Refutation of intuitionistic logic on a "transfinite argument"

Abstract: We evaluate intuitionistic logic via Hilbert's "transfinite argument" and Komogorov's implementation. None of the axioms is tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q: A, B; ~ Not, ¬; & And; > Imply, →;
% possibility, for one or some, ∃; # necessity, for all or every, ∀;

From: Coquand, T. (2004). "Kolmogorov's contribution to intuitionistic logic". Chapter 2. in Charpentier, E. et al (Eds). Komogorov's heritage in mathematics. Springer-Verlag. sciencedocbox.com/Physics/65775934-Kolmogorov-s-heritage-in-mathematics.html

"**2.1.2 Kolmogorov's formalization of intuitionistic logic**

Kolmogorov's first contribution in this paper is a complete formalization of *minimal* propositional calculus (a strict subset of intuitionistic logic which is usually attributed to Johansson [Joh36]) and minimal predicate calculus. As indicated by Wang, Kolmogorov's formalization is no less remarkable than Heyting's [Hey30]. The very possibility of such a formalization is already quite surprising, if we reflect that the motivations behind intuitionism were opposed to the process of formalization[9].

[9]According to Wang[Wan87], Brouwer considered this result to be more remarkable and surprising than Gödel's celebrated incompleteness theorem [Göd31].

Kolmogorov's work is final concerning propositional calculus, but less precise with respect to predicate calculus.

The formalization is directly inspired from Hilbert [Hil23], who had suggested the following axioms for implication and negation:

1.A→B→A (1.1)
(p>q)>p ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (1.2)

2. (A→A→B)→A→B (2.1)
(((p>p)>q)>p)>q ; **FF**TT **FF**TT **FF**TT **FF**TT (2.2)

3. (A→B→C)→B→A→C (3.1)
((((p>q)>r)>q)>p)>r ; T**F**T**F** TTTT T**F**T**F** TTTT (3.2)

4. (B→C)→(A→B)→A→C (4.1)
(((q>r)>(p>q))>p)>r ; T**F**T**F** TTTT T**F**T**F** TTT (4.2)

5.$A\rightarrow\neg A\rightarrow B$ (5.1)

(p>~p)>q ; **F**TTT **F**TTT **F**TTT **F**TTT (5.2)

6. $(A\rightarrow B)\rightarrow(\neg A\rightarrow B)\rightarrow B$" (6.1)

(p>q)>(~p>q))>q ; T**F**TT T**F**TT T**F**TT T**F**TT (6.2)

Remark 1.-6.: Eqs. 1.2-6.2 as rendered are *not* tautologous. This means Kolmogorov's adaptation of Hilbert's intuitionistic logic is similarly flawed.

"Hilbert's article [Hil23] raises the problem of justifying the rules of quantification (both existential and universal) over an infinite domain, in particular the following principle

$(\neg\forall x.A)\rightarrow\exists x.\neg A$, (7.3.1)

(~#p&q)>(%p&~q) ; TTFN TTFN TTFN TTFN (7.3.2)

which follows from the Principle of Excluded Middle, and may be used to deduce the existence of an element

$\exists x.\neg A$ (7.2.1)

%p&~q ; CT**FF** CT**FF** CT**FF** CT**FF** (7.2.2)

from a proof of the impossibility of its non-existence

$\neg\forall x.A$[6]. (7.1.1)

~#p&q ; **FF**TC **FF**TC **FF**TC **FF**TC (7.1.2)

This is a typical instance of what Hilbert calls a *transfinite argument*, a terminology which is also used in Kolmogorov's paper (these terms may be somewhat surprising, since the adjective "transfinite" is associated nowadays with the use of the class of countable ordinals, or more generally of uncountable classes)."

Remark 7.: Eqs. 7.1.2, 7.2.2, and 7.3.2 as rendered are *not* tautologous. This refutes Hilbert's use of the term "transfinite argument".

Eqs. 1.-7. are *not* tautologous. This means that Hilbert's intuitionistic logic, and as implemented by Kolmogorov, and by association Heyting, is refuted.