Measurement Quantization Accounts for Galactic Rotational Velocities and Obviates Dark Matter

Jody A. Geiger

Informativity Institute, 30 E Huron Street, Chicago, IL 60611; jodygeiger@informativity.org
Phone: (312) 898-7988; ORCID: 0000-0001-5389-0447

Submitted 2019-01-06

Abstract

A physical description of the orbital dynamics of stars around a galactic core has proved difficult. Notably, there is insufficient mass to account for the observed star velocities. The missing-mass mystery is one of few in modern science that defy the known laws of physics. It has been conjectured that there exists a new form of matter that interacts gravitationally while otherwise remaining undetectable. In this paper, we offer an alternative resolution of this mystery. Its expression does not modify the known laws of physics, contains no free variables or fittings, and is entirely classical in nature. Using the notion of counts of the fundamental measures—length, mass, and time—it is shown that measure is bounded. Accounting for this bound and the expansion of space reveals that the dark matter conjecture is unnecessary while resolving the missing-mass mystery.

Key words: dark energy, dark matter, expanding universe, galaxy dynamics, galaxy rotation, gravitational equilibrium

1. INTRODUCTION

The focus of this paper is a discussion of the rotation velocity of galaxies and the processes that affect and constrain gravity on galactic scales. The effect is an outcome of physically significant smallest units of measure, each of the three fundamental measures constrained by an upper and lower count bound with respect to the two other measures. A framework of countable units of measure—the fundamental measures—provides a mathematical foundation with which to describe phenomena with quantum precision. Most importantly, when a count bound is exceeded, additional mass counts overlap; this is what constrains gravity on galactic scales.

The natural units of measure are most commonly known as Planck's Units, which we identify by appending a subscript p to a quantity's symbol: length l_p, mass m_p, and time t_p. Planck's Units differ slightly from that resolved with measurement quantization, the units of which are referred to as fundamental units and are distinguished with a subscript f.

By first describing gravity using the Pythagorean Theorem and measurement quantization, an approach to bounded measure may be applied. Resolving the upper bound to mass counts with respect to counts of the remaining two measures allows us to describe the orbital dynamics of galaxies. Each relation is tightly constrained, being a function of constants. When applied to the Milky Way, the minimum mass density, the crossover point between Newtonian and non-Newtonian behavior, and the associated mass and velocity curves are resolved. The expansion of space is also included. Most importantly, a classical description is presented that does not require the existence of dark matter.

After applying the approach to the existing Milky Way data, the expressions are then modeled with a smooth mass distribution to demonstrate what an average of thousands of galaxies would look like. As expected, orbital velocities flat-line. The magnitude of the orbital velocity is correlated to the excess mass above the mass frequency bound (i.e. the upper count bound of mass with respect to time).

The presentation addresses the ΛCDM [1] dark matter distribution presently considered the leading candidate with respect to this phenomenon. Expressions for each distribution are presented, but ΛCDM is not used to resolve the distribution values. Instead measurement quantization [2] is used in an approach that differs from the Standard Model only in its recognition of the physical significance of the smallest units of measure. The advantage of this approach is that a base expression with no free variables may be resolved. The approach allows an inspection that resolves concisely an understanding of the characteristics and differences in the distributions.

Also addressed are existing proposals. For one, MOND models have provided a good correlation with observed star velocities. Alternatives such as that used by McCulloch modify the inertial mass by assuming it is caused by Unruh radiation [3]. Each of these approaches incorporate some element of data dependence, but it is their dependence on less established mass distributions and expansion expressions (i.e., ΛCDM) that present conflicts. The expressions herein clarify the physical description of each mass distribution and why existing applications to galactic phenomena are in conflict.

2. METHODS

2.1. Quantum Gravity

Quantum gravity is a consequence of measurement quantization. Informativity—a term that describes the application of measurement quantization to the description of phenomena—rests on evidentiary support for the physical significance of fundamental units of measure. This property of observation differs from what might be understood with respect to observations first proposed by Planck. Specifically, the fundamental measures do not imply that nature is discrete, only that measure is discrete. Thus, while nature is infinitely divisible in length, mass, and time, there are physically significant count bounds to what can be measured. Those bounds constrain the behavior of matter as much as they constrain the behavior of gravity.

We shall discuss the evidence only briefly and refer the reader to the paper *"Measurement Quantization Unites Classical and Quantum Physics"* [2] for a more complete treatment of the subject. We also refer the reader to the paper *"Measurement Quantization Unifies Relativistic Effects, Describes Inflation/Expansion Transition, Matches CMB Data"* [4] for examples of the application of measurement quantization to the distortion of measure, quantum inflation, and the transition event which ends quantum inflation, initiates expansion, and marks the formation of a Cosmic Microwave Background (CMB). Those familiar with these papers may skip directly to Section 3.

For those new to Informativity, we review gravity as described in the first paper [2]. We begin with the premise, that there exists a physically significant smallest unit of measure (i.e., a reference), which will then be supported with observational data. A reference is the source thing used to ascertain and describe some other thing. By example, a marked ruler with its fundamental measure of length l_f is the reference that may be used to measure any length. This is accomplished as a whole-unit count of the reference. A fraction of a reference violates the definition of a reference indicating that the identified source is not the reference. In such instances, the new thing becomes the reference until no smaller candidates are found. We describe this mathematically.

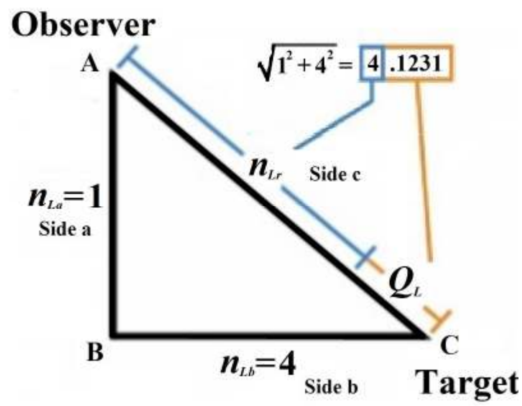

Figure 1. Count of distance measures between an observer and target where n_b=*4* (not to scale).

Suppose we wish to describe an unknown distance on side **c** of a right-angle triangle (Figure 1) as a count of a reference. For long side **c** and short sides **a**=1 (the reference) and **b** (a count of the reference) of any chosen integer count of this triangle, we may resolve a count representing the uncertain distance,

$$c = \left(1 + n_{Lb}^2\right)^{1/2} l_f \tag{1}$$

Any non-whole-unit count describes a change in distance and may be described by rounding up (repulsion) or down (attraction). The remainder lost to rounding is denoted by Q_L. Notably, Q_L is less than half and thus attractive. The model describes a count of the reference that is closer by

$$Q_L = \left(1 + n_{Lb}^2\right)^{1/2} - n_{Lb}\,, \tag{2}$$

at every instant in time. To demonstrate the mathematics, if n_{Lb}=4, then Q_L/n_{Lb}=(($\sqrt{}$17)-4)/4=0.1231/4. Because side **c** always rounds down, we find that n_{Lr} always equals n_{Lb}. As such, we always refer to the 'observed measure count' as n_{Lr}. Moreover, note that the reference measure against which all counts are measured is defined by n_{La}=1. With this we conjecture that we have composed an expression for gravity such that the loss of the remainder relative to the whole-unit count is Q_L/n_{Lr}.

We proceed with that hypothesis by presenting the ratio in meters per second squared (m s^{-2}). We multiply by l_f for meters and divide by t_f^2 together describing the distance loss at the maximum sample rate of one sampling every t_f seconds per second,

$$\frac{Q_{Lf}l_f}{n_{Lr}t_f^2}, \tag{3}$$

Also note that this quantity is scaled and hence requires a scaling constant; we multiply by the speed of light c and divide by a scaling constant S. Setting $r{=}n_{Lr}l_f$ and $c{=}l_f/t_f$, then

$$\frac{Q_L l_f}{n_{Lr}t_f^2}\frac{c}{S}=\frac{Q_L c^2}{n_{Lr}t_f S}=\frac{Q_L l_f c^2}{n_{Lr}l_f t_f S}=\frac{Q_L c^3}{rS}. \tag{4}$$

The ratio c/S may be understood as kg^{-1} or a maximum count of m_f per kilogram; it may also be thought of as the corresponding mass frequency associated with gravity. If S=3.26239, this expression is now equivalent to G/r^2 to five significant digits for all distances greater than $10^3\ l_f$; where quantum differences are not a consideration, we may set the expression equal to G/r^2 and thus

$$\frac{Q_L c^3}{rS}=\frac{G}{r^2}, \tag{5}$$

$$Q_L rc^3 = GS. \tag{6}$$

The expression may be reduced in the limit $lim_{r\to\infty}f(Q_L n_{Lr})$=1/2, as demonstrated in Appendix A1. With $r{=}n_{Lr}l_f$, then the expression becomes

$$\frac{c^3}{G}=\frac{S}{Q_L r}=\frac{S}{Q_L n_{Lr}l_f}=\frac{2S}{l_f}. \tag{7}$$

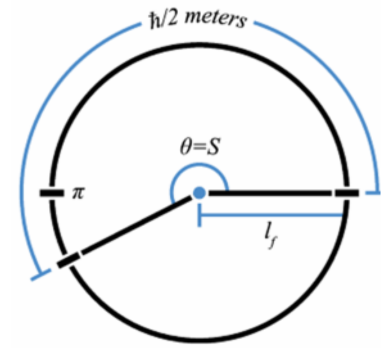

Figure 2. Arc length of a circle of radius l_f and subtending angle θ=S radians.

Our focus now turns to the scaling constant. What is it and how do we measure it? There are two physically significant phenomena where S may be measured. First, we may measure S as a momentum; hence the units for these expressions will match accordingly. However, as illustrated in Figure 2, S is also an angular measure given by the expression

$$S=\frac{l_f}{2}\left(\frac{c^3}{G}\right)=\frac{l_f}{2}\left(\frac{\hbar}{l_f^2}\right)=\frac{\hbar}{2l_f}. \tag{8}$$

If $S{=}\hbar/2l_f$, then the arc length of a circle of radius l_f and angle S is

$$L=r\theta=l_f\left(\frac{\hbar}{2l_f}\right)=\frac{\hbar}{2}. \tag{9}$$

With respect to this reference, the units for S are radians. Explicitly, the units for S depend on the frame of reference. For this reason, we use θ_{si} throughout all Informativity expressions, not because the term always denotes a radian measure, but to emphasize that the value of θ_{si} is invariant for all frames of reference. The subscripts s and i exist for a historical purpose denoting the signal and the idler measures in the Shwartz and Harris quantum entanglement experiments [5], both of which are precisely 3.26239 radians.

When θ_{si} is described with respect to other measures in the local inertial frame, either units of momentum or radians apply depending on what is being measured. When θ_{si} is described with respect to a measurement bound (i.e., the age or diameter of the universe), the term is dimensionless. This is most evident in a unity expression for which an example is presented later. In each case, the value of θ_{si} is the same. Most expressions are with respect to a bound, but where there is an exception, notes will be provided. A more complex example is examined in Appendix A3.

