5 different superposition principles with/without test charge, retarded waves/advanced waves applied to dynamic equation of the photon

Shuang-ren Zhao (shrzhao@gmail.com)

November 27, 2018

Abstract

In electromagnetic theory and quantum theory, there are superposition principle. The author found that there are 5 different kinds of superpositions. The superposition principles have some differences. The research about these differences is a key to open the the door of many physical difficulties. For example the particle and wave duality problem, and to judge which interpretation of the quantum mechanics is a correct one. The first two superposition principles are the superpositions with and without the test charges. The slight different superposition principles are the superposition with a retarded wave alone and the superposition with the advanced wave alone. According to theory of this author, the emitter sends the retarded wave, the absorber sends the advance wave. Hence, normal electromagnetic field actually is consist of retarded wave and advanced wave. This two wave together become the normal electromagnetic field. This kind of electromagnetic field can be seen approximately as retarded wave, this kind wave also has its own superposition. This kind of superposition is also different with the superposition when we consider the retarded wave and also the advanced wave. In this article this author will discuss the differences of these different situations of superpositions. This author will also discuss the different physical result with a few different superposition principles. In this article this author will prove only when the self-energy principle is accept, all kinds of superposition can be accept. Otherwise only the superposition with test charge or the superposition with only one kind wave either retarded waves or advanced waves can be accepted. Hence, the discussion about the superposition also support the concept of the self-energy principle which means there must exist the time reversal waves. That also means the waves do not collapse but collapse back. Wave collapse means collapse to target of the wave, for example, the retarded wave will collapse to a absorber and the advanced wave will collapse to a emitter. Wave collapse back means the retarded wave sent from emitter will collapse back to emitter; The advanced wave sent from the absorber will collapse back to an absorber. Hence, one purpose of this article is to clarify the superposition principles, and another purpose is to support this author's electromagnetic field theory which is started from two new axioms the self-energy principle and the mutual energy principle.

Keywords: Poynting; Maxwell; Self-energy; Mutual energy; Mutual energy flow; Time reversal; Photon; Electromagnetic; Action-at-a-distance; Advanced wave; Advanced potential; Advanced field. Absorber theory; macroscopic, microscopic;

1 Introduction

Superposition principle is usually thought a self-explanatory principle. However, this author found that there are differences for different kinds of superpositions. Especially if we accept the advanced wave also as electromagnetic field similar to the retarded wave. It is naturally to ask also whether or not the retarded fields can be superposed to the advanced field? It is know that the retarded wave and advance wave all satisfy the Maxwell equations, hence, they are all electromagnetic field, hence it should be possible to superposed the retarded wave and the advanced wave. Whether or not the superposition principle can be applied, we can assume it can do it in the beginning, if this assumption doesn't conflict with any other theorem in electromagnetic field theory, the superposition can be accept. Other wise the superposition can not be accept.

1.1 Maxwell theory

In Maxwell theory the field can be superposed. That means for example for each charge the field satisfy the Maxwell equations,

$$\nabla \cdot \boldsymbol{D}_i = \rho_i \tag{1}$$

$$\nabla \cdot \boldsymbol{B}_i = 0 \tag{2}$$

$$\nabla \times \boldsymbol{E}_i = \boldsymbol{J}_i + \frac{\partial \boldsymbol{D}_i}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{H}_i = -\frac{\partial \boldsymbol{B}_i}{\partial t} \tag{4}$$

where ρ_i, $\boldsymbol{J}_i$ are charge intensity and current intensity of i-th charge. Assume there are N charges, then the field can be superposed as following,

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i \tag{5}$$

$$\boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \tag{6}$$

or

$$\xi = \sum_{i=1}^{N} \xi_i \tag{7}$$

where $\xi = [\boldsymbol{E}, \boldsymbol{H}]$ and $\xi_i = [\boldsymbol{E}_i, \boldsymbol{H}_i]$. This is the superposition principle. This superposition are without any additional conditions.

1.2 Absorber theory

However there are lot of arguments about the above superposition principle. That is electromagnetic field is defined on the force on the charge, the magnetic field is defined on the force to a moved charge. The charge play a rule as a test charge. They have the argument that if the test charge is here, it is clear the force can be superposed, and hence the field also can be superposed. Because the electric field is defined by the electric force divided the charge.

$$\boldsymbol{F}_e = q\boldsymbol{E} \tag{8}$$

$$\boldsymbol{F}_h = q\boldsymbol{v} \times \boldsymbol{B} \tag{9}$$

where q is the charge amount of the test charge. $\boldsymbol{F}_e$ is the force assumed to the charge. $\boldsymbol{F}_h$ is the magnetic force exerted to the charge. However this doesn't mean that if you can superpose the two electric fields or two magnetic fields even if the test charge isn't there.

In the absorber theory of Wheeler and Feynman[1], they said that the electromagnetic fields have no their own freedom. They are adjunct fields to the action-at-a-distance[2].

We know that action take place among at least two objects for example an emitter and an absorber. Hence the retarded field of the emitter can be defined on the place of the absorber because the absorber can be applied as a test charge. The advanced field of the absorber can be defined on the place of the emitter, because the emitter can be applied as a test charge to the advanced field. When there is only one charge, there is no another charge as the test charge, the action between two charges cannot be built and hence the action cannot be existed. If the action cannot be existed, because the electromagnetic fields are adjunct fields to the action, which cannot be existed too. Any way without test charge it is possible the field cannot be defined correctly, if fields themselve cannot be defined how they can be superposed?

The absorber theory of Wheeler and Feynman [1, 2] involved the field of advanced field. Advanced field, has also been said as advanced wave or advanced potential. In electromagnetic field theory advanced potential is used more often, in quantum physics, the advanced wave is used more often. The following we have to review the theories about the advance wave.

1.3 Review of the theory for photon and electromagnetic fields which the advanced wave is involved

Maxwell equations have two solutions, the retarded potential/wave and the advanced potential/wave. But it is not clear about the relationship of the retarded wave and the advanced wave from the Maxwell's theory. The theory of the action-at-a-distance from Schwarzs, Child-Tetrode and Fokker [15, 8, 16], the absorber theory of Wheeler-Feynman[1, 2] and the transactional interpretation of Cramer [5, 6] all support the existence of the both retarded wave and advanced wave. According to the absorber theory, the current will send the retarded wave and advanced wave in the same time. Each has the half value of the total field. That means

$$\xi = \xi_r + \xi_a \tag{10}$$

Where $\xi = [\boldsymbol{E}, \boldsymbol{H}]$ is the total electromagnetic field. $\xi_r = [\boldsymbol{E}_r, \boldsymbol{H}_r]$ is the retarded field, $\xi_a = [\boldsymbol{E}_a, \boldsymbol{H}_a]$, is the advanced field.

W.J. Welch has introduced time-domain reciprocity theorem[17] in 1960. In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula[14]. In early of 1987 this author has introduced the concept of mutual energy and the mutual energy theorem [10, 19, 18]. In the end of 1987 Adrianus T. de Hoop has introduced the time domain cross-correlation reciprocity theorem[7]. It can be proven that the cross correlation reciprocity theorem of de Hoop is the inverse Fourier transform of the mutual energy theorem of this author. Welch's reciprocity theorem is a special situation of the cross correlation reciprocity theorem. Rumsey reciprocity theorem has the same formula with the mutual energy theorem. Hence all these 4 theorems are a same theorem in different domain: Fourier domain or time domain. In these theorems, the advanced waves were involved. These theorems are clear a reciprocity theorem, but this author believe it is also a energy theorem and hence will call all these theorem as mutual energy theorems.

