Refutation of the two-sided page paradox

Abstract: We evaluate the two-sided page conjecture that: if either the front page implies the back page is false or the back page implies the front page is true is a paradox (contradiction). The conjecture is a theorem and hence refuted as a paradox.

We assume the method and apparatus of Meth8/VŁ with Tautology as the designated proof value, \( F \) as contradiction, \( N \) as truthity (non-contingency), and \( C \) as falsity (contingency).

The 16-valued truth table is row-major and horizontal.

LET:  \(~\) Not;  \(+\) Or;  \(=\) Equivalent;  \(\neq\) Not Equivalent;
\%
\# possibility, for one or some;  \# necessity, for all or every;
p, \(~p\): page front side, page back side (not page front side);
(p=p) Tautology;  (pp) \( F \) as contradiction;
(%p>#{p}) \( N \) as truthity (non-contingency);  (%p<#{p}) \( C \) as falsity (contingency).


[C]onsider the double contradiction represented by a single sheet of paper with contradictory signs on each face:

The statement on the other side is false.  \( (1.0) \)
The statement on the other side is true. \( (2.0) \)

If we accept either in its entirety, we are in a double-bind, for each leads us into a state of global contradiction, when the other is taken into account. \( (3.0) \)

We write Eqs. 1.0 and 2.0 as:

Front page implies back page is false.
\( (p>\neg p)>(p@p) \); \( F F T T F T F T F F T T \)  \( (1.2) \)

Back page implies front page is true
\( (\neg p>p)>(p=p) \); \( T T T T T T T T T T T T \)  \( (2.2) \)

If either front page implies back page is false or back page implies front page is true implies contradiction.
\( (((p>\neg p)>(pp))+((\neg p>p)>(p=p)))>(p@p) \); \( F F F F F F F F F F F F \)  \( (3.2) \)

Remark 3.2: Eq. 3.2 is not tautologous as asserted in Eq. 3.1, is contradictory, and hence is refuted.
We rewrite Eqs. 1.0, 2.0, and 3.0 to replace false and true respectively with falsity and truthity so as to weaken the assertions.

Front page implies back page is falsity.  \[ (p \rightarrow \neg p) \rightarrow (%p < \#p) ; \quad \text{CTCT CTCT CTCT CTCT} \quad (4.1) \]

Back page implies front page is truthity.  \[ (\neg p \rightarrow p) \rightarrow (%p > \#p) ; \quad \text{TNTN TNTN TNTN TNTN} \quad (5.1) \]

If either front page implies back page is falsity or back page implies front page is truthity implies contradiction.  \[ (((p \rightarrow \neg p) \rightarrow (%p < \#p)) + ((\neg p \rightarrow p) \rightarrow (%p > \#p))) \rightarrow (p \land p) ; \quad \text{FFFF FFFF FFFF FFFF} \quad (6.1) \]

**Remark 6.2:** Eq. 6.2 is not tautologous as asserted in Eq. 6.1, is contradictory, and hence is refuted.

From Eqs. 3.2 and 6.2 as rendered, no paradox (contradiction) exists, and in fact the conjectures in Eqs. 3.1 and 6.1 are theorems.