Lastly, a notable example of cross-referenced expressions combines both momentum and angular expressions. Planck's expression for Planck length is then

$$\frac{l_f c^3}{2G} = \frac{\hbar}{2l_f}, \tag{10}$$

$$l_f = \left(\frac{\hbar G}{c^3}\right)^{1/2}. \tag{11}$$

Planck's mass and time expressions are also in the same class of cross-referenced expressions and as such all of his unit work is a derivative of two frames of reference. Mixing frames of reference may seem inappropriate but doing so also offers physically significant descriptions of nature. With that, care must be taken with each Informativity expression to track units and resolve them.

Evidence does not rest on one or even several experimental results. There are, at present, more than 20 verifiable predictions of the model [2,4] in disciplines that include quantum physics (Table 1), quantum gravity equation (6), classical physics, the distortion of measure (i.e., also described by relativity) ([4], Section 3.1), quantum inflation ([4], Section 3.14–3.15), expansion ([2], Section 3.12), and cosmology ([2], Section 3.10). One measure of θ_{si} is published in Shwartz and Harris's 2011 paper, *'Polarization Entangled Photons at X-Ray Energies'* [5]. Using Informativity, their measures can be described to the same precision as calculated in Table 1.

TABLE 1. Angle setting in radians of the **k** vectors of the pump, signal, and idler for maximally entangled states at the degenerate frequency with corresponding Shwartz and Harris values (Ref. [5]).

Bell's State	k vector angle		
	θ_p	θ_s	θ_i
$(\vert H_s, V_i\rangle + \vert V_s, H_i\rangle)/\sqrt{2}$	$(l_f c^3/2G)-\pi$ (0.1208)	π - $(l_f c^3/2G)$ (-0.1208)	π - $(l_f c^3/2G)$ (-0.1208)
	2π - $(l_f c^3/2G)$ (3.02079)	$(l_f c^3/2G)$ (3.26239)	$(l_f c^3/2G)$ (3.26239)

Most importantly, in recognition of the physically significant units of measure, Informativity provides an approach that mathematically correlates measurement quantization to gravity. It follows, where bounds to measure are found, a corresponding bounding effect must also be found with respect to gravity. In the next section we explore further the reference measures to build a tool set necessary for describing how gravity is bounded.

2.2. Fundamental Measures

The physical significance of fundamental units of measure is instrumental to describing galactic rotation. It is because the fundamental units are countable, having a smallest and greatest count with respect to the remaining two measures, that gravity is constrained. A review of the fundamental units, their values and definitions provide the foundation for the expressions to follow. Thus, with equation (7) and a measured value of θ_{si} equal to 3.26239, each of the fundamental measures can be resolved. When defined with respect to the fundamental measures, the units for θ_{si} are that of momentum kg m s^{-1}. Thus,

$$l_f = \frac{2G\theta_{si}}{c^3} = \frac{2\times 6.67408\times 10^{-11}\times 3.26239}{(299792458)^3} = 1.61620\times 10^{-35} \text{ m,} \tag{12}$$

$$t_f = \frac{l_f}{c} = \frac{2G\theta_{si}}{c^4} = \frac{2\times 6.67408\times 10^{-11}\times 3.26239}{(299792458)^4} = 5.39106\times 10^{-44} \text{ s,} \tag{13}$$

$$m_f = t_f \frac{c^3}{G} = \frac{2\theta_{si}}{c} = \frac{2\times 3.26239}{299792458} = 2.17643\times 10^{-8} \text{ kg.} \tag{14}$$

To describe a count of l_f, m_f, and t_f with respect to time, divide the rate by the respective measure.

$$n_L = 2.99792458\times 10^8 / l_f = 1.85492\times 10^{43} \text{ units s}^{-1}, \tag{15}$$

$$n_M = 4.0371111\times 10^{35} / m_f = 1.85492\times 10^{43} \text{ units s}^{-1}, \tag{16}$$

$$n_T = 1/t_f = 1.85492\times 10^{43} \text{ units s}^{-1}, \tag{17}$$

The term mass frequency as used throughout describes a count of mass units relative to a count of time units. The upper count bound of mass units per second is 1.85492×10^{43}. The same count applies also to length frequency and frequency, the rate of time itself. Mass-to-length frequency is distinctly different and important to an understanding of galactic orbital dynamics.

Another often used expression in Informativity is the *fundamental expression*. This may be resolved from equation (14) $m_f=2\theta_{si}/c$ where $c=l_f/t_f$,

$$l_f m_f = 2\theta_{si} t_f . \tag{18}$$

Lastly, while we have demonstrated the importance of θ_{si} in describing gravity, in resolving the fundamental units, in describing momentum, in defining Planck's constant, and in resolving Planck's Unit expression for length, we have not discussed specifically the evidence for a physical significant measure. To that end, consider Heisenberg's Uncertainty Principle applied to the position and momentum of a particle. With $r=n_{Lr}l_f$ multiplied by $Q_L n_{Lr}$ (i.e., $\lim_{r\to\infty} f(Q_L n_{Lr})=1/2$) to place the distance measure in quantum form, $m=n_M\theta_{si}/Q_L n_{Lr}c$ generalized from the *fundamental expression*, and $v=n_L l_f/n_T t_f$, then Heisenberg's expression reduces to counts n_L, n_M, n_T, and the length count between a target and a center of mass n_{Lr} such that

$$\sigma_X \sigma_P \ge \frac{\hbar}{2}, \tag{19}$$

$$f(r)f(mv) = (n_{Lr} l_f Q_L n_{Lr})\left(\frac{n_M \theta_{si}}{Q_L n_{Lr} c} v\right) \ge \theta_{si} l_f , \tag{20}$$

$$(n_{Lr})\left(\frac{n_M v}{c}\right) \ge 1 , \tag{21}$$

$$(n_{Lr})\left(n_M \frac{n_L l_f}{n_T t_f}\frac{t_f}{l_f}\right) \ge 1 , \tag{22}$$

$$n_M n_{Lr} n_L \ge n_T . \tag{23}$$

In passing, suppose gravity is described as a loss of the fractional length count Q_L above and beyond the whole-unit count of the reference. The description of Q_L arises from the Pythagorean Theorem, a mathematical description of the measure of length. Thus, the interpretation implies two qualities: that nature is infinitely divisible or at least to the extent as described by all solutions to Q_L, and that measure is discrete.

Consider now a description of the speed of light $c=n_L l_f/n_T t_f$, a whole-unit count of the reference such that $n_L=n_T=1$ in Heisenberg's reduced expression. It follows that the remaining counts of n_M and n_{Lr} must also be one. The expression confirms the conjecture. Where we find support for the Heisenberg Uncertainty Principle, we also find the fundamental measures to be of physical significance, defining the threshold. The threshold between certainty and uncertainty is precisely at $n_M=1$, $n_L=1$ and $n_T=1$ such that $n_{Lr}=1$.

2.3. Nomenclature

In the description of phenomena, Informativity uses a distinct nomenclature to describe length, mass, time, unit counts of those measures, and the measure of several other quantities. Let us take this moment to discuss nomenclature.

The description of fundamental units with respect to the three measures are denoted as l_f for length, m_f for mass, and t_f for time. The description of counts of the fundamental measures is denoted by symbol n, each measure recognized by a corresponding capitalized subscript, L for length, M for mass, and T for time. To avoid confusion between length descriptions of motion and those of gravitational fields, a subscript r (i.e., n_{Lr}) is used when describing a count of l_f between a static frame of reference and a center of gravity. Similarly, a subscript m (i.e., n_{Lm}) is used when describing a change in the count of l_f with respect to a target in motion with respect to the observer.

With respect to mass distributions associated with the universe, there are several categories. The total mass of the universe is distinguished with the term Ω_{tot} and may be divided into two parts, dark mass Ω_{dkm} and observable mass Ω_{obs}. The dark mass distribution is presently attributed to dark energy, but as presented in Section 3.1, it is also the mass that can never be known because it exists at such a distance that the expansion of the universe prevents information regarding its existence from ever reaching the observer.

Of importance also, if we now subtract the visible Ω_{vis} from the observable mass Ω_{obs}, we resolve that which will be observed, the unobserved mass Ω_{uobs}. The unobserved mass is that matter which will eventually be known given sufficient elapsed time. The distribution is typically attributed to dark matter.

To convert a distribution from a percentage to kilograms, multiply the total mass of the universe in kilograms by the percentage. Many expressions in Informativity use the corresponding symbols M_{tot}, M_{dkm}, M_{obs}, M_{vis} and M_{uobs} with the understanding that the value taken may be either a percentage of M_{tot} or kilograms. This practice is common as the ΛCDM distributions (i.e. percentages) are resolved in the first paper [2] using fundamental mass which is a value in kilograms.

This leads us to the final term, the fundamental mass M_f. This mass is associated with the mass frequency bound ([2], equation (93))

$$M_f = A_U \theta_{si} \frac{m_f}{t_f}, \tag{24}$$

and is instrumental in calculations of all mass distributions. While the distribution values are the same as those resolved with ΛCDM, the two approaches differ significantly. The Informativity approach is an outcome, a prediction of Informativity implicit to physically significant quantized measure. There are no free variables and as such the precision is constrained only by the measure of θ_{si}, i.e., to six significant digits. Each expression is presented in Section 3.1 along with a discussion as to their meaning, differences, and why one may conclude that the only physical difference between the distributions is if and when mass is visible.

Lastly, the expansion of the universe can be described with respect to two measures. Stellar expansion, the measure of increasing distance between galaxies, follows the traditional understanding in modern theory. When discussing stellar expansion, we describe the effect using Hubble's constant H_o which is quoted in kilometers per second per megaparsec. Conversely, the universal expansion H_U describes the expansion of the universe when defined with respect to the universe. SI units are used, but the reference is fixed with respect to the age A_U and diameter D_U of the universe. Universal expansion refers to the isotropical increase in space.

A listing of symbols used and their definitions may be found in Appendix A8.

2.2. Terminology

There are several terms often used when describing galaxies. Having introduced the nomenclature for describing expansion, we consider now the expression for universal expansion ([2], equation (87)),

$$D_U = 2\theta_{si} A_U = 2 \times 3.26239 \times 13.799 = 90.035 \text{ bly.} \tag{25}$$

The rate of expansion follows from the definition $1/A_U$ and may be resolved in the customary units,

$$H = \frac{{}_{\text{km/Mpc}}}{A_U} = \frac{3.08567758 \times 10^{19}{}_{\text{km/Mpc}}}{13.799 \times 10^{9}{}_{\text{y}} \times 3.15576 \times 10^{7}{}_{\text{s/y}}} = 70.860 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{26}$$

When defined with respect to the universe D_U/A_U, the expansion is invariant ([2], equation (81))

$$H_U = 2\theta_{si} . \tag{27}$$

Resolving a description of phenomena with respect to the universe can provide a perspective that is straight-forward with which to build a cohesive understanding of many presently unsolved physical phenomena.