This author noticed that the energy transfer in the empty space is more accurate by using the mutual energy theorem instead of the Poynting theorem. The Poynting theorem often lead to wrong results. For example, if we calculate the receiving energy or scattering energy of a small object, we should use the Poynting vector times the object's section area. However, that often offers a wrong result. Hence, some one create a concept "the effective scattering section area" which is much larger than the object's real section area. We use the Poynting vector times the effective scattering section area to calculate the received or scattered energy of the object. The effective scattering section area can be thousand times larger than the object's real section area. That actually means there is thousand times error if we use the Poynting theorem to calculate the energy received or scattered by a object.

This author also noticed that if the Poynting theorem together with superposition principle applied to a situation with N charges, there are conflicts to the energy conservation law [11].

This author combined the concept of mutual energy and the mutual energy theorem[10] with the the concepts in absorber theory[1, 2] introduced the the mutual energy principle. The theory of the mutual energy principle is further proven by introducing the energy conservation condition to the classical electromagnetic field theory of Maxwell, which also need to introduced the self-energy principle. The two new principles can be applied as two axioms for the electromagnetic field theory [11].

The self-energy principle tells us there should be 4 waves in the electromagnetic field theory instead of 1 or 2 waves. The 4 waves are the retarded wave, the advanced wave and the two time-reversal waves. The two time-reversal waves are the time-reversal wave corresponding to the retarded wave and the time-reversal wave corresponding to the advanced wave. The 4 waves completely balanced out or canceled. And hence, the waves cannot transfer energy in space.

The mutual energy principle tell us the retarded wave and the advanced wave must synchronized. Only one wave for example the retarded wave can not satisfy the mutual energy principle, and hence, it is not a wave has real physical effect. The retarded wave of the emitter and the advanced wave of the absorber can be superposed to produce the mutual energy flow which can carry energy in the space. There is also a time-reversal mutual energy flow. The energy of the photon can be described by the mutual energy flow. The time-reversal mutual energy flow is responsible to bring the half photon back to the emitter if a race case happens and the two absorbers each received a half photon energy. These new electromagnetic field theory is suitable not only the electromagnetic fields but also to the photonics.

1.4 The Maxwell equations and the superpositions

We often heard that the Maxwell field are linear, hence, the fields can be superposed. This author do not think this is true without any limitation. Hence in this article the Maxwell equations for single charge and the Maxwell equations for superposed fields are believed as different Maxwell equations. This means this author does not believe Maxwell equations implied the superposition principle. This author will explicitly distinguish Maxwell equations and the superposition principle. This author will explicitly distinguish the Maxwell equations for a single charge or the Maxwell equation for superposed fields. In the later section the reader will found in this article the superposition will also be divided as different superpositions according to whether the test charge appears or not. The superpositions also will be distinguished as the fields of the retarded waves alone, the fields of the advanced waves alone or the fields of traditional fields. The traditional field is a field used in the electromagnetic field theory where the advanced wave do not accept and hence the advanced wave has been merged to the retarded field.

Since in the author's electromagnetic field theory the advanced field is accept, the superposition needs to be distinguished with a few different situations.

2 Superposition principle

2.1 Superposition without test charge

Accounting the introduction we have known that about the superposition principle there are different opinions. Even they have different opinions, both sides claim they are the corrected superposition principle. In order to distinguish the two situations, the superposition without a test charge and the superposition

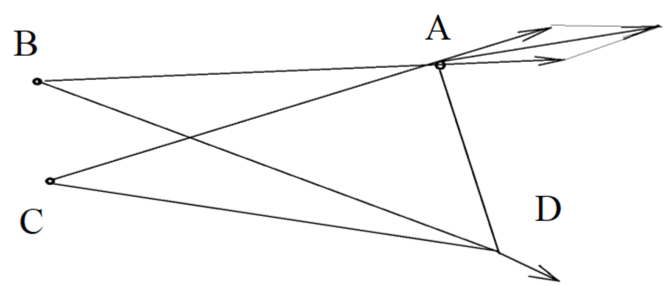

Figure 1: Assume there are 3 charges in the empty space. Charge A, Charge B and Charge C. We discuss the superposition on place D there are no charges and the superposition on the place A, B or C where there is a charge.

with a test charge are explicitly given two names.

See Figure 1, there are 3 charges A, B, and C in the empty space (a empty space is a assumed space where is nothing else). Assume D is a at a place there is no any charge. It is clear the superposition of the 3 charges on the place D will be,

$$\xi_D = \xi_{AD} + \xi_{BD} + \xi_{CD} \tag{11}$$

where $\xi_D = [\boldsymbol{E}_D, \boldsymbol{H}_D]$ is the electromagnetic field on the place D. $\xi_{AD} = [\boldsymbol{E}_{AD}, \boldsymbol{H}_{AD}]$ is the field at the place D and produced by the charge A. $\xi_{BD} = [\boldsymbol{E}_{BD}, \boldsymbol{H}_{BD}]$ is the field at place D produced by the charge B. $\xi_{CD} = [\boldsymbol{E}_{CD}, \boldsymbol{H}_{CD}]$ is the field at the place D produced by the charge C. Hence, we have,

$$\boldsymbol{E}_D = \boldsymbol{E}_{AD} + \boldsymbol{E}_{BD} + \boldsymbol{E}_{CD} \tag{12}$$

$$\boldsymbol{H}_D = \boldsymbol{H}_{AD} + \boldsymbol{H}_{BD} + \boldsymbol{H}_{CD} \tag{13}$$

This kind of superposition is referred as superposition without test charge. Hence, the corresponding of the superposition principle is referred as the superposition principle without the test charge.

It should be notice, in this author's electromagnetic field theory, the retarded wave and the advanced wave are all acceptable and hence, this superposition means the superposition with all kind fields include the retarded waves and also the advanced waves.

2.2 Superposition with test charge

In the place of A, B and C, there are charges, hence the self-field are infinite. The self-field is the infinite big,

$$\begin{cases} \boldsymbol{E}_{AA} = \infty \\ \boldsymbol{E}_{BB} = \infty \\ \boldsymbol{E}_{CC} = \infty \end{cases} \tag{14}$$

And hence in any place with a charge, for example A, the field can only be superposed with the fields of the other two charges, for example,

$$\begin{cases} \boldsymbol{E}_A = \boldsymbol{E}_{BA} + \boldsymbol{E}_{CA} \\ \boldsymbol{H}_A = \boldsymbol{H}_{BA} + \boldsymbol{H}_{CA} \end{cases} \tag{15}$$

It is same to the fields of $\xi_B = [\boldsymbol{E}_B, \boldsymbol{H}_B]$, $\xi_C = [\boldsymbol{E}_C, \boldsymbol{H}_C]$ they can also only superposed with two charges. According to the absorber theory of Wheeler and Feynman, they support this kind of superposition. Because, there is a test charge, the field can be defined according to the action. For the superposition without test charge, according the absorber theory[1, 2] that this kind of superposition is not really correct because without test charge we cannot define the action, without action, the field cannot be defined correctly, if field cannot be defined correctly, the superposition can not be done too.

2.3 Superposition all retarded fields or all advanced wave

In subsection 2.1. The superposition include all retarded wave and advanced wave. However some time we need the superposition only with the retarded wave or only with advanced wave. Hence we need to define

$$\xi_r = \sum_{i=1}^{N} \xi_{ri} \tag{16}$$

$$\xi_a = \sum_{j=1}^{M} \xi_{aj} \tag{17}$$

$\xi_r = [\boldsymbol{E}_r, \boldsymbol{H}_r]$ is the retarded wave. $\xi_a = [\boldsymbol{E}_a, \boldsymbol{H}_a]$ is the advanced wave, $\xi_{ri} = [\boldsymbol{E}_{ri}, \boldsymbol{H}_{ri}]$ is the retarded wave of the i-th charge. $\xi_{aj} = [\boldsymbol{E}_{aj}, \boldsymbol{H}_{aj}]$ is the advanced wave of the j-th charge.

