Elements 2 : The Integral Formula

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abstract

This note presents a elementary integral formula.

Formula

\[
\int_0^{1/4} \frac{\sqrt{1-2x-2x^2+\sqrt{1-4x}}}{x^2} dx - \int_{(\sqrt{2}-1)/2}^{1/4} \frac{\sqrt{1-2x-2x^2-\sqrt{1-4x}}}{x^2} dx =
\]

\[
= \frac{\sqrt{2} \left( \Gamma \left( \frac{1}{4} \right) \right)^2}{8\sqrt{\pi}} - \frac{\pi}{4\sqrt{2}} + \frac{1}{4\sqrt{2}} \ln \left( \frac{2+\sqrt{2}}{2-\sqrt{2}} \right) + \frac{\sqrt{2} \left( \sqrt{2} - 1 \right)}{2}
\]

Remarks:

\[
\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592...
\]

\[
\Gamma \left( \frac{1}{4} \right) = 4 \prod_{n=1}^{\infty} \left( 1 + \frac{1}{n} \right)^{1/4} \left( 1 + \frac{1}{4n} \right)^{-1} = 3.625609...
\]

References