We consider the universe, in this application, as a frame of reference. As there is no outside reference to the universe, the universe is recognized as a *self-defining* frame. Terms that describe the universe are part of a class recognized as system bounds. For instance, the age and diameter of the universe describe the upper bound to elapsed time and length. Conversely, a thing defined relatively with respect to some other thing is called *self-referencing*. One's choice of frame in no way identifies a physically significant difference. Nevertheless, *self-defining* expressions are often invariant (i.e., H_U=$2\theta_{si}$). *Self-referencing* expressions often vary (i.e., H=((km/Mpc)/A_U). Also, the units for θ_{si} depend on which frame is chosen.

While not as central to our discussion, we remark that the *system constant* $2\theta_{si}$ is often present in physical descriptions. The value is fundamental to the description of matter. For example, we may describe the expansion of the universe with respect to measure or as a function of $2\theta_{si}$ and use the *fundamental expression* to convert between them;

$$\left(\left(\frac{t_f}{l_f m_f}\right)^{1/3}\right)^2 + \left(\frac{n_{Lm}}{\mathrm{n_{Lc}}}\right)^2 = 1\,, \tag{28}$$

$$\frac{1}{(2\theta_{si})^{2/3}} + \frac{n_{Lm}^2}{\mathrm{n_{Lc}^2}} = 1\,. \tag{29}$$

Many expressions are modifications of these *unity expressions*. There are two classes. *Relations* are expressions that may be reduced to the *fundamental expression. Boundary expressions* describe upper and lower count bounds relatively between measures.

It should not go unqueried as to what anchors measure, the fundamental measures—$(l_f m_f / t_f) = 2\theta_{si}$—or the corresponding rate of universal expansion n_{Lm}? This can be a difficult query as measure is relatively defined. However, their relation is fixed and therefore distinguishes θ_{si} as perhaps the most fundamental constant. Many of the known constants may be reduced to include only θ_{si}, the fundamental measures or counts thereof. Several examples are ([2], equations 36, 49, 81)

$$\hbar = 2\theta_{si} l_f\,, \tag{30}$$

$$E_f = 2\theta_{si} l_f / t_f\,, \tag{31}$$

$$H_U = 2\theta_{si}\,. \tag{32}$$

As noted before, θ_{si} has units of kg m s^{-1} in the first two examples, but the latter is a system bound and thus dimensionless. Conversely, the speed of light and the gravitational constant (see Appendix A2) are examples of *boundary expressions*,

$$\mathrm{c} = l_f / t_f\,, \tag{33}$$

$$G = \frac{l_f}{t_f}\frac{l_f}{t_f}\frac{l_f}{t_f}\frac{t_f}{m_f}\,. \tag{34}$$

Lastly, the terms quantum and quantized are often used. Neither should be understood as having a relation with respect to quantum mechanics. Rather, the term quantum is intended to mean small as in a few tens, hundreds or thousands of fundamental units of measure. The term quantized is intended to mean that expressions are composed of terms that are whole-unit counts of the fundamental measures.

A quantized expression possesses qualities that are immensely valuable in our effort to describe nature. For one, quantized expressions are defined for the entire measurement domain. For another, quantized expressions are nondimensionalized. Nondimensionalization is not in itself a valuable endeavor but demonstrating that all phenomena may be expressed entirely with nondimensionalized whole-unit counts of the fundamental measures; it contributes to a new understanding of measure that is finite and discrete.

A listing of terms used in Informativity may be found in Appendix A7.

3. RESULTS

In the sections that follow, we use Informativity to present expressions describing the motion of stars in galaxies. As noted at the outset, when averaging hundreds or thousands of galactic rotational curves, the curve is nearly constant from a given radius outward. Star velocities are in conflict with Newton's law of gravitation, which predicts a decreasing velocity with increasing distance.

A second anomaly concerns the magnitude of these velocities, a value that is significantly higher than expected. To describe these phenomena, incorporation of the effects of expansion and a new constraint to the behavior of matter is entertained. While expansion is a seemingly straight-forward application, the constraint—mass frequency—is a new concept to modern theory. Like length frequency, $c=l_f/t_f$, mass frequency describes that bound where counts of m_f greater than 1.85492×10^{43} units per second, equation (16), may no longer be distinguished.

The upper bound to mass frequency is physically significant and cannot be exceeded any more than a length frequency greater than one-to-one (i.e., $n_L l_f/n_T t_f>c$). As we work through an understanding of mass frequency, we shall demonstrate how counts above and beyond this bound correspond to measures smaller than the reference. Not only does a mass frequency above a frequency bound (i.e., a smaller value for m_f in the expression $1/m_f$) describe a point in space-time subject to an indistinguishable count of m_f, it also describes a faster-than-light relationship between length and time, identifiable using the *fundamental expression*, $l_f m_f=2\theta_{si} t_f$ (i.e., a smaller value for m_f implies a larger value for l_f where $c=l_f/t_f$ is then a faster-than-light relation).

3.1. Mass Distribution

Galactic rotations follow classical theory with adjustments made for the effects described by relativity, the *Informativity differential* (Appendix A1), and the universal expansion. To simplify the expressions, the first two effects are not integrated into the results. In contrast, expansion is significant in magnitude. As described in the first paper, we begin with a review of expansion followed by mass distribution.

Stellar expansion—the modern understanding of expansion as a function of the universal expansion plus those forces of interaction since the earliest epoch—is not discussed. Universal expansion, conversely, describes the spatial increase of the universe. The rate when defined with respect to the universe, equation (27), is

$$H_U = 2\theta_{si}\,. \tag{35}$$

The constant $2\theta_{si}$ is referred to as the *system constant*. With it, universal expansion may be described using familiar terms ([2], equation (87)) such as the diameter D_U of the universe in billions of light-years and the age A_U of the universe in billions of years.

$$D_U = 2\theta_{si} A_U = 2\times3.26239\times13.799 = 90.035\text{ bly.} \tag{36}$$

With these parameters we may now summarize the mass-distribution expressions starting with the fundamental mass ([2], equation (93)) which is then used to derive the distributions,

$$M_f = A_U \theta_{si} \frac{m_f}{t_f}\,. \tag{37}$$

Because our frame of reference is the universe, θ_{si} carries no units. A complete derivation is provided in the first paper ([2], Section 3.12). The advantage of this approach is that each distribution is clearly defined. The total is divided such that dark mass Ω_{dkm} is that mass sufficiently distant that expansion prevents the light (i.e., information) from ever reaching the observer. The observable mass Ω_{obs} makes up the remainder. The observable may then be divided into two categories, that which is presently visible Ω_{vis} and the unobserved Ω_{uobs} which will be visible given sufficient elapsed time. Each distribution ([2], equations (109), (110), (113), and (115)) precisely matches the ΛCDM results. We learn here that each is invariant,

$$\Omega_{dkm} = \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 68.3624\,\%, \tag{38}$$

$$\Omega_{obs} = \frac{4}{\theta_{si}^2 + 2} = 31.6376\,\%, \tag{39}$$

$$\Omega_{vis} = \frac{1}{2\theta_{si}}\frac{\Omega_{obs}}{\Omega_{tot}} = \frac{\Omega_{obs}}{2\theta_{si}} = 4.84884\,\%, \tag{40}$$

$$\Omega_{uobs} = \Omega_{obs} - \Omega_{vis} = 31.6376 - 4.84884 = 26.7888\,\%. \tag{41}$$

In modern theory Ω_{dkm} is identified as dark energy; Ω_{uobs} is identified as dark matter. As neither reflect the calculations leading to the above expressions as carried out in the first paper, the terms are accordingly replaced by dark mass and unobserved mass, each a description of the matter associated with the distributions described above.

This brings us to an important observation as intimated in equation (40),

$$\Omega_{obs} = 2\theta_{si}\Omega_{vis}\,. \tag{42}$$

If the visible mass is that which is presently visible and the unobserved mass is that which becomes visible with elapsed time, then with respect to the earliest epoch nearly all the visible mass we see today was previously unobserved (i.e., dark matter). The idea that dark matter is different than what we presently identify as visible mass is in conflict. Further technical details regarding the treatment of mass distributions are provided in Appendix A6.

Consider now that the Informativity distributions precisely match the ΛCDM calculations. This is accomplished with only an understanding of the fundamental mass M_f . Combining the distributions (i.e. such that all values are in kilograms, multiply each distribution by M_{tot} in kilograms), we find that ([2], equation (118))

$$M_f = \frac{M_{tot}M_{obs}}{2M_{tot} - M_{obs}}\,, \tag{43}$$

but this seemingly reveals a problem. If the expressions are invariant, why are the distributions properly resolved while mass is moving from the unobserved to the visible? From another perspective, if M_f is a function of time, must M_{tot} also increase? Yes, evidence that the total mass of the universe is increasing follows.

The CMB calculations are just one inevitable outcome of mass accretion M_{acr} ([2], equation (135)). The age, quantity, density, and temperature from the CMB data may each be calculated such that

$$M_{acr} = \frac{\theta_{si}^3}{2} = 17.3611 \text{ units } m_f/\text{unit } t_f. \tag{44}$$

Most importantly, *"there are no free variables"*, in the calculation. Density and temperature, are naturally functions of the elapsed time we identify as being the present,

$$A_U = e^{\sqrt{3}\theta_{si}^3/2} = 1.14652\times10^{13}\,\text{s} = 363,309 \text{ y}, \tag{45}$$

$$M_{tot} = n_{Tu} m_f \frac{\theta_{si}^3}{2} = 1.50159\times10^{50} \text{ kg}, \tag{46}$$

$$\rho = \frac{M_{tot}c^2}{V_U} = 4.17041\times10^{-14} \text{ J/m}^3, \tag{47}$$

$$T = \left(\frac{\rho}{a}\right)^{1/4} = 2.72468 \text{ K}. \tag{48}$$

The calculations are a direct result of M_{acr}. Being a quantum bubble, the universe is unable to expand at the speed of light because there exists no means to resolve a point outside of the bubble until the universe reaches a radius of $\sqrt{3}l_f$. This trigger ends quantum inflation precisely at 363,309 years, releases the accreted mass/energy (which occurs at the noted rate of $\theta_{si}^3/2$ units of m_f per unit of t_f) as the CMB and initiates expansion as we see it today. There is no faster-than-light inflationary period and the results match our best observational data precisely. The calculations and details were published in Ref. [2] (Section 3.15) with additional explanation of the effects of relativity following ([4], Section 3.6).

A graphical representation of the distributions is also presented in Figure 3. The mass values are constrained to the precision of the age of the universe, 13.799 billion years [6], as our most accurate measure of the universe. For a more complete list of mass distribution conversions refer to Appendix A5.