2.4 Superposition for the total field of the electromagnetic fields

We know that according to this author electromagnetic field theory [12], there is the advanced wave and the retarded wave. However in our traditional electromagnetic field theory only the retarded field is applied. We should say that, this traditional electromagnetic field theory is also very successful in most situations. However this electromagnetic field actually are superposed field of the advanced field and the retard field, that means,

$$\xi_t = \xi_r + \xi_a \tag{18}$$

where ξ_t is traditional electromagnetic field. ξ_r is the retarded wave, ξ_a is the advanced wave.

If there are two different traditional field, this two fields should be possible to be superposed which means that,

$$\xi_t = \sum_{i=1}^{N} \xi_{ti} \tag{19}$$

ξ_{ti} are transitional field which is composed with the retarded wave and the advanced wave. The superposition principle to this kind of field is certainly different with the superposition principle described in a few last sub-sections. We will discussion this kind of superposition in details later.

2.5 Summarization

When we talk about superposition and superposition principles, we have to be clear that what kind of superposition is involved. We also need to know the differences of these superpositions. This article will discuss all the differences of these superposition and the superposition principles.

3 Conflict between the superposition principle without test charge and the energy conservation law

Wheeler and Feynman has noticed the problem of the definition of the field[1, 2]. Defined a field need a test charge. If field cannot be defined properly how it can be superposed? Hence, according their discussion, the superposition principle without test charge is problematic. This kind of superposition superpose all field of charges include the retarded wave and the advanced wave.

But Wheeler and Feynman did not offer a clear solution to solve the problem. They only ask the reader should avoid that kind of wrong field and hence the wrong superposition (superposition without a test charge). Because according there absorber theory[2], the field has no its own freedom, the electromagnetic fields are only the adjective field of the action-at-a-distance. but action-at-a-distance needs at least two charges, one is emitter to produce the retarded wave, another is used as the test charge to measure the electromagnetic field. Hence to define the electromagnetic field a test charge is needed. If define a electromagnetic field need a test charge. According their discussion, to superpose the electromagnetic field also need a test charge.

In this article this author will try to find the reason of the problem and offer a clear solution to this kind of superposition. Hence this author will assume the superposition without test charge is allowed now. If this assumption lead to a conflict, this author will solve the problem in that time.

3.1 The energy conservation law

Let us assume there are N charges in the empty space. Empty space is a abstract space there are nothing in it, no star no sun and no earth. We first calculate the total work have done by this N charges. This is clear that the work has done by N charge can be expressed as,

$$W = \int_{t=-\infty}^{\infty} \iiint_V \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV dt = 0 \tag{20}$$

where $\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$ is the current of the i-th charge exerted a power to the j-th charge.

The above formula tell us, the total work is done for all charges are 0. This is because if a charge offer some work to other charge, then the other charge's energy will increase, but in the same time the charge itself will lose equal part of the energy. To the total work has been done, somewhere the work increase is canceled by the work decrease at the other place. We will assume the above N-charge energy conservation law is self-explanatory.

3.2 Superposition principle without test charge

Assume that the superposition principle without test charge is correct. In the beginning I will assume this kind of superposition is correct, however I will point out this can lead a conflict to the energy conservation law and hence is not really correct. Substitute Eq.(5,6) to the Maxwell equations Eq.(1, 2, 3 and 4) we obtained the Maxwell equations for superposed field,

$$\nabla \cdot \boldsymbol{D} = \rho \tag{21}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{22}$$

$$\nabla \times \boldsymbol{E} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{23}$$

$$\nabla \times \boldsymbol{H} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{24}$$

where

$$\xi = \sum_{i=1}^{N} \xi_i \tag{25}$$

where$\xi = [\boldsymbol{E}, \boldsymbol{H}]$ and $\xi_i = [\boldsymbol{E}_i, \boldsymbol{H}_i]$. The above formula is the superposition without test charge.

3.3 The Poynting theorem

The Poynting theorem can be derived from the above Maxwell equations Eq.(21-24). Assume the Poynting theorem is correct for the superposed field $\xi = [\boldsymbol{E}, \boldsymbol{H}]$, i.e.,

$$-\iint_S (\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma$$

$$=\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV+\iiint_V(\boldsymbol{E}\cdot\partial\boldsymbol{D}+\boldsymbol{H}\cdot\partial\boldsymbol{B})dV \tag{26}$$

In the above we have written $\frac{\partial}{\partial t}$ as ∂ for simplicity. This simplification will be used in the whole article. Apply the supposition principle without test charge Eq.(5,6) to the above formula we have,

$$-\oint\!\!\!\oint_\Gamma \sum_{i=1}^N\sum_{j=1}^N \boldsymbol{E}_i\times\boldsymbol{H}_j\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^N\sum_{j=1}^N \boldsymbol{E}_i\cdot\boldsymbol{J}_jdV+\iiint_V\sum_{i=1}^N\sum_{j=1}^N(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \tag{27}$$

The above formula is referred as the Poynting theorem of N charges.

3.4 The 3 conditions to prove from the above N charges Poynting theorem to the energy conservation law

Let us analysis, how we can derive the energy conservation law from the above Poynting theorem of N charges to the energy conservation law Eq.(20). There are 3 steps to lead the energy conservation law.

I. Self-energy terms do not contributed to the energy transfer between the charge,

$$-\oint\!\!\!\oint_\Gamma \sum_{i=1}^N \boldsymbol{E}_i\times\boldsymbol{H}_i\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^N \boldsymbol{E}_i\cdot\boldsymbol{J}_idV+\iiint_V\sum_{i=1}^N(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_i)dV \tag{28}$$

Hence, the above formula can be subtracted from the N charge of Poynting theorem Eq.(27), we obtain,

$$-\oint\!\!\!\oint_\Gamma \sum_{i=1}^N\sum_{j=1,j\neq i}^N \boldsymbol{E}_i\times\boldsymbol{H}_j\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^N\sum_{j=1,j\neq i}^N \boldsymbol{E}_i\cdot\boldsymbol{J}_jdV+\iiint_V\sum_{i=1}^N\sum_{j=1,j\neq i}^N(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \tag{29}$$

This formula is referred as mutual energy formula. In the above formula if we can prove that

II.

$$\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma = 0 \tag{30}$$

This means that the surface integral terms are 0.
III.

$$\int_{t=-\infty}^{\infty} dt \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV = 0 \tag{31}$$

This means the space energy terms are 0.
If the 3 conditions are met, we obtain,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV = 0 \tag{32}$$

That will be the energy conservation law. Hence, it is clear the proof of energy conservation law needs the above 3 steps I., II. and III..

We cannot prove steps I, we cannot prove all items in Eq.(28) are 0. However we just assume it is 0 for the time being, see what will happen.

