~q&p))>(((p&t)=(~q&p))&(s=(~q&s)))) +
(((p&t)=(q&p))>(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s)))))) ;
TTTT TTTT TT**F**T TT**F**T, TTTT TTTT TTTT TTTT (1.1.3.2)

Alice replaces Bob's system S with her own. (1.1.4.1)

s=((((p&t)=(~q&p))>(((p&t)=(~q&p))&(s=(~q&s)))) +
(((p&t)=(q&p))>(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s))))))) ;
FFFF FFFF TTTT TTTT, **FFFF FFFF** TT**F**T TT**F**T (1.1.4.2)

T 2: **Remark 2:** We follow *T* 1 by replacing Alice with Bob and R with S, but exclude Eq. 1.1.3.1 for Bob.

S = $|0>_{S}$, $|1>_{S}$ (2.1.1.1)

s=((~q&s)+(q&s)) ;
TTTT TTTT TTTT TTTT(128) (2.1.1.2)

Bob records the result in his memory B. (2.1.2.1)

(v&t)=((~q&v)+(q&v));
TTTT TTTT TTTT TTTT(4), **FFFF FFFF FFFF FFFF** (2.1.2.2)

Bob prepares R :
if outcome b = 0 then his memory is $|0>_B$ and R is $|0>_R$; or
if outcome b = 1 then his memory is $|1>_B$ and R is
$(1/\sqrt{2})((|0>_R)+|1>_R))$ (2.1.3.1)

(((v&t)=(~q&v))>(((v&t)=(~q&v))&(r=(~q&r))))
+
(((v&t)=(q&v))>(((v&t)=(q&v))&(r=((x\y)&((~q&r)+(q&r)))))) ;
TTTT TTTT TTTT TTTT, TTTT TT**FF** TTTT TT**FF** (2.1.3.2)

T 3: Ursula measures and records the result of Alice's lab as:

RA = $|ok>_{RA}$, $|fail>_{RA}$ where

$|ok>_{RA} = \sqrt{(1/2)}(|0>_R|0>_A - |1>_R|1>_A)$
$|fail>_{RA} = \sqrt{(1/2)}(|0>_R|0>_A + |1>_R|1>_A)$ (3.1.1.1)

(((r&p)&(p=p))=((x\y)&(((~q&r)&(~q&p))-((q&r)&(q&p))))) +
(((r&p)&(p@p))=((x\y)&(((~q&r)&(~q&p))+((q&r)&(q&p))))) ;
TTTT T**F**T**F** TTTT T**F**T**F**, TTTT TTTT TTTT TTTT (3.1.1.2)

T 4: Wigner measures and records the result of Bob's lab as:

SB = $|ok>_{SB}$, $|fail>_{SB}$ where

$|ok>_{SB} = \sqrt{(1/2)}(|0>_S|0>_B - |1>_S|1>_B)$
$|fail>_{SB} = \sqrt{(1/2)}(|0>_S|0>_B + |1>_S|1>_B)$ (4.1.1.1)

(((s&v)&(p=p))=((x\y)&(((~q&s)&(~q&v))-((q&s)&(q&v))))) +
(((s&v)&(p@p))=((x\y)&(((~q&s)&(~q&v))+((q&s)&(q&v))))) ;
TTTT TTTT TTTT TTTT, TTTT TTTT **FFFF FFFF** (4.1.1.2)

T 5: Ursula and Wigner compare their recorded measurements.
If both are ok, then the experiment ends, otherwise initial settings and timers are reset to repeat. (5.0)

Excepting the obvious theorems of Eqs. 1.1.1.2 and 2.1.1.2, Eqs. 1.2-4. as rendered are *no*t tautologous. This means the model conjectured is not a contradiction (paradox) *and* not a tautology (theorem). Therefore the model is indeterminate.

The authors invoke the Born rule. (We refute the Born rule elsewhere in Everettian quantum mechanics (EQM) as the probability of the wave function squared.) The authors halt the experiment at an injected *T* 2.5 to give a probability of 1/12. In fact, the model cannot halt because at each iteration, initial values are reset .

We ask what is the logical table result of the entire system as rendered, combining each step as an antecedent to imply the next step as a consequent. This amounts to:

If Eqs.1.0, then if T 1 and T 2 then T 3 and T 4. (6.1)

(((r=(((x\y)&(~q&r))+((y\z)&(q&r))))&(s=(~q&s)))&((((p&t)=
(~q&p))&((v&t)=(~q&v)))&(((u&t)=(~q&u))&((w&t)=
(~q&w)))))
>
((((r=((~q&r)+(q&r)))>((p&t)=((~q&p)+(q&p))))>(s=((((p&t)=
(~q&p))>(((p&t)=(~q&p))&(s=(~q&s))))+(((p&t)=(q&p))>
(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s)))))))))
&
(((s=((~q&s)+(q&s)))>((v&t)=((~q&v)+(q&v))))>
(((((v&t)=(~q&v))>(((v&t)=(~q&v))&(r=(~q&r))))+(((v&t)=
(q&v))> (((v&t)=(q&v))& (r=((x\y)&((~q&r)+(q&r)))))))
>
(((((r&p)&(p=p))=((x\y)& (((~q&r)&(~q&p))-((q&r)&(q&p)))))+
(((r&p)&(p@p))=((x\y)&(((~q&r)&(~q&p))+((q&r)&(q&p))))))
&
((((s&v)&(p=p))=((x\y)&(((~q&s)&(~q&v))-((q&s)&(q&v)))))+
(((s&v)&(p@p))=((x\y)&(((~q&s)&(~q&v))+((q&s)&
(q&v)))))))))) ;

FT**F**T **F**T**FF** TTTT TTTT, **FFF**T **FFF**T TTTT T**F**TT,
FFFT **FF**TT TTTT T**F**TT, **FFF**T TT**F**T TTTT TTTT,
FFFT TTTT TTTT TTTT, **FF**TT **FF**TT **FF**TT **F**FTT,
FFTT **FF**TT TTTT T**F**TT, **FF**TT TTTT TTTT TTTT,
FT**F**T **F**TTT TTTT TTTT, **F**T**F**T TT**F**T TTTT TTTT,
FT**F**T TTTT TTTT TTTT, TT**F**T TT**FF** TTTT TTTT,
TT**F**T TT**F**T TTTT TTTT, TT**F**T TTTT TTTT TTTT

(317 steps) (6.2)

Eq. 6.2 is *not* tautologous, meaning the thought experiment is not a paradox (contradiction) *and* not a theorem (tautology).