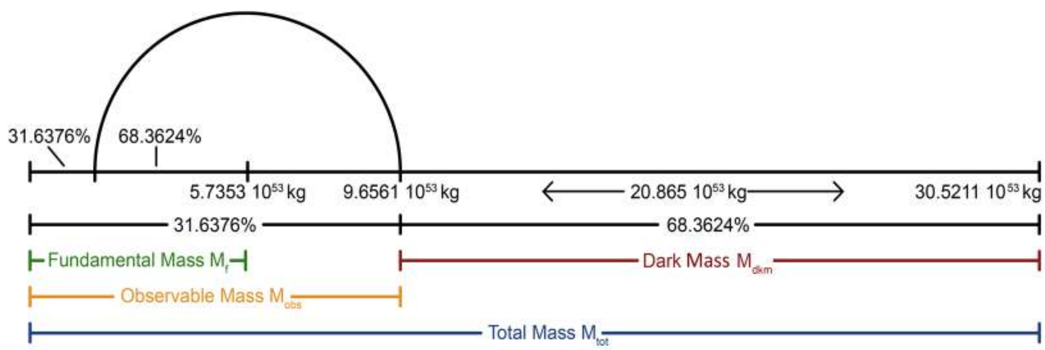

Figure 3. Relative measure of mass.

Finally, to provide a reference for the expressions to follow, we use the Milky Way as our target galaxy. The calculations consider only the mass within the first 84,000 light-years. The corresponding value for observable mass is then

$$M_{obs} = 8.56060\times10^{41} \text{ kg}. \tag{49}$$

All the mass, density, and velocity data for the Milky Way comes from Stacy McGaugh's 2018 Milky Way mass models [7].

3.2. Orbital Velocity Bound

Count bounds are an important and physically significant attribute in describing the behavior of matter. Length frequency is the most well-known count bound $c=l_f/t_f$; for each count of fundamental time there can be at most one count of fundamental length. Any count of l_f greater than t_f would correspond to a velocity greater than the speed of light. The physical significance of fundamental units of measure is what distinguishes measurement quantization from an unbounded description of nature.

There also exist upper and lower count bounds for m_f/t_f and m_f/l_f. We respectively call these bounds mass frequency and mass-to-length frequency. The orbital velocity of a star is subject to all three bounds in addition to the effects of expansion. A description may be resolved starting with the classical expression for orbital velocity,

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{l_f^3}{t_f^3}\frac{t_f}{m_f}\frac{n_M m_f}{n_{Lr} l_f}} = \sqrt{\frac{l_f^2}{t_f^2}\frac{n_M}{n_{Lr}}} = c\sqrt{\frac{n_M}{n_{Lr}}} \ . \tag{50}$$

As described in Appendix A2, the upper mass-to-length count bound with respect to orbital velocity is one-to-one, $n_M < n_{Lr}$. However, the relation we seek is the mass-to-length count bound with respect to the escape velocity,

$$2n_M < n_{Lr} \ . \tag{51}$$

Consider now that the smallest count of m_f with respect to l_f may not be less than the precision offered by the reference m_f=2.17647×10^{-8} kg. To translate this to a whole-unit count of the reference scale requires the ratio,

$$\frac{2.17643\times10^{-8}\ \text{units}\, m_f}{1\ \text{unit}\ l_f} = \frac{1\ \text{unit}\ m_f}{4.59468\times10^{7}\ \text{units}\ l_f} = \frac{1}{1/m_f} = \mathrm{n_{Mb}} \ . \tag{52}$$

Combining both bounds, the ratio is then 2 units of m_f per unit of l_f giving $1/(1/m_f)$. Therefore, $2(1/(1/m_f))=2m_f$. If $\mathrm{n_{Mb}}$ and m_f are equal in value and have no units, then the classical velocity bound is

$$v_{bc} = c\sqrt{\frac{n_M}{n_{Lr}}} = c\sqrt{2m_f} \ . \tag{53}$$

The expression does not account for the expansion of space $H_U=2\theta_{si}$, equation (27). Such that H_U is relative to the diameter of the universe, divide by 2. In respect to orbital and escape velocities, the radial expansion may be written in two ways using the *fundamental expression* to convert between them:

$$v_b = \theta_{si} c\sqrt{2m_f} = 204.054\ \text{km s}^{-1}, \tag{54}$$

$$v_b = c m_f \sqrt{\theta_{si} c} = 204.054\ \text{km s}^{-1}. \tag{55}$$

As a reminder, both θ_{si} and our substitution of m_f for $\mathrm{n_{Mb}}$ carry no units. This is the velocity bound corresponding to the upper count bound of m_f that may be discerned at a point in space. To resolve a corresponding mass bound, set v_b equal to the same as expressed with Newton's expression and reduce with the *fundamental expression*. The derivation may be found in Appendix A3 along with an explanation of units,

$$M_{b-f(R)} = R\theta_{si}\frac{m_f^3}{t_f} \ . \tag{56}$$

The mass bound $M_{b\text{-}f(R)}$ is a function of the mass within the target orbital radius *f(R)*. By example, a galaxy with a radius of 84,000 light-years R=7.94157×10^{20} m would need more than

$$M_{b-f(R)} = 4.95454\times10^{41}\ \text{kg} \tag{57}$$

of mass, 2.49×10^{11} solar masses to display behavior associated with a measurement quantization bound. Such a mass represents $2.49\times10^{11}/4.30\times10^{11}$=57.9% of the mass of the estimated mass of the Milky Way. Equation (54) describes the upper bound to measurable mass unadjusted for the total mass and a mass density profile. If mass density exceeds this bound, the upper mass count bound exceeds the mass frequency bound causing any additional mass count to be indistinguishable.

Lastly, consider what a higher or lesser velocity bound implies. We may demonstrate by reorganizing the *fundamental expression* $m_f l_f = 2\theta_{si} t_f$ into a form that resolves the length count presented in the denominator of equation (52), $1/m_f = 4.59468\times10^7$. Thus, a count of 100 units greater implies a corresponding speed of

$$\frac{l_f}{t_f} = c = 2\theta_{si}\frac{1}{m_f} \approx 2\theta_{si}\left(4.59468\times10^7 + 100\right) = 299{,}793{,}110\ \text{m s}^{-1}, \tag{58}$$

a 652 m s^{-1} increase above the speed of light. The increase also corresponds to a velocity bound of

$$v_b = \theta_{si} c\sqrt{2\text{n}_{\text{Mb}}} \approx \theta_{si} c\sqrt{\frac{2}{\left(\left(1/m_f\right)+100\right)}} = 204.053\ \text{km s}^{-1}, \tag{59}$$

a decrease of 1 m s^{-1}. This does not mean that the speed of a star may not fall below 204.054 km s^{-1}. The expression describes an upper bound with which to discern mass counts and as such an upper bound to the gravitational pull on a star. When the mass count exceeds the mass count bound, the target is unable to distinguish additional mass and as such the gravitational effect of mass on a star reaches a maximum.

This analysis also does not imply that stars cannot have velocities greater than 204.054 km s^{-1}. While these expressions are invariant, we have not integrated the effects of an uneven mass distribution typical of a galaxy. This is the subject of the next section.

3.3. Galactic Rotation and the Milky Way

Using the velocity bound, an expression may now be developed as a function of mass distribution in a galaxy. The relation follows the same form as that which describes visible Ω_{vis} and unobserved Ω_{uobs} (i.e., dark matter) mass, equation (A5.6),

$$\Omega_{uobs} = \Omega_{vis}\left(2\theta_{si} - 1\right). \tag{60}$$

Replacing the dimensionless speed parameter θ_{si} with the ratio of observed over bound v_o/v_b (i.e., in relativity, v/c) provides the corresponding relation between the effective and mass bound. However, an understanding of the geometry of the substitution is difficult. For that reason, we follow an algebraic approach that resolves the speed parameter β as a relative percent difference $\Delta\%_{o-b}$ of the bound,

$$\Delta\%_{o-b} = \frac{\left(v_o - v_b\right)}{v_b} = \frac{v_o}{v_b} - 1. \tag{61}$$

The reduction also needs the velocity bound v_b from equation (54), $v_b = \theta_{si}c(2m_f)^{1/2}$. The expression for mass is then the product of the mass bound $M_{b-f(R)}$ and $2\Delta\%_{o-b}$, hereafter referred to as the effective mass $M_{e-f(R)}$,

$$M_{e-f(R)} = M_{b-f(R)}\left(2\Delta\%_{o-b}+1\right) = M_{b-f(R)}\left(2\frac{v_o}{v_b}-2+1\right), \tag{62}$$

$$M_{e-f(R)} = M_{b-f(R)}\left(2\frac{v_o}{v_b}-1\right). \tag{63}$$

The subscripted symbol *f(R)* is to indicate that the mass considered is that within the orbital radius *R* from the galactic center. Like the relation presented in equation (54), the speed parameter $\Delta\%_{o-b}$ is doubled to describe mass in terms of the bound mass for the escape velocity.

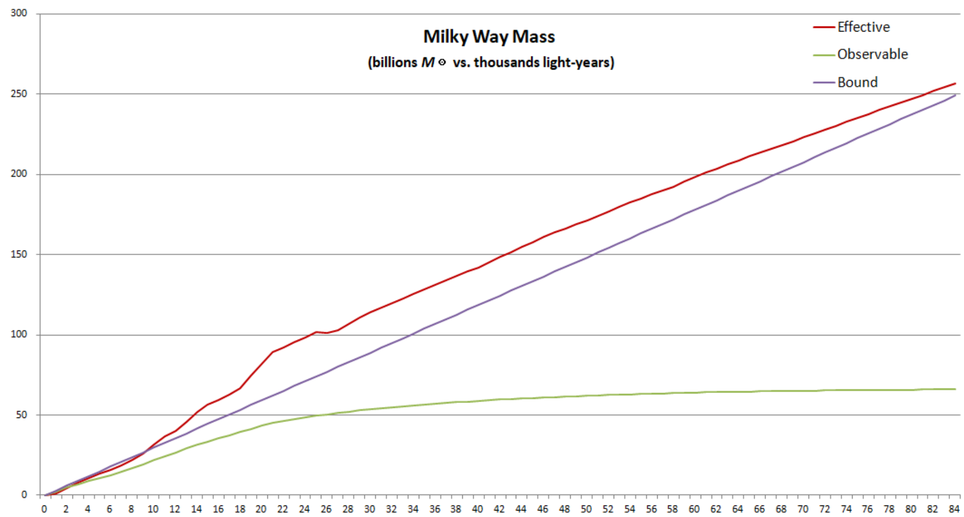

Figure 4. Galactic mass corresponding to actual (green), effective mass (red), and mass frequency bound (purple).

When incorporating an expansion, we realize that the observer's view of the universe is skewed; the effect suggests the presence of more mass than is actually present. In Figure 4, the observable $M_{o\text{-}f(R)}$, bound $M_{b\text{-}f(R)}$, and effective $M_{e\text{-}f(R)}$ masses are displayed. Where the effective mass is less than the bound mass, the orbital velocity of the star follows a classical behavior. Conversely, an effective mass greater than the bound presents a mass count greater than the mass frequency bound. Some count of m_f will be indistinguishable leading to a constraining effect on gravity and the corresponding star velocities. The crossover from classical to non-classical behavior occurs at 9.32848×10^3 light-years.