3.5 Two charge situation

The above equation Eq.(29) can be rewritten as

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1,j<i}^{N} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j<i}^{N} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_j \cdot \boldsymbol{J}_i) dV$$

$$+ \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j<i}^{N} [(\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) + (\boldsymbol{E}_j \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_j \cdot \partial \boldsymbol{B}_i)] dV \tag{33}$$

Assume $N = 2$, we obtain,

$$-\oiint_{\Gamma} \sum_{i=1}^{2} \sum_{j=1,j<i}^{2} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{2} \sum_{j=1,j<i}^{2} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_j \cdot \boldsymbol{J}_i) dV$$

$$+ \iiint_{V} \sum_{i=1}^{2} \sum_{j=1,j<i}^{2} [(\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) + (\boldsymbol{E}_j \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_j \cdot \partial \boldsymbol{B}_i)] dV \tag{34}$$

or

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\iiint_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV$$

$$+\iiint_V[(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2)+(\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)]dV \qquad (35)$$

Corresponding differential Formula is,

$$-\nabla\cdot(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)$$

$$=\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1$$

$$+\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1 \qquad (36)$$

When $N=2$, there is only two charges, For example one is the emitter another is the absorber. The emitter sends the retarded wave which is $\xi_1=[\boldsymbol{E}_1,\boldsymbol{H}_1]$. The absorber sends the advanced wave which is $\xi_2=[\boldsymbol{E}_2,\boldsymbol{H}_2]$. Eq.(35, 36) are mutual energy formula for two charges. Now we assume the two charge are also possible tow emitters or two absorbers. Any way there are two charges.

It is easy to discuss the situation there are only two charges. The results can be easily widen to the situation where we have N charges.

4 Welch's reciprocity theorem and the mutual energy theorem

Let us following the proof of Welch's reciprocity[17] to prove the two condition II. and III. We will only do this in two charge situation, the result can be easily widen to N charges.

4.1 The surface integral is 0

Proof for

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=0 \qquad (37)$$

Assume the surface Γ is a sphere with infinite big radius. Assume the fields $\xi_1=[\boldsymbol{E}_1,\boldsymbol{H}_1]$ and $\xi_2=[\boldsymbol{E}_2,\boldsymbol{H}_2]$ are fields of two charges close to the center of the big sphere Γ. Welch assume that the two fields ξ_1 and ξ_2 one is retarded field and another is advanced field. When the retarded wave and the advanced wave reach the sphere, one is in a future time and another is in a past time. Hence the two fields can not nonzero in the same time at the surface of Γ. Hence

$\boldsymbol{E}_1 \times \boldsymbol{H}_2$ and $\boldsymbol{E}_2 \times \boldsymbol{H}_1$ the two products must zero at the surface Γ and hence the surface integral above is zero. The proof finished.

Only when the two fields $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ and $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ one is retarded field and another is advanced field we can have the surface integral as 0. If the two field all the retarded field or all advanced field we cannot prove the surface integral as 0 in general.

4.2 The energy in the empty space are 0

proof

$$\int\limits_{t=-\infty}^{\infty} \iiint\limits_V [(\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1)] dV dt = 0 \quad (38)$$

We have,

$$\begin{aligned} &\int\limits_{t=-\infty}^{\infty} \iiint\limits_V [(\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1)] dV dt \\ &= \int\limits_{t=-\infty}^{\infty} \partial U dt \\ &= U|_{t=-\infty}^{\infty} \\ &= 0 \end{aligned} \quad (39)$$

where

$$\begin{aligned} \partial U &= \iiint\limits_V [(\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1)] dV \\ &= \partial \iiint\limits_V (\epsilon_0 \boldsymbol{E}_1 \cdot \boldsymbol{E}_2 + \mu_0 \boldsymbol{H}_1 \cdot \boldsymbol{H}_2) dV \end{aligned} \quad (40)$$

where $\boldsymbol{D}_2 = \epsilon_0 \boldsymbol{E}_2$, $\boldsymbol{B}_2 = \mu_0 \boldsymbol{H}_2$, where ϵ_0 and μ_0 are permittivity and permeability constant. Space energy is,

$$U = \iiint\limits_V (\epsilon_0 \boldsymbol{E}_1 \cdot \boldsymbol{E}_2 + \mu_0 \boldsymbol{H}_1 \cdot \boldsymbol{H}_2) dV \quad (41)$$

$U(-\infty)$ and $U(\infty)$ are too stable energy stats, they should be equal to each other, and hence, there is $U|_{t=-\infty}^{\infty} = 0$.

4.3 The reciprocity of Welch

Integral with time to Eq.(35) we obtain,

$$-\int_{t=-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt$$

$$+ \int_{t=-\infty}^{\infty} \iiint_{V} [(\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1)] dV dt \quad (42)$$

Substitute the Eq.(37,38) to the mutual energy formula above, we obtained,

$$\int_{t=-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt = 0 \quad (43)$$

or

$$-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt = \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \quad (44)$$

This is the Welch's reciprocity theorem[17].

4.4 The new reciprocity of Rumsey

V.H. Rumsey has introduced the new reciprocity theorem in 1961[14], which is similar to the Welch's reciprocity theorem, but it is in the Fourier domain.

$$-\iiint_{V} \boldsymbol{E}_2(\omega) \cdot \boldsymbol{J}_1(\omega) dV = \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2(\omega) dV \quad (45)$$

In this article, we will use same symbol to the electromagnetic field in the time-domain and in the Fourier domain. The difference can be distinguished by the the two different variable t and ω.

4.5 The mutual energy theorem

The above formula is re-derived by this author in beginning of the 1987. The formula is referred as Mutual energy theorem[10, 19, 18]. The important is that this author realized that this formula is some kind of energy theorem instead of just a reciprocity theorem.

4.6 The cross-correlation reciprocity theorem of de Hoop

de Hoop derived the cross-correlation reciprocity theorem in the end of 1987 based on the Welch's reciprocity reciprocity theorem. The proof of Welch can be widen to the cross-correlation reciprocity theorem of de Hoop.

$$-\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_2(t+\tau)\cdot\boldsymbol{J}_1(t)dVdt=\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t+\tau)dV \tag{46}$$

It is clear that Welch's reciprocity is a spatial situation of de Hoop's reciprocity theorem, in which $\tau=0$. It is also clear that the Rumsey's reciprocity theorem or the mutual energy theorem of this author are Fourier transform of the the de Hoop's reciprocity theorem. Hence these 4 theorems can be seen as one theorem.

This author will call this 4 theorem as the mutual energy theorem, the reason is these theorems are reciprocity theorem no problem. But they are not only a pure reciprocity theorem like the Lorentz reciprocity theorem[3, 4, 13], they are also a energy theorem.

5 The mutual energy flow and mutual energy flow theorem

5.1 The mutual energy flow theorem

Substitute Eq.(39) to the mutual energy formula Eq.(42) we obtain,

$$-\int_{t=-\infty}^{\infty}\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt$$
$$=\int_{t=-\infty}^{\infty}\iiint_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt \tag{47}$$

In this time we do not take the surface Γ at infinite big sphere, hence the surface integral is not zero in general. Consider the volume V is at V_1 in which only the current J_1. See Figure 2

$$-\int_{t=-\infty}^{\infty}\oiint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt$$
$$=\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{E}_2\cdot\boldsymbol{J}_1dVdt \tag{48}$$

similarly we have,

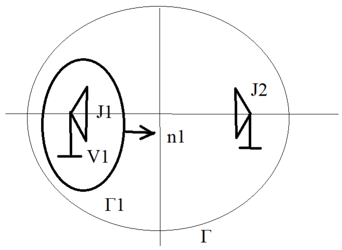

Figure 2: If the V_1 is taken in which only has the current $\boldsymbol{J}_1 \neq 0$.

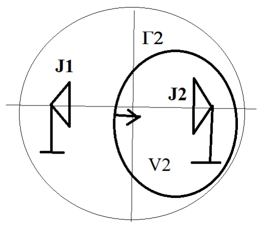

Figure 3: If the V_2 is taken in which only has the current $\boldsymbol{J}_2 \neq 0$.

$$-\int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$
$$= \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dV dt \tag{49}$$

or

$$\int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$
$$= \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dV dt \tag{50}$$

See Figure 3. In equation Eq.(50) we have changed the direction of the normal vector $\hat{n}$ surface integral. Originally the direction $\hat{n}$ is direct to the outside of the the sphere Γ_2. After the change, the direction is from V_1 to V_2 hence, obtained a negative sign.

Substitute Eq.(38,39) to the Welch's reciprocity theorem Eq.(44), we obtain,

$$-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$$

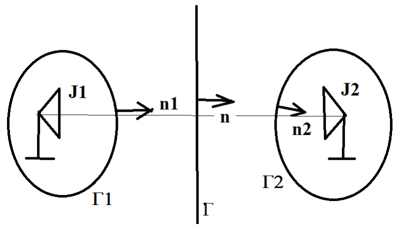

Figure 4: mutual energy flow can be calculate at Γ_1,Γ,Γ_2.

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \qquad (51)$$

See Figure 4. We have add a surface Γ which is a infinite big plane, Actually the surface can be taken in any place between the current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. The surface can be a complete surface like Γ_1 and Γ_2. The surface is also can be a infinite open space like Γ. All the direction of the norm vector of the surfaces are from V_1 to V_2.

It has been proved [10, 11]that

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt \qquad (52)$$

is a inner product, that means,

$$(\xi_1, \xi_2) = (\xi_2, \xi_1) \qquad (53)$$

$$(\xi_1, \gamma\xi_2) = \gamma(\xi_2, \xi_1) \qquad (54)$$

$$(\xi_1, \xi_{21} + \xi_{22}) = (\xi_1, \xi_{21}) + (\xi_1, \xi_{22}) \qquad (55)$$

$$(\xi, \xi) = 0 \Longleftrightarrow \xi = 0 \qquad (56)$$

Hence the mutual energy flow theorem can be re-written as,

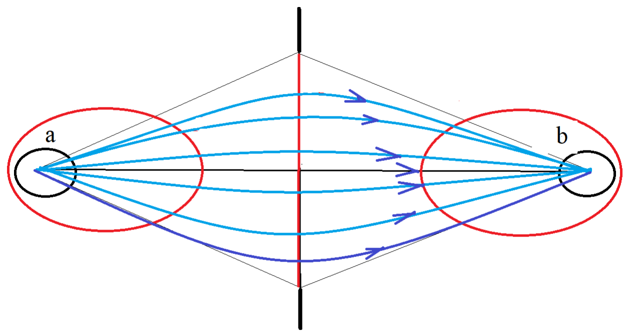

Figure 5: mutual energy flow can be calculate at Γ_1,Γ,Γ_2.

$$-\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt$$

$$=(\xi_1,\xi_2)$$

$$=\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dV \tag{57}$$

In the above formula, the inner product (ξ_1,ξ_2) can be taken in any surface between $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$.

We can assume that $\boldsymbol{J}_1$ is the source which sends the retarded field. $\boldsymbol{J}_2$ is a sink or absorber which sends the advance wave. $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ one must send retarded wave and another must sends the advanced wave, these are requirement of Eq.(37). We can see the mutual energy flow in Figure 5.

5.2 Orthogonality with same waves

We can see, the retarded wave sends from the emitter to the absorber can produce the mutual energy flow. What about the same wave for example the two retarded waves or the two advanced waves send from the two current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$.

We assume the current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ are not too close to each other, we assume the currents are all very short signals. Assume the two currents all send the retarded waves. Since in this situation, the two waves cannot be synchronized, That means the field cannot no zero in the same time at all the surface of Γs, The Γ is the any surface between $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$.

For example assume the retarded wave of ξ_1 which is sent by current $\boldsymbol{J}_1$ reached the $\boldsymbol{J}_2$ at the time t, in this time $\boldsymbol{J}_2$ send a retarded wave ξ_2. The two wave can only no zero in the same time at the time t and at the place of the $\boldsymbol{J}_2$, after time t or before time t the two field ξ_1 and ξ_2 are not nonzero in the same time in and surface Γ, hence we have,

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt = 0 \tag{58}$$

This can be referred as second mutual energy flow theorem, that means the two retarded waves are orthogonal in the surface Γ where Γ is any surface between the currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. It should be noticed that if there is a surface the above integral is 0, there will no any energy can be sent from the emitter to the the another emitter. In the above we have proved there is only one surface the above integral is possible not zero other surface are all zero. And hence, in this situation there is no any energy can be send an emitter to another emitter. This is also true for the two advanced waves.

If the mutual energy flow between two retarded wave are zero, that is clear we have,

$$-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$$

$$= (\xi_1, \xi_2) = 0$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \tag{59}$$

That means a retarded wave cannot send any energy to another emitter. Similarly, an advanced wave cannot send any energy to an absorber.

Please notice, a current actually send the retarded wave and advance wave in the same time. They send half retarded potential and half advanced potential[1, 2]. However we can think that the currents are two currents one is a emitter which sends out the retarded wave, the another is a absorber which send the advanced wave.

The sun can been seen to have many emitters, the black clothes can be seen to have many absorbers. Actually the sun can receive energy also and has lot of absorbers in it. The black clothes have also many emitters inside which can send retarded waves, however, we can see the sun as all emitters for simplification. We can see that the black clothes are only absorbers for simplification.

It should be noticed that in the later section when the self-energy principle is introduced, there is the time-reversal waves. Hence if the charge with current send half retarded wave and half advance wave, if the advanced wave cannot meet another retarded wave from another charge, this advanced wave will be canceled by its corresponding time-reversal wave, and hence, no any physical effect to the space and hence, can be omit. In this situation this charge is a pure emitter. It only sends out the retarded wave.

6 The mutual energy principle

6.1 A clear look at the proof of Welch's reciprocity theorem

If we have a clear look to the proof of Welch's reciprocity theorem we can found it is can be started from the Poynting theorem

$$-\iint_S (\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma$$

$$=\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV+\iiint_V(\boldsymbol{E}\cdot\partial\boldsymbol{D}+\boldsymbol{H}\cdot\partial\boldsymbol{B})dV \tag{60}$$

and also for the two charges we have

$$-\iint_S (\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma$$

$$=\iiint_V \boldsymbol{E}_i\cdot\boldsymbol{J}_i dV+\iiint_V(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_i)dV \qquad i=1,2 \tag{61}$$

Assume the superposition principle without test charge is correct, that means

$$\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{E}_2 \tag{62}$$

$$\boldsymbol{H}=\boldsymbol{H}_1+\boldsymbol{H}_2 \tag{63}$$

Substitute Eq.(62,63) to the Eq.(60) we have,

$$-\iint_\Gamma \sum_{i=1}^{2}\sum_{j=1}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^{2}\sum_{j=1}^{2}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dV+\iiint_V\sum_{i=1}^{2}\sum_{j=1}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \tag{64}$$

Subtracting Eq.(61) from the above formula we obtain,

$$-\iint_\Gamma \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dV+\iiint_V\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \tag{65}$$

or

$$-\iint\limits_{\Gamma}\sum_{i=1}^{2}\sum_{j=1,j<i}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_j+\boldsymbol{E}_j\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma$$

$$=\iiint\limits_{V}\sum_{i=1}^{2}\sum_{j=1,j<i}^{2}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\boldsymbol{E}_j\cdot\boldsymbol{J}_i)dV$$

$$+\iiint\limits_{V}\sum_{i=1}^{2}\sum_{j=1,j<i}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j+\boldsymbol{E}_j\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_j\cdot\partial\boldsymbol{B}_i)dV \tag{66}$$

or

$$-\oint\!\!\!\oint\limits_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\iiint\limits_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV$$

$$+\iiint\limits_{V}[(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2)+(\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)]dV \tag{67}$$

This is the mutual energy formula of 2 charges. After this, similar to last section we can obtained Welch's reciprocity theorem. Actually the Welch's reciprocity theorem is largely this way to prove. The only thing different is I started from Poynting theorem, Welch started from the Maxwell equations directly.