Note that the observable and effective mass differ by a factor of 3.9 at $R=84\times10^3$ light-years, i.e., 74% of the mass is missing. The magnitude of this effect depends on the total mass of the galaxy or galaxies considered. A second notable factor regards mass distribution. As discussed, an excess mass count is indistinguishable creating a mitigating gravitational effect. Which mass counts are lost? This is presently unknown, but also less significant in a well-organized system such as a galaxy. Conversely, consideration of an uneven distribution (i.e., several galaxies) present a center-of-mass offset regarding the indistinguishable mass count.

Both effects are notably evident in the Bullet Cluster. For one, the cluster exhibits a missing mass of approximately 90%. The cluster also exhibits a center-of-mass offset as would be expected with a lost mass count, the latter being of considerable interest for future research.

Using Newton's expression for velocity, $v_b=(GM_{b\text{-}f(R)}/R)^{1/2}$ and the mass-bound expression $M_{b\text{-}f(R)}=\theta_{si}m_f^3R/t_f$ from equation (56), we may now resolve the effective star velocity

$$v_e = \left(\frac{GM_{e-f(R)}}{R}\right)^{1/2} = \left(\frac{GM_{b-f(R)}}{R}\left(2\frac{v_o}{v_b} - 1\right)\right)^{1/2}, \tag{64}$$

$$v_e = \left(\frac{G}{R}\theta_{si}\frac{m_f^3}{t_f}R\left(2\frac{v_o}{\theta_{si}c\sqrt{2m_f}} - 1\right)\right)^{1/2}, \tag{65}$$

$$v_e = 2\theta_{si}\left(2\frac{v_o}{\sqrt{2m_f}} - c\theta_{si}\right)^{1/2}. \tag{66}$$

While it may seem more appropriate to use a mass or mass density dataset, the choice is irrelevant. One may modify the expression to enter velocity, mass or mass density and still arrive at the same expression. For example, as resolved in Appendix A4, we may substitute the observable velocity v_o in equation (66) for this equivalent function written in terms of the effective mass,

$$v_o = \sqrt{\frac{m_f}{2}}\left(\frac{M_{e-f(R)}}{R}\frac{l_f}{m_f^3} + \theta_{si}c\right). \tag{67}$$

More importantly, Newton's expression for orbital velocity does not produce the observable velocity curve. Informativity succeeds because the expression for effective velocity is a function of the mass count bound, equation (54), an invariant expression with no free variables. To highlight that fact, we retain the corresponding velocity bound v_b in Figure 5 to demonstrate the natural tendency for stars to approach the bound when the mass count reaching a star exceeds the effective bound. The remaining curves are: the effective velocity v_e plotted in red, the observable velocity v_o plotted in green, and the classical velocity v_c (Newton's expression) plotted in blue.

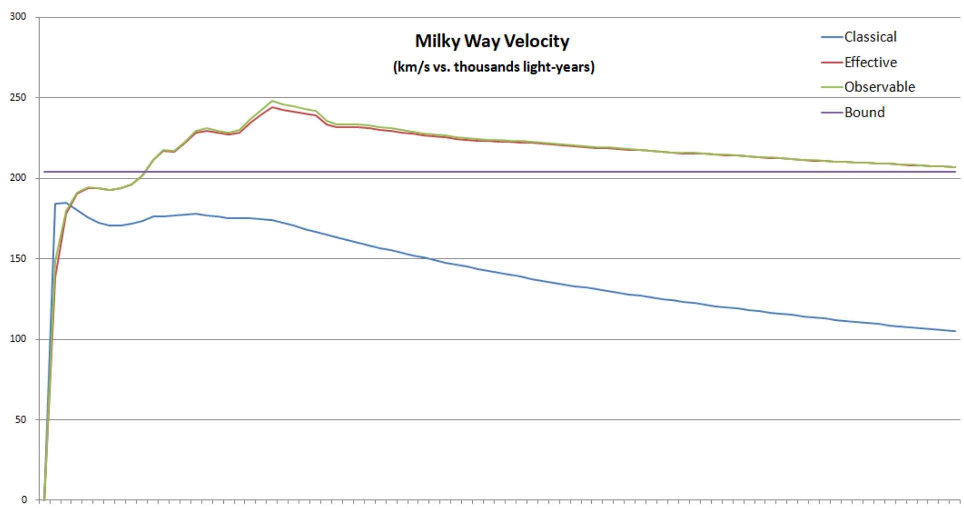

Figure 5. Stellar velocities corresponding to observed (green), effective (red), mass frequency bound (purple) and Newton's expression (blue).

There are two points of view in conflict. That is, the classical velocity implies that what we observe is moving too fast. The curve also suggests that there is dark matter holding the stars in orbit. At the same time, the observable velocity suggests a correspondence to variations in mass density.

The Informativity approach resolves the discrepancy describing an effective velocity that follows the bound when the effective mass exceeds the mass bound. When effective mass does not exceed the bound, orbital velocities follow a classical behavior.

Although the bound is invariant—204.054 km s^{-1}—variations in galactic mass density do affect the gravitational pull on a star. These effects may be smoothed out upon taking an average of thousands of galaxies. Except near galactic cores, where the crossover between classical and Informativistic behavior varies from one galaxy to the next, the velocity curve levels out reflecting an averaging of mass profiles.

An unexpected effect of mass count bounds is apparent between 4 and 8 thousand light-years where star velocities level out, until otherwise affected by increasing mass density. The cause of this effect is a subject of interest. Although perhaps physically insignificant, star velocities may trend toward classical behavior at the crossover between the effective and mass bounds.

The mass bound delineates two behaviors. Recall from equation (56), $M_{b\text{-}f(R)}=R\theta_{si}m_f^3/t_f$ that the mass bound is a function of how much mass is within a given radius. Variations in mass density imply increases or decreases in the spherical space prescribed by R for a fixed amount of mass. If we fix R in the consideration of a region of greater mass density, then the effective velocity increases, describing measured velocities that rise above the bound (i.e., 204.054 km s^{-1}). The opposite effect applies for less dense regions in that velocities lessen.

To demonstrate this effect further, consider a model galaxy with the same mass as the Milky Way (Figure 6) but with a mass distribution that has been smoothed as though we were averaging the mass

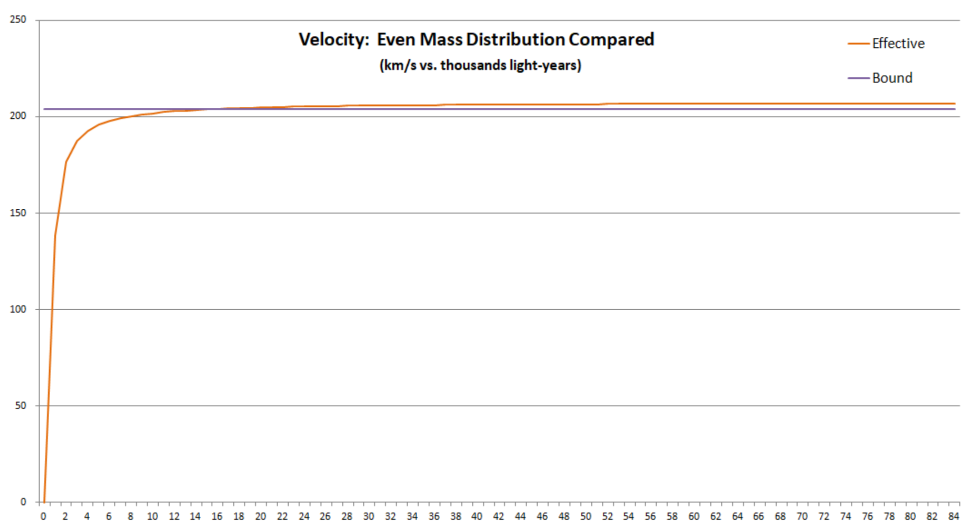

Figure 6. Star velocities corresponding to a smooth mass distribution (orange) and the mass frequency bound (purple).

profile of thousands of galaxies. Specifically, a mass equal to that for R<1000 light-years from the Milky Way center is taken. Then the remaining mass (where the total considered is only the mass in the first 84,000 light-years) is evenly divided across the remaining 83 thousand light-years. The corresponding effective velocity (orange) is drawn. As expected, the curve levels out just above the bound velocity (purple) with a magnitude that is in proportion to the excess mass above the bound. An average of thousands of galaxies exhibits a level velocity curve with a magnitude that corresponds to the mass in excess of the mass bound.

As a final note, a separation of the velocity term in equation (63) from the data can be challenging. It is the mass density data that characterizes the galaxy under consideration. The argument may be extended to demonstrate that it is also irrelevant what dataset is chosen: mass, mass density or velocity. As each measure is mathematically related, an argument for data independence by favoring any dataset over another cannot be made.

There are two remaining considerations that are data independent. Notably, an expression must describe a phenomenon with the correct magnitude. The Informativity expression properly accommodates the effects of a mass count bound in an expanding universe. Where Newton's expression does not provide the observed magnitude in describing orbital velocity, the Informativity expression does.

Also providing support is the bound itself, specifically, the purple line denoting an invariant velocity of 204.054 km s^{-1}. The bound expression contains no measurement data, no free variables, and as such no 'fitting', $v_b=\theta_{si}c(2m_f)^{1/2}$. Referring to Figure 5, star velocities favor the bound, although that is not always clearly evident. What is clear is that the bound is the baseline measure from which the magnitude of the Informativity expression is calculated. If the bound were not physically significant, the magnitude would be incorrect, and the resulting curve would not match the observational data.

Returning to our initial discussion, our goal was to develop a mass expression defined with respect to a bound. For this purpose, we compared the effective and unobserved mass expressions, each taking the form $M_1=M_2(2\beta-1)$.

$$M_{e-f(R)} = M_{b-f(R)}\left(2\frac{v_o}{v_b}-1\right), \tag{68}$$

$$M_{uobs} = M_{vis}\left(2\theta_{si}-1\right). \tag{69}$$

The details of the speed parameter depend on the masses being compared. For orbital velocity, the parameter is found on the right-side of this expression ([4], equation (68))

$$\beta^2 = \frac{v^2}{\mathrm{c}^2} = \left(\frac{n_{Lm}l_f}{n_T t_f}\frac{n_T t_f}{\mathrm{n_{Lc}}l_f}\right)^2 = \frac{n_{Lm}^2}{\mathrm{n_{Lc}^2}} = 2\frac{n_M}{n_{Lr}}, \tag{70}$$

which predicts and demonstrates the equivalence between the phenomena of motion and gravitation. Respecting the difference between orbital motion $v=(GM/R)^{1/2}$ and escape velocity $v=(2GM/R)^{1/2}$, we remove the factor of 2. Then, equation (50) may be completed,

$$v = \sqrt{\frac{GM}{R}} = c\sqrt{\frac{n_M}{n_{Lr}}} = c\sqrt{\frac{v^2}{\mathrm{c}^2}} = v, \tag{71}$$

and recognized as the same speed parameter β found commonly in relativistic expressions. Finally, a comparison of equations (50) and (68) successfully confirms the correlation between motion and gravitation (i.e., mass) as expected.