We can see in the proof there is a step to prove Welch's reciprocity theorem which is the subtraction. From Eq.(64) subtract Eq.(61), this has no problem. That is because we just need to prove a reciprocity theorem. The reciprocity is only formula in the a electromagnetic field theory. However if we need to proof the Welch's reciprocity theorem is energy conservation law or energy theorem that is not enough.

You cannot subtract Eq.(61) unless all self-energy terms in Eq.(61) are zero, that means, we need

$$-\iint\limits_{S}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma=0 \tag{68}$$

$$\iiint\limits_{V}\boldsymbol{E}_i\cdot\boldsymbol{J}_idV=0 \tag{69}$$

$$\iiint\limits_{V}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_i)dV=0 \tag{70}$$

where,

$$i=1,2 \tag{71}$$

if the above terms are not zero, there is always some energy is lost in the empty space, and hence energy is not conserved. In case there is N charges, the corresponding formula is Eq.(28). Hence if we need to prove the energy conservation law of Eq.(20), all term in Eq(28) must be as zero.

6.2 The mutual energy principle

In order to obtained the formula Eq.(68,69,70), the author assume the problem is at the Maxwell equations. If the Maxwell equations has the problem, we can give up the Maxwell equations, in that case, the Poynting theorem can be give up too. In this case, we do not need the Eq.(61) to be satisfied, and hence do not need each terms in Eq.(61) all zero. In this way perhaps we can obtained the energy conservation law of Welch's reciprocity Eq.(59).

In case we thought the Maxwell equations have problem, there need something else can replace Maxwell equation as axioms of the electromagnetic field theory. The author thought the mutual energy formula Eq.(67) for $N = 2$ situation and the formula Eq.(29) for N is very large situation are a good candidate. This formula is referred as the mutual energy principle. The mutual energy principle now can be applied as a axiom of the electromagnetic field theory.

6.3 Starting from the mutual energy principle

We rewrite the formula Eq.(67),

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$

$$+ \iiint_V \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{72}$$

and

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV$$

$$+ \iiint_V [(\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1)] dV \tag{73}$$

Now they are axioms, both for $N > 2$ and $N = 2$ situations. We started from $N = 2$ situation, the differential formula of Eq.(73) can be written as,

$$-\nabla \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1)$$
$$= (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1)$$
$$+ (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2) + (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) \tag{74}$$

considering the vector differential formula

$$\nabla \cdot \boldsymbol{E}_1 \times \boldsymbol{H}_2 = \nabla \times \boldsymbol{E}_1 \cdot \boldsymbol{H}_2 - \boldsymbol{E}_1 \cdot \nabla \times \boldsymbol{H}_2 \tag{75}$$

$$\nabla \cdot \boldsymbol{E}_2 \times \boldsymbol{H}_1 = \nabla \times \boldsymbol{E}_2 \cdot \boldsymbol{H}_1 - \boldsymbol{E}_2 \cdot \nabla \times \boldsymbol{H}_1 \tag{76}$$

we have

$$-(\nabla \times \boldsymbol{E}_1 \cdot \boldsymbol{H}_2 - \boldsymbol{E}_1 \cdot \nabla \times \boldsymbol{H}_2 + \nabla \times \boldsymbol{E}_2 \cdot \boldsymbol{H}_1 - \boldsymbol{E}_2 \cdot \nabla \times \boldsymbol{H}_1)$$
$$= \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1$$
$$+ \boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1 \tag{77}$$

or

$$\boldsymbol{E}_1 \cdot (\nabla \times \boldsymbol{H}_2 - \boldsymbol{J}_2 - \partial \boldsymbol{D}_2) + \boldsymbol{E}_2 \cdot (\nabla \times \boldsymbol{H}_1 - \boldsymbol{J}_1 - \partial \boldsymbol{D}_1)$$

$$+ \boldsymbol{H}_1 \cdot (-\nabla \times \boldsymbol{E}_2 - \partial \boldsymbol{B}_2) + \boldsymbol{H}_2 \cdot (-\nabla \times \boldsymbol{E}_1 - \partial \boldsymbol{B}_2) = 0 \tag{78}$$

Considering the $\boldsymbol{E}_1 \neq 0$, $\boldsymbol{E}_2 \neq 0$ and $\boldsymbol{H}_1 \neq 0$, $\boldsymbol{H}_2 \neq 0$ we obtain,

$$\nabla \times \boldsymbol{E}_1 = -\partial \boldsymbol{B}_2 \tag{79}$$

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \partial \boldsymbol{D}_1 \tag{80}$$

$$\nabla \times \boldsymbol{E}_2 = -\partial \boldsymbol{B}_2 \tag{81}$$

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \partial \boldsymbol{D}_2 \tag{82}$$

We obtains two group Maxwell equations.

6.4 The two wave must synchronized

Now let us to see whether we can obtained only one Maxwell equations from the mutual energy principle. Assume

$$\boldsymbol{E}_1 \neq 0, \boldsymbol{H}_1 \neq 0, \boldsymbol{J}_1 \neq 0 \tag{83}$$

$$\boldsymbol{E}_2 = 0, \boldsymbol{H}_2 = 0, \boldsymbol{J}_2 = 0 \tag{84}$$

Substitute the above to Eq.(78) we obtain,

$$\nabla \times \boldsymbol{H}_1 - \boldsymbol{J}_1 - \partial \boldsymbol{D}_1 < \infty \tag{85}$$

$$-\nabla \times \boldsymbol{E}_1 - \partial \boldsymbol{B}_2 < \infty \tag{86}$$

That means

$$\boldsymbol{E}_1 < \infty, \boldsymbol{H}_1 < \infty \tag{87}$$

This is not a reasonable electromagnetic field. It is same if we assume

$$\boldsymbol{E}_2 \neq 0, \boldsymbol{H}_2 \neq 0, \boldsymbol{J}_2 \neq 0 \tag{88}$$

$$\boldsymbol{E}_1 = 0, \boldsymbol{H}_1 = 0, \boldsymbol{J}_1 = 0 \tag{89}$$

we will obtained,

$$\boldsymbol{E}_2 < \infty, \boldsymbol{H}_2 < \infty \tag{90}$$

This is also not a reasonable electromagnetic field. These tell us, the mutual energy principle Eq.(78) or Eq.(74) do not allow a solution there is only one electromagnetic field. That means that, the two electromagnetic fields must exist together. That also means the two electromagnetic field which satisfy the two Maxwell equations Eq.(79-82) must be synchronized.

In the subsection 4.1 we have shown that in order to make the surface integral vanish, see Eq.(37) the two electromagnetic fields must one is retarded field and another is advanced field. Eq.(37) is established is a condition for Welch's theorem Eq.(44) or the mutual energy theorem Eq.(45). This means that the synchronized two electromagnetic field needs to be one is the retarded wave and another needs to be advanced wave. This way we can prove the mutual energy theorem or Welch's reciprocity theorem from the mutual energy principle. Now the mutual energy theorem or Welch's reciprocity theorem are energy conservation law. Hence, we have proved the energy conservation law from the mutual energy principle.

6.5 Wave collapse

The author has assumed the mutual energy principle is the axiom and give up the Maxwell equations as axioms. However from the mutual energy principle we obtained the Maxwell equations again. This means that Maxwell equations can be give up as axiom, but even we started from new axiom which is the mutual energy principle, the Maxwell equations are still established. The Maxwell equations can be seen as theorems now.

Maxwell equations are established, that means the Poynting theorem also need to be established. Hence, Eq.(61) is also established. And the terms in the Eq.(61) cannot be zero. That means we cannot obtained Eq.(68-70). This means that we still cannot prove the energy conservation law Eq.(44) or Eq.(20) for $N = 2$ and $N > 2$ situations.

We often heard that wave can be collapsed. In case Eq.(68-70) cannot be satisfied, we begin to think perhaps wave collapsed, that means the retarded wave send from the emitter will collapsed to the absorber. The advanced wave sends from the absorber will collapsed to the emitter.

Wave collapse cannot be accept, because if wave is collapsed, the self-energy terms will contribute to additional energy terms to the Welch's reciprocity theorem Eq.(44) or the mutual energy theorem Eq.(45). Widen from the charges $N=2$ to $N>2$ we cannot obtain the energy conservation law Eq.(20).

Hence wave collapse cannot be accept, this author thought another possibility is the wave is collapse back, that means the wave is not collapse to its target but its source. That means the retarded wave sends out from the emitter will collapse back to the emitter. The advanced wave sends out from the absorber will collapse beck to the absorber.

6.6 Generalized mutual energy theorem or energy conservation law

We have proved the mutual energy theorem for N=2, situation. This is easy to generalized to where $N>2$ situation. Generalized mutual energy theorem is,

$$W=\int_{t=-\infty}^{\infty}\iiint_V \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dVdt=0 \tag{91}$$

proof: We have the mutual energy principle,

$$-\oint\!\!\oint_\Gamma \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\boldsymbol{E}_i\times\boldsymbol{H}_j\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dV$$

$$+\iiint_V \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \tag{92}$$

or

$$-\oint\!\!\oint_\Gamma \sum_{i=1}^{N}\sum_{j=1,j<i}^{N}(\boldsymbol{E}_i\times\boldsymbol{H}_j+\boldsymbol{E}_j\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma$$

$$=\iiint_V \sum_{i=1}^{N}\sum_{j=1,j<i}^{N}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\boldsymbol{E}_j\cdot\boldsymbol{J}_i)dV$$

$$+\iiint_V \sum_{i=1}^{N}\sum_{j=1,j<i}^{N}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j+\boldsymbol{E}_j\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_j\cdot\partial\boldsymbol{B}_i)dV \tag{93}$$

Considering,

$$\int\limits_{t=-\infty}^{\infty} dt \iiint\limits_{V} \sum_{i=1}^{N} \sum_{j=1,j<i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j + \boldsymbol{E}_j \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_j \cdot \partial \boldsymbol{B}_i) dV$$

$$= \int\limits_{t=-\infty}^{\infty} \partial U dt U|_{-\infty}^{\infty} = U(\infty) - U(-\infty) = 0 \tag{94}$$

where

$$U = \boldsymbol{E}_i \cdot \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \boldsymbol{B}_j \tag{95}$$

Now let us to prove,

$$\oiint\limits_{\Gamma} \sum_{i=1}^{N} \sum_{j=1,j<i}^{N} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{96}$$

We need to prove if

$$\oiint\limits_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{97}$$

for any given group i, j. Assume$\xi_i = [\boldsymbol{E}_i, \boldsymbol{H}_i]$, $\xi_j = [\boldsymbol{E}_j, \boldsymbol{H}_j]$ has 3 situations, both are retarded wave, bother are advanced wave, one is retarded wave, one is an advanced wave. We have proved that if one is retarded wave and one is advanced wave the above formula is 0. Now let us to prove if the two fields both are retarded field the above formula is also 0.

Assume $\xi_i = [\boldsymbol{E}_i, \boldsymbol{H}_i]$ and $\xi_j = [\boldsymbol{E}_j, \boldsymbol{H}_j]$ are all retarded field. Let use assume that $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ and $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ are all retarded field, we have the mutual energy principle of two charge which is,

$$-\int\limits_{t=-\infty}^{\infty} \iiint\limits_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$$

$$= (\xi_1, \xi_2) = 0$$

$$= \int\limits_{t=-\infty}^{\infty} \iiint\limits_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \tag{98}$$

This means,

$$\int\limits_{t=-\infty}^{\infty} \iiint\limits_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt = 0 \tag{99}$$

$$\int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt = 0 \tag{100}$$

The mutual energy principle tell us

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV$$

$$+ \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV \tag{101}$$

we have,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV$$

$$= \int_{t=-\infty}^{\infty} \partial U dt U|_{-\infty}^{\infty} = U(\infty) - U(-\infty) = 0 \tag{102}$$

where

$$U = \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \boldsymbol{B}_2) dV \tag{103}$$

considering

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV \tag{104}$$

Considering Eq.(99,100), the right side of the Equation is 0. Hence, the left site of the above formula is also 0. That means if any two pair retarded fields the surface integral

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{105}$$

The same reason we can obtained that if the two fields are all advanced field we have also above formula. The two fields only can have 3 situations, the are all retarded field, the two are all advanced field and the two are one is retarded and one is advanced, in the 3 situation we all have,

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{106}$$

or

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{107}$$

Substituting Eq.(107, 94) to Eq.(92) we obtain,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV = 0 \tag{108}$$

We obtain the energy conservation law.

7 Discussion the conflict again started from mutual energy principle

From the above discussion we have known that if we started from Poynting theorem and the superposition without test charge we cannot derive the energy conservation law. However if we add the self-energy principle. The conflict can be eliminate and hence the superposition without test charge is allowed. This lead us to introduce the self-energy principle. In this section we started from mutual energy principle but do not assume the self-energy principle, let us seen what situation will happen.

7.1 Assume mutual energy principle is assumed

In this section we assume the mutual energy principle is established. But we do not assume the self-energy principle.

From above discussion we have know that if the mutual energy principle is established, for example we have,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$
$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$
$$+ \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{109}$$

For N=2, we have

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV$$

$$+ \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV \quad (110)$$

$$\nabla \times \boldsymbol{E}_1 = -\partial \boldsymbol{B}_2 \quad (111)$$

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \partial \boldsymbol{D}_1 \quad (112)$$

$$\nabla \times \boldsymbol{E}_2 = -\partial \boldsymbol{B}_2 \quad (113)$$

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \partial \boldsymbol{D}_2 \quad (114)$$

This two group Maxwell equation must synchronized. This two group Maxwell equation lead to two Poynting theorem,

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \iiint_{V} \boldsymbol{E}_1 \cdot \boldsymbol{J}_1 dV + \iiint_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_1) dV \quad (115)$$

$$-\oiint_{\Gamma} (\boldsymbol{E}_2 \times \boldsymbol{H}_2) \cdot \hat{n} d\Gamma = \iiint_{V} \boldsymbol{E}_2 \cdot \boldsymbol{J}_2 dV + \iiint_{V} (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_2) dV \quad (116)$$

or

$$-\oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \iiint_{V} \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \iiint_{V} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV \quad (117)$$

In general we have this also means,

$$-\sum_{i=1}^{N} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \sum_{i=1}^{N} \iiint_{V} \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \sum_{i=1}^{N} \iiint_{V} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV \quad (118)$$