3.4. What Does the 26.7888% Distribution Describe?

We next discuss why the dark matter phenomenon has been so closely associated with the ΛCDM distribution also distinguished by the same name. We shall not use the ΛCDM approach to discuss the mass distribution but instead use Informativity expressions, such that each distribution is a function of one physical constant θ_{si}, which has been accurately measured to six significant digits [5].

We begin with the unobserved mass,

$$\Omega_{uobs} = \Omega_{obs} - \Omega_{vis} = 31.6376 - 4.84884 = 26.7888\,\%, \tag{72}$$

which describes the mass that will be observable Ω_{obs}, equation (39), but is not presently visible mass Ω_{vis}, equation (41). This is one interpretation. Using equation (40), $\Omega_{obs}=2\theta_{si}\Omega_{vis}$, we may also resolve this distribution as

$$\Omega_{uobs} = 2\theta_{si}\Omega_{vis} - \Omega_{vis}\,. \tag{73}$$

With $H_U=2\theta_{si}$, equation (27), then $\Omega_{uobs}=\Omega_{vis}(H_U-1)$. The dark matter distribution Ω_{uobs} is then the energy of the expansion as a function of the visible mass $H_U\Omega_{vis}$ minus the energy associated with the visible mass Ω_{vis}.

The two interpretations—mass and energy—while mathematically equivalent have led to significant confusion. Additionally, mass distributions are defined with respect to the universe, but the rate of expansion is much less than $2\theta_{si}$ in a region the size of a galaxy. As such, the application of a distribution such as dark matter to the description of a galaxy is questionable.

With respect to the existing observational support, Informativity does not imply that the mass we measure in a galaxy is all the mass present. There are studies that suggest there is additional non- or low-light-absorbing fine dust [8]. While there is a great deal to learn about galactic mass composition, Informativity constrains the magnitude of this mass to the observable distribution Ω_{obs}. In addition, gravitational lensing studies are not an indicator of missing mass. Rather, these studies need to incorporate the effective mass, which accounts for the expansion and the mass frequency bound. With this approach, the bending of light conforms to the effective mass as is demonstrated by the effective velocity curve.

3.5. Interpretation of Mass

At this point, we have a better understanding of the unobserved distribution and its relationship to expansion, but have not resolved a clear understanding of the bound.

We present three expressions, each describing the mass bound against which the effective mass is measured. Equating equation (54) with Newton's orbital velocity expression describes the mass bound in terms of the fundamental measures (see Appendix A3).

$$v_b = \theta_{si} c \sqrt{2m_f}\,, \tag{74}$$

$$v_b = \left(\frac{GM_{b-f(R)}}{R}\right)^{1/2}, \tag{75}$$

$$M_{b-f(R)} = v_b^2 \frac{R}{G} = 2\theta_{si}^2 c^2 m_f \frac{R}{G} = \left(\frac{l_f m_f}{t_f}\right)\theta_{si} c^2 m_f \frac{R m_f}{c^3 t_f}\,, \tag{76}$$

$$M_{b-f(R)} = \theta_{si} R \frac{m_f^3}{t_f}\,. \tag{77}$$

For the next two expressions, consider one point on the mass bound curve such that the radius is that of the universe (equation A5.20),

$$M_{Ub} = M_f m_f^2 \theta_{si} \,. \tag{78}$$

In a similar fashion, consider the mass bound in terms of mass distributions in kilograms (equation A5.10),

$$2M_{tot}M_f = M_{obs}(M_{tot} + M_f)\,, \tag{79}$$

$$M_{Ub} = \theta_{si} m_f^2 \frac{M_{tot}M_{obs}}{2M_{tot} - M_{obs}} = \theta_{si} m_f^2 \frac{M_{tot}M_{obs}}{M_{tot} + M_{dkm}}\,. \tag{80}$$

While each approach offers the opportunity to present the mass bound as a function of mass, energy or physical constants, equation (77) presents the clearest description, a line. The relation demonstrates that geometry is at work.

With the bound more clearly understood, we return to equation (63) and resolve the effective mass,

$$M_{e-f(R)} = M_{b-f(R)}\left(2\frac{v_o}{v_b} - 1\right), \tag{81}$$

$$M_{e-f(R)} = \theta_{si} R \frac{m_f^3}{t_f}\left(2\frac{v_o}{\theta_{si} c\sqrt{2m_f}} - 1\right), \tag{82}$$

$$M_{e-f(R)} = v_o 2R \frac{m_f^3}{l_f\sqrt{2m_f}} - \theta_{si} R \frac{m_f^3}{t_f}\,. \tag{83}$$

Built on equation (77), velocity v_o is the only new variable, a data-dependent value that characterizes the target. The result is quantum in detail and valid for the entire measurement domain. When the effective mass rises above or falls below the mass bound, so does the velocity. When the effective and bound masses are equal, then the velocities are as well.

We may summarize effective mass as having one of two states. The first serves as the *reference*, defined where the effective and bound masses are equal, a purely geometric description $M_{b-f(R)}=\theta_{si}Rm_f^3/t_f$. The second state, we call the *offset* equation (83). Collectively the two states describe the observed velocity as a function of the effective mass that characterizes the target.

Several studies of galaxies and galaxy clusters have suggested the presence of a gravitational force that does not coincide with visible matter. Notably, the effective mass of a given matter field describes a force that is unexpected from our point of view. While we shall not review the specific calculations of existing investigations, it is expected that, with respect to this model, the effects of a mass frequency bound when integrated with that of the expansion must produce an offset and that offset is even more pronounced when describing random targets.

Importantly, expansion does not explain the dark matter mass discrepancy because mass alone does not determine the orbital velocity as Newton had surmised. Several effects are at work. Thus, Newton's expression is correct so long as expansion, mass frequency, measurement distortion (i.e., also described by relativity), and the *Informativity differential* are not significant factors.

Modern physical descriptions use mass to describe gravity, but the effective mass is significantly greater in magnitude than the observed mass described by Newton. The bound describes a geometric *reference* with an *offset* swinging from one side to the other like a weight on a rubber band. While the expressions of Informativity and Newton coincide for systems having less mass density, velocity is not solely a function of mass. Thus, the question, where is the missing mass, is not valid.

Bounded gravity may also be applied to the early universe when mass density was significant. When expressions that incorporate bounded gravity are used, some doors may open with early-universe modeling. While not the focus of this paper, a detailed account of quantum inflation, the trigger event that ends this epoch, and the ensuing expansion are described in the first paper ([1], Section 3.15) with an additional explanation of the effects of measurement distortion described in the second paper ([4], Section 3.6). Notably, the solution as presented in equations (45)–(48) is a function of one physical constant.

A final question is why should the measured mass of a galaxy be attributed to the observable and not the visible distribution? The answer is primarily subjective as mass distributions may not be used to describe a galaxy. That said, one may note with elapsed time that a specific amount of observable mass becomes visible. Because the mass of a galaxy does not increase, the label 'observable' is more appropriate.

In practice, the issue with scaling distributions is that the scaling process changes the properties that the distributions are defined against. To succeed, any application must retain each property of the initial definition. For instance, the scaling would require that the outer edge of the galaxy expand at the speed of light. Loss of this property is immediately obvious. For one, dark mass cannot even exist. As well, the visible and observable distributions are always the same.

3.6. Kinetic Energy

As a follow-up to mass frequency, we may provide one final confirmation of our understanding of $n_M/n_{Lr}=2m_f$ by reducing the Informativity interpretation to demonstrate the equation for kinetic energy. Notably, the classical expression does not include the radial expansion parameter θ_{si} which is defined with respect to a bound and therefore carries no units. Hence, we start with the static radial form. Given $m_f=2\theta_{si}/c$ from the *fundamental expression* and the expression for half a fundamental unit of mass, $E_f=2\theta_{si}c$ ([2], equation (49)), then the static velocity bound is

$$v = c\sqrt{2m_f} = c\sqrt{\frac{4\theta_{si}}{c}} = \sqrt{4\theta_{si}c} = \sqrt{2E_f}\ . \tag{84}$$

The generalized expression such that $m_f=2\theta_{si}/c$ and $n_M<=(1/m_f)$ is

$$v = c\sqrt{2m_f} = c\sqrt{\frac{2}{1/m_f}} = c\sqrt{\frac{2}{n_M}} = \sqrt{\frac{2c^2}{n_M}}\ , \tag{85}$$

$$v = \sqrt{\left(\frac{2\theta_{si}}{c}\frac{1}{m_f}\right)\frac{2c^2}{n_M}} = \sqrt{\frac{4\theta_{si}c}{n_M m_f}} = \sqrt{\frac{2E}{m}}\ , \tag{86}$$

and may then be reduced to resolve the kinetic energy associated with any mass,

$$E = \frac{mv^2}{2}\ . \tag{87}$$

One may compare the first and last velocity expressions and wonder why the latter has a mass value in the denominator. The mass value is what generalizes the expression for any mass, velocity, and energy. The initial expression is an invariant description of the smallest unit of energy E_f corresponding to a mass count bound of n_M=1/m_f. That ratio is precisely 1 leaving us with 2E_f under the square root operator.

4. DISCUSSION

Perhaps the most significant outcome of this research is not a model of galactic orbital dynamics, but the inclusion of expansion, the mass frequency bound, and mass density into a single description of orbital velocity. Since the time of Newton, mass has been considered the primary factor describing the effects of gravitation. There have been modifications to that understanding (i.e., relativity), but such modifications have been a fine-tuning of the broader expressions set forth by Newton, mass and radial distance being the variables that determine orbital motion. However, with the expressions set forth here, mass is one of several factors. The mass frequency bound now designates the demarcation point; it is quantum and valid for the entire measurement domain.

Finally, where there have been several proposals describing solutions to galactic orbital dynamics [9], the traditional approach is one of resolving data-dependent expressions. Informativity takes a uniquely different view of the universe, in that the physical expression is an outcome of bounds to measure. The mass frequency bound is an outcome of this axiom, a geometric expression that identifies a *reference* and an *offset* against which the effective mass is resolved. A mathematics of counts of the fundamental measures is all that is needed to unravel the motion of stars.

APPENDIX

A1. Numerical Limits to $Q_L n_{Lr}$

The term $Q_L n_{Lr}$ is referred to as the *Informativity differential* in recognition of the central role it plays in describing how fractional values less than the reference measure reflect a distorting effect in distance measurement. Knowing the limits to $Q_L n_{Lr}$ is essential in resolving the fundamental measures.

Equation (2) multiplied by n_{Lb} yields

$$Q_L n_{Lr} = \left(\sqrt{1+n_{Lb}^2} - n_{Lb}\right) n_{Lb}\,. \tag{A1.1}$$

Note, what is measured always equals a whole-unit count of a fundamental measure, and with a=1, we find that n_{Lb}=n_{Lr} for all values. This is easily verified in that the highest value for Q_L is obtained for n_{Lb}=1 where $(1+1^2)^{0.5}-1$= 0.414 and the 'observed' distance of c presented as a count n_{Lr} is always rounded down to the highest integer value equal to the count n_{Lb} with Q_L=0.414 at its highest and quickly approaches 0 with increasing n_{Lb}. Therefore,

$$Q_L n_{Lr} = \left(\sqrt{1+n_{Lr}{}^2} - n_{Lr}\right) n_{Lr}\,. \tag{A1.2}$$

The lower limit where n_{Lr}=1 is easily produced, $lim_{r=1} f(Q_L n_{Lr})$ =$\sqrt{}$2−1. Conversely, if we divide by n_{Lr}, then add n_{Lr}, square, subtract $n_{Lr}{}^2$, and divide by 2, we find that

$$\frac{Q_L{}^2}{2} + Q_L n_{Lr} = \frac{1}{2}\,. \tag{A1.3}$$

Q_L decreases with increasing n_{Lr} until the left term drops out. Distance does not need to be significant to reduce the *Informativity differential*. At just 10^4 l_f, $Q_L n_{Lr}$ rounds to 0.5 to nine significant digits.

A2. Upper Bound Relationship between Length and Mass

To resolve the upper bound relation between length and mass, we begin with the expression for escape velocity, setting the velocity equal to the speed of light being the upper bound, and then substituting the fundamental units for each of the terms. The expression for G follows notably from equation (6) as

$$G = \frac{Q_{Lf} r c^3}{\theta_{si}} = \frac{Q_{Lf} r_{Lf} l_f c^3}{\theta_{si}} = \frac{c^3 l_f}{2\theta_{si}} = \frac{c^3 t_f}{m_f} = \frac{l_f}{t_f}\frac{l_f}{t_f}\frac{l_f}{t_f}\frac{t_f}{m_f}. \quad \text{(A2.1)}$$

Likewise, a generalized mass count n_M of m_f follows from the *fundamental expression* $l_f m_f = 2\theta_{si} t_f$. Because $lim_{r\to\infty} f(Q_L n_{Lr}) = 1/2$, as resolved in Appendix A1, then

$$m_f = \frac{2\theta_{si}}{c} = \frac{\theta_{si}}{Q_{Lf} r_{Lf} c}. \quad \text{(A2.2)}$$

With $c = l_f/t_f$, the expression for escape velocity may be reduced to show that

$$v = \left(\frac{2GM}{r}\right)^{1/2}, \quad \text{(A2.3)}$$

$$c > \left(\frac{2}{r}\frac{Q_L r c^3}{\theta_{si}}\frac{n_M \theta_{si}}{Q_L n_{Lr} c}\right)^{1/2} > \left(\frac{2 n_M c^2}{n_{Lr}}\right)^{1/2}, \quad \text{(A2.4)}$$

$$n_{Lr} > 2 n_M. \quad \text{(A2.5)}$$

Using the escape velocity, the upper bound of a count of n_M with respect to n_{Lr} is resolved. Conversely, for orbital velocity, the expression is $v = (GM/r)^{1/2}$. The relation differs by a factor of two,

$$n_{Lr} > n_M. \quad \text{(A2.6)}$$

A3. Observable Mass Bound

The observable mass may be resolved by setting the bound velocity equal to the classical velocity and reducing. With $G = c^3 t_f/m_f$, then

$$\theta_{si} c \sqrt{2 m_f} = \sqrt{\frac{G M_{b-f(R)}}{R}}, \quad \text{(A3.1)}$$

$$M_{b-f(R)} = 2\theta_{si}^2 R c^2 m_f \frac{1}{G} = 2\theta_{si}^2 R c^2 m_f \frac{m_f}{c^3 t_f}, \quad \text{(A3.2)}$$

$$M_{b-f(R)} = 2\theta_{si}^2 R c^2 m_f \frac{m_f}{c^3 t_f} = 2\theta_{si}^2 R \frac{m_f^2}{l_f}, \quad \text{(A3.3)}$$

$$M_{b-f(R)} = 2\theta_{si}^2 R \frac{m_f^2}{l_f} = 2\theta_{si} R \frac{m_f l_f}{2 t_f}\frac{m_f^2}{l_f}, \quad \text{(A3.4)}$$

$$M_{b-f(R)} = \theta_{si} R \frac{m_f^3}{t_f} . \quad \text{(A3.5)}$$

Recall, the left-hand side of the v_b expression in equation (A3.1) has a value of m_f, which is a dimensionless substitute for n_{Mb}. There are no units. This is fine until equation (A3.4) where R in meters cancels with l_f in meters leaving one of the two m_f^2 with a single kilogram unit describing $M_{b-f(R)}$. However, in equation (A3.1), we introduce the dimensionless expression $\theta_{si}=m_f l_f/2t_f$. Several cancellations leave both R and t_f, as well as an additional m_f , dimensionless. The result is in kilograms,

$$M_{b-f(R)} = 2\theta_{si}^2 R \frac{m_f^2}{l_f} m \frac{kg}{m} = \theta_{si} R \frac{m_f^3}{t_f} kg . \quad \text{(A3.6)}$$

A4. Resolving Effective Velocity as a Function of Mass

The effective and observed velocities may have the same value, yet each term is identified separately. This calls into question the use of one to identify the other if they are not physically different. They are.

The two terms operate as a limit where the estimated value for v_o constrains the resulting value for v_e. The correct physical description is where the modeled value for v_o produces a value for v_e that is closer than any other combination,

$$v_e = 2\theta_{si} \left(2 \frac{v_o}{\sqrt{2m_f}} - c\theta_{si} \right)^{1/2} . \quad \text{(A4.1)}$$

There may exist a theoretical argument in which the use of an observed velocity to resolve the effective velocity is still in principle problematic. For that reason, an alternative is offered whereby the observable velocity is replaced by a function with effective mass as the only free variable,

$$M_{e-f(R)} = M_{b-f(R)} \left(2 \frac{v_o}{v_b} - 1 \right), \quad \text{(A4.2)}$$

$$\frac{v_o}{v_b} = \frac{1}{2} \left(\frac{M_{e-f(R)}}{M_{b-f(R)}} + 1 \right), \quad \text{(A4.3)}$$

$$2v_o = v_b \left(\frac{M_{e-f(R)}}{M_{b-f(R)}} + 1 \right), \quad \text{(A4.4)}$$

$$2v_o = \theta_{si} c \sqrt{2m_f} \left(\left(M_{e-f(R)} / \theta_{si} \frac{m_f^3}{t_f} R \right) + 1 \right), \quad \text{(A4.5)}$$

$$v_o = \sqrt{\frac{m_f}{2}} \left(\frac{M_{e-f(R)}}{R} \frac{l_f}{m_f^3} + \theta_{si} c \right). \quad \text{(A4.6)}$$

While it is not possible to produce a data-independent expression that characterizes a galaxy, the initial expression is now resolved as a function of the effective mass, which is resolved as a function of observed mass. In short, the effective velocity is resolved as a function of the observed mass.

A5. Mass Distribution Conversions

Following is a list of commonly used universal mass conversion expressions. Several are resolved from the first paper ([2], equations (113), (110), (109), and (108)). Notably, many of the expressions in the first paper are percentage expressions of a total mass. To resolve distribution values in kilograms, multiply the distribution percentage by M_{tot} in kilograms. In that all expressions are appropriate when using the units of kilograms, we often express these relations using the same units. Notably, units of kilograms are required for some expressions such as those that incorporate fundamental mass M_f or that incorporate counts of the fundamental units.

$$M_{obs} = 2\theta_{si} M_{vis} , \tag{A5.1}$$

$$M_{obs} = M_{tot} \frac{4}{\theta_{si}^2 + 2} , \tag{A5.2}$$

$$M_{dkm} = M_{tot} \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} , \tag{A5.3}$$

$$M_{tot} = M_{obs} + M_{dkm} , \tag{A5.4}$$

$$M_{uobs} = M_{obs} - M_{vis} . \tag{A5.5}$$

From the above, we resolve

$$M_{uobs} = M_{vis} \left(2\theta_{si} - 1\right) , \tag{A5.6}$$

$$2M_{tot} = M_{vis} \theta_{si} \left(\theta_{si}^2 + 2\right) , \tag{A5.7}$$

$$M_{dkm} = M_{obs} \frac{\left(\theta_{si}^2 - 2\right)}{4} , \tag{A5.8}$$

$$M_{dkm} = M_{vis} \frac{\theta_{si} \left(\theta_{si}^2 - 2\right)}{2} , \tag{A5.9}$$

and from the first paper ([2], equation (118)) we may also resolve

$$2M_{tot} M_f = M_{obs} (M_{tot} + M_f) , \tag{A5.10}$$

$$M_{tot} M_f = \theta_{si} M_{vis} (M_{tot} + M_f) , \tag{A5.11}$$

$$M_f = \frac{M_{tot} \theta_{si} M_{vis}}{M_{tot} - \theta_{si} M_{vis}} . \tag{A5.12}$$

We may also derive the relationship between the total and fundamental mass using the expression for the total mass ([2], equation (134)), and the expression for fundamental mass ([2], equation (128)),

$$M_{tot} = n_{Tu} m_f \frac{\theta_{si}^3}{2} , \tag{A5.13}$$

$$M_f = n_{Tu} m_f \theta_{si} , \tag{A5.14}$$

$$M_{tot} = M_f \frac{\theta_{si}^2}{2} , \tag{A5.15}$$

$$\frac{M_f}{M_{tot}} = \frac{2}{\theta_{si}^2} . \tag{A5.16}$$

Next, the fundamental mass from equation (24) is reduced with D_U=$2R_U$=$2\theta_{si}A_U$ from equation (25) and set equal to the bound mass in equation (77). The mass bound for the universe M_{Ub} is then

$$M_f = A_U \theta_{si} \frac{m_f}{t_f} = R_U \frac{m_f}{t_f} \tag{A5.17}$$

$$M_{b-f(R)} = R\theta_{si} \frac{m_f^3}{t_f} \tag{A5.18}$$

$$\frac{M_{Ub}}{m_f^2 \theta_{si}} = R\frac{m_f}{t_f} = M_f \tag{A5.19}$$

$$M_{Ub} = M_f m_f^2 \theta_{si} \tag{A5.20}$$

Lastly, given the observable v_{obs} and visible v_{vis} velocities and equation (A5.1), then

$$\frac{v_{obs}}{v_{vis}} = \frac{\sqrt{GM_{obs-f(R)} / R}}{\sqrt{GM_{vis-f(R)} / R}}, \tag{A5.21}$$

$$\frac{v_{obs}}{v_{vis}} = \sqrt{\frac{M_{obs-f(R)}}{M_{vis-f(R)}}} = \sqrt{\frac{2\theta_{si} M_{vis-f(R)}}{M_{vis-f(R)}}} = \sqrt{2\theta_{si}} . \tag{A5.22}$$

In regard to mass, visible mass corresponds to the 4.84884% distribution as described in equation (40). The observable mass corresponds to the 31.6376% distribution as described in equation (39), and incorporates universal expansion, Ω_{obs}=$H_U\Omega_{vis}$.

A6. Clarifying Interpretation of Mass Distributions

Distribution expressions may take a percentage or mass value. An expression demonstrating percentages may be converted to kilograms by multiplying the result by M_{tot} in kilograms. Depending on the substitutions selected, a resulting expression can lead to an incorrect interpretation. One notable complication, fundamental mass M_f is always in kilograms. To demonstrate the issue, consider equation (42),

$$2\theta_{si} = \frac{M_{obs}}{M_{vis}}, \tag{A6.1}$$

$$\frac{2}{\theta_{si}^2} = \frac{1}{\theta_{si}^3} \frac{M_{obs}}{M_{vis}} . \tag{A6.2}$$

Then set the two expressions equal to one another,

$$\frac{M_f}{M_{tot}} = \frac{1}{\theta_{si}^3} \frac{M_{obs}}{M_{vis}} = \frac{1}{\theta_{si}^3} \frac{2\theta_{si} M_{vis}}{M_{obs} / 2\theta_{si}} = \frac{4}{\theta_{si}} \frac{M_{vis}}{M_{obs}}, \tag{A6.3}$$

$$\theta_{si} M_{obs} M_f = 4 M_{vis} M_{Tot} . \tag{A6.4}$$

And finally, as

$$M_{tot} = n_{Tu} m_f \frac{\theta_{si}^3}{2} \quad \text{(A6.5)}$$

is a known function of time ([2], equation (134)), we may reduce equation (A6.4) such that time is the only free variable.

$$2M_{tot}M_f = M_{obs}(M_{tot} + M_f)\,, \quad \text{(A6.6)}$$

$$2M_{tot}2\frac{M_{Tot}}{\theta_{si}^2} = M_{obs}(M_{tot} + 2\frac{M_{Tot}}{\theta_{si}^2})\,, \quad \text{(A6.7)}$$

$$\theta_{si}^2 M_{tot} M_{obs} + 2M_{Tot}M_{obs} - 4M_{Tot}^2 = 0\,, \quad \text{(A6.8)}$$

$$\theta_{si}^2 n_{Tu} m_f \frac{\theta_{si}^3}{2} M_{obs} + 2n_{Tu} m_f \frac{\theta_{si}^3}{2} M_{obs} - 4n_{Tu}^2 m_f^2 \frac{\theta_{si}^6}{4} = 0\,, \quad \text{(A6.9)}$$

$$n_{Tu} m_f \frac{\theta_{si}^5}{2} M_{obs} + n_{Tu} m_f \theta_{si}^3 M_{obs} - n_{Tu}^2 m_f^2 \theta_{si}^6 = 0\,, \quad \text{(A6.10)}$$

$$\frac{\theta_{si}^2}{2} M_{obs} + M_{obs} - n_{Tu} m_f \theta_{si}^3 = 0\,, \quad \text{(A6.11)}$$

$$M_{obs}\left(\frac{\theta_{si}^2}{2} + 1\right) = n_{Tu} m_f \theta_{si}^3\,, \quad \text{(A6.12)}$$

$$M_{obs} = 2n_{Tu} m_f \frac{\theta_{si}^3}{\theta_{si}^2 + 2}\,. \quad \text{(A6.13)}$$

With elapsed time n_{Tu}, one might assume that the observable mass distribution M_{obs} is increasing. This is not a complete picture. The observable and total masses (A6.5) are both increasing while the distributions remain invariant,

$$M_{obs} = \left(n_{Tu} m_f \frac{\theta_{si}^3}{2}\right)\frac{4}{\theta_{si}^2 + 2}\,, \quad \text{(A6.14)}$$

$$\Omega_{obs} = \Omega_{tot} \frac{4}{\theta_{si}^2 + 2}\,. \quad \text{(A6.15)}$$

The result was demonstrated in the first paper ([2], equation (110)).

A7. Glossary of Terms

Framework
A frame of reference against which a system of measure is applied. Frameworks are commonly discussed in Informativity and are typically either that of the observer's inertial frame, the observed target or that of the universe.

Fundamental Expression
The simplest expression correlating the three fundamental measures, $l_f m_f = 2\theta_{si} t_f$.

Fundamental Mass
The fundamental mass of the universe distinguishes a specific amount of mass whereby from a point in space-time additional mass would cause overlapping mass events that could not be distinguished due to physically significant bounds to the measure of fundamental units of mass. Understanding and resolving

fundamental mass in turn allows one to solve for all the mass distributions presently understood only with ΛCDM.

Fundamental Measure
One of the measures length l_f, mass m_f, and time t_f along with their correlation called the *fundamental expression*. Using measurement data from the Shwartz and Harris experiments in combination with Heisenberg's Uncertainty Principle, each are macroscopically defined and physically significant.

Informativity Differential
The Informativity differential $Q_L n_{Lr}$ describes a new form of length contraction associated with the lower bound to measure. The loss of immeasurable space at each increment of t_f describes gravity.

Observable Mass
The observable mass includes the mass which is visible in the present and the mass which will be visible at some point in the future. The observable mass represents all the mass that can be known in the universe. This is as opposed to mass that exists sufficiently distant that it is beyond the horizon and as such, due to the expansion of the universe, the light from that mass will never reach the observer.

Quantum
The term quantum is intended to mean a small measure such as a few tens, hundreds or thousands of fundamental units of measure.

Quantized
The term quantized is intended to mean that expressions are composed of terms that are whole-unit counts of the fundamental units and that those units are physically significant.

Visible Mass
The visible mass is that mass which is presently visible. In relation to the universe this would be the mass of those stars, dust or other forms of mass that are visible in the present as opposed to the mass corresponding to light that will be visible in the future.

A8. Symbol Definitions

H_U is the expansion of the universe defined with respect to the universe (diameter). This differs slightly from stellar expansion (i.e. Hubble's description).
l_f, m_f and t_f are effectively Planck's Units for length, mass, and time, but not precisely the same.
θ_{si}, is 3.26239 radians or kg m s^{-1} (momentum) or no units at all a function of the chosen frame of reference. This is a new constant to modern theory and exists in nearly every equation of the model. It may be measured macroscopically given specific Bell states necessary for quantum entanglement of X-rays such as those carried out by Shwartz and Harris.
β is the speed parameter typically found in relativistic expressions. The parameter varies depending on the measures being compared.
$A_{s\text{-}ref}$ is the dilated age of the universe as measured from our point of view inside an expanding universe.
$A_{s\text{-}def}$ is the non-dilated age of the universe as would be measured if the universe were not expanding.
Ω_{vis} is the matter distribution that is presently seen from a point in space. M_{vis} is the same matter in kilograms.
Ω_{obs} is the matter distribution that is presently or will eventually be known from a point in space. M_{obs} is the same matter in kilograms.
Ω_{dkm} is the matter distribution that is beyond the observable matter, matter which will never be known from a given point in space. M_{dkm} is the same matter in kilograms.

Ω_{uobs} is the matter distribution that will eventually be known from a point in space, but is not presently known. M_{uobs} is the same matter in kilograms.
Ω_{tot} is all the matter in the universe. M_{tot} is the same matter in kilograms.
M_f is the fundamental mass. Mass in excess of the fundamental mass exceeds the number of mass events per unit of time that can be distinguished at a point in space.
M_{acr} is the rate of mass accretion with respect to the universe.
$M_{o\text{-}f(R)}$ is the observable mass within a given radial orbit of a target galaxy
$M_{e\text{-}f(R)}$ is the Informativity effective radial mass within of a target galaxy. The value incorporates Newton's expression and the effects of universal expansion.
$M_{b\text{-}f(R)}$ is the Informativity mass frequency bound radial mass which corresponds to upper mass bound of mass events that equals but does not exceed the upper mass-to-length frequency bound.
$M_\odot$ is one solar mass.
A_U is the age of the universe.
R_U is the radius of the universe.
D_U is the diameter of the universe.
H_U is the rate of universal expansion with units light-years per year.
n_{Mu} is a count of m_f equal to the total of mass/energy in the universe.
n_{Tu} is a count of t_f equal to the age of the universe.
n_{Lu} is a count of l_f equal to the diameter of the universe.
n_{Lo} is a count of l_f that is being observed.
n_{Lr} is a count of l_f from the observer to a center of gravity.
n_{Ll} is a count of l_f as measured in the local frame of reference.
n_{Tl} is the count of t_f as measured in the local frame of reference.
n_{To} is the count of t_f that is being observed.
n_{Lm} is the change in position of the target as a count of l_f as measured in the local frame of reference.
n_{Lc} is the change in position of light as a count of l_f as measured in the local frame of reference.
n_M is a count of m_f representing the mass corresponding to a gravitational field.
n_L is a count of l_f representing the length between an observer and the target.
n_T is a count of t_f representing the time elapsed between two events.
n_{Lf} is a known count of l_f typically used when describing distance with respect to an observer.
Q_L is the fractional portion of a count of l_f when engaging in a more precise calculation.
n_{Lb} is a known distance, a count of the reference l_f.
v_n is the radial velocity of a star plotted with respect to Newton's expression for gravity
v_o is the observed radial velocity of a star when accounting for all well-established effects
v_e is the Informativity effective velocity of a star in orbit around a galactic core. The expression may resolved using Newton's expression and the effective radial mass for a given radius.
v_b is the Informativity mass frequency bound velocity which corresponds to upper mass bound of mass events that equals but does not exceed the upper mass-to-length frequency bound.
G is Newton's gravitational constant.
S is the symbol assigned to the unknown constant when resolving a description of gravity. The symbol is replaced with θ_{si}.
c is the speed of light which may also be written as $c=l_f/t_f$.
v is velocity measured between an observer and a target.
r is some unknown distance between an observer and a target.
h is Planck's constant adjusted to reflect the quantum effects of the *Informativity differential*.
$\hbar$ is Planck's reduced constant adjusted for the Informativity differential as a function of distance to target.
σ_x is a description of the uncertainty in the position of a particle
σ_P is a description of the uncertainty in the momentum of a particle
k is the Boltzmann constant.
ρ is the energy density of mass/energy accumulated at a given age of the universe.

a is the total energy radiated as described with respect to blackbody radiation (i.e. the Stefan-Boltzmann law).
T is the temperature of the Cosmic Microwave Background

Acknowledgments

We thank Edanz Group (www.edanzediting.com/ac) for editing a draft of this manuscript.

References

[1] Lonappan, A.I., Ruchika, & Sen, A.A., 2017, arXiv: 1705.07336
[2] Geiger, J.A., 2018, JHEP, 4, 2, 262
[3] McCulloch, M.E., 2012, ApSS, 342 575
[4] Geiger, J.A., 2018 JHEP 4, 4
[5] Shwartz, S., & Harris, S.E., 2011, PRL, 106, 080501
[6] Cox, N.A., 2002, Allen's Astrophysical Quantities, (4th ed.; Springer, p. 294), ISBN 0-387-98746-0
[7] McGaugh, S.S., 2008, ApJ, 683, 137
[8] Madhavacheril, M., et. al. (Atacama Cosmology Telescope Collaboration), 2015, PRL, 114, 151302
[9] Hossenfelder, S., & McGaugh, S.S., 2018, SciAm 319, 36