Add the above formula to Eq.(109) we have,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint\limits_V \sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$

$$+ \iiint\limits_V \sum_{i=1}^{N}\sum_{j=1}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{119}$$

This formula is obtained from the mutual energy principle Eq.(109), hence, it should be equal to Eq.(109). This will mean their difference will be 0, i.e., all terms of following

$$-\sum_{i=1}^{N} \oiint\limits_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \sum_{i=1}^{N} \iiint\limits_V \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \sum_{i=1}^{N} \iiint\limits_V (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV \tag{120}$$

Will be 0. That is,

$$- \oiint\limits_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \iiint\limits_V \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \iiint\limits_V (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV \tag{121}$$

will be 0. Or,

$$- \oiint\limits_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{122}$$

$$\iiint\limits_V \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV = 0 \tag{123}$$

$$\iiint\limits_V (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV = 0 \tag{124}$$

However the above cannot be 0, because if it is 0, then

$$\xi_i = [\boldsymbol{E}_i, \boldsymbol{H}_i] = 0 \tag{125}$$

This means all electromagnetic field will be 0. This is clear wrong. All electromagnetic field cannot be 0. This conflict will lead us to introduce the self-energy principle in the following section.

8 Self-energy principle

8.1 Time-reversal transform

According the end of last section, this author assume the wave collapse back, that means the wave should satisfy time-reverse equations. That means there is a new kind of electromagnetic field $[\boldsymbol{e}, \boldsymbol{h}, \boldsymbol{j}]$ which describes the collapse back process. The time-reversal transform, should satisfy,

$$\mathrm{x} \longrightarrow \mathrm{x} \tag{126}$$

$$t \longrightarrow -t \tag{127}$$

$$\frac{\partial}{\partial t} \longrightarrow \frac{\partial}{\partial(-t)} = -\frac{\partial}{\partial t} \tag{128}$$

$$\boldsymbol{v} = \frac{d\boldsymbol{x}}{dt} \longrightarrow \frac{d\boldsymbol{x}}{d(-t)} = -\frac{d\boldsymbol{x}}{dt} = -\boldsymbol{v} \tag{129}$$

$$\boldsymbol{E} \longrightarrow \boldsymbol{e} \tag{130}$$

$$\boldsymbol{H} \longrightarrow \boldsymbol{h} \tag{131}$$

$$\boldsymbol{J} \longrightarrow -\boldsymbol{j} \tag{132}$$

Apply this to Maxwell equations,

$$\begin{cases} \nabla \times \boldsymbol{E} = -\partial \boldsymbol{B} & \longrightarrow & \nabla \times \boldsymbol{e} = \partial \boldsymbol{b} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D} & \longrightarrow & \nabla \times h = -\boldsymbol{j} - \partial \boldsymbol{d} \end{cases} \tag{133}$$

In Eq.(133) offers the equations the time-reversal electromagnetic field should satisfy. This equations is referred as time-reversal Maxwell equations. According to this equations, the Poynting theorem can also have the time-reversal Poynting theorem,

$$\begin{cases} -\iint_S (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV + \iiint_V (\boldsymbol{E} \cdot \partial \boldsymbol{D} + \boldsymbol{H} \cdot \partial \boldsymbol{B}) dV \\ \\ \iint_S (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = \iiint_V \boldsymbol{e} \cdot \boldsymbol{j} dV + \iiint_V (\boldsymbol{e} \cdot \partial \boldsymbol{d} + \boldsymbol{h} \cdot \partial \boldsymbol{b}) dV \end{cases} \longrightarrow \tag{134}$$

When we have the time-reversal waves, even equation Eq.(68-70) cannot be satisfy but we can have,

$$-\iint_S (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma + \iint_S (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = 0 \tag{135}$$

$$\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \iiint_V \boldsymbol{e} \cdot \boldsymbol{j} dV = 0 \tag{136}$$

$$\iiint_V (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV + \iiint_V (\boldsymbol{e} \cdot \partial \boldsymbol{d} + \boldsymbol{h} \cdot \partial \boldsymbol{b}) dV = 0 \tag{137}$$

This means that the corresponding energy terms of the time reversal field just canceled the corresponding energy terms of the normal electromagnetic fields.

Eq.(135-137) tell us also the self-energy terms do not transfer energy.

8.2 Self-energy principle

Since now we have the new electromagnetic field which is time reversal electromagnetic field, the original electromagnetic field will referred as normal electromagnetic field. Here the normal fields satisfy the Maxwell equations. The time-reversal field satisfy the time-reversal Maxwell equations Eq.(133).

There exist time reversal fields which can cancel the corresponding electromagnetic field is referred as self-energy principle. This is the starting point of the author's new electromagnetic field theory. It will be applied as axiom. According to this new principle, there are 4 kinds of waves: the retarded wave, the advanced wave, the time-reversal wave corresponding to the retarded wave, the time-reversal wave corresponding to the advanced wave. The retarded wave and the advanced wave are normal electromagnetic fields. The time reversal electromagnetic fields are unnormal electromagnetic fields.

8.3 The mutual energy principle for time-reversal wave

We can apply the time-reversal transform to the mutual energy principle and obtain the mutual energy principle for time-reversal waves, which are

$$\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{e}_i \times \boldsymbol{h}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{e}_i \cdot \boldsymbol{j}_j dV + \iiint_V \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\boldsymbol{e}_i \cdot \partial \boldsymbol{d}_j + \boldsymbol{h}_i \cdot \partial \boldsymbol{b}_j) dV \tag{138}$$

for $N > 2$ and

$$\oiint_{\Gamma} (\boldsymbol{e}_1 \times \boldsymbol{h}_2 + \boldsymbol{e}_2 \times \boldsymbol{h}_1) \cdot \hat{n} d\Gamma$$

$$= \iiint_V (\boldsymbol{e}_1 \cdot \boldsymbol{j}_2 + \boldsymbol{e}_2 \cdot \boldsymbol{j}_1) dV$$

$$+ \iiint_V [(\boldsymbol{e}_1 \cdot \partial \boldsymbol{d}_2 + \boldsymbol{h}_1 \cdot \partial \boldsymbol{b}_2) + (\boldsymbol{e}_2 \cdot \partial \boldsymbol{d}_1 + \boldsymbol{h}_2 \cdot \partial \boldsymbol{b}_1)] dV \tag{139}$$

For the time-reversal wave we also do not apply the time-reversal Maxwell equation as the axioms, but apply above time-reversal mutual energy principle as the axioms. Started from the above time-reversal mutual energy principle we can obtained the time-reversal Maxwell equations, similar to Eq.(79-82)

$$\nabla \times \boldsymbol{e}_1 = \partial \boldsymbol{b}_2 \tag{140}$$

$$\nabla \times \boldsymbol{h}_1 = -\boldsymbol{j}_1 - \partial \boldsymbol{d}_1 \tag{141}$$

$$\nabla \times \boldsymbol{e}_2 = \partial \boldsymbol{b}_2 \tag{142}$$

$$\nabla \times \boldsymbol{h}_2 = -\boldsymbol{j}_2 - \partial \boldsymbol{d}_2 \tag{143}$$

Similar to the mutual energy principle, the time reversal mutual energy principle can prove a mutual energy theorem for time reversal mutual energy flow. This author believe the energy of the photon is transferred by the mutual energy flow, hence, the photon is the mutual energy flow. This author also believe that the time reversal mutual energy flow can eliminate the half photon. In case two advanced waves from two absorbers race a retarded wave sent from a emitter, each absorber is possible to obtain only a energy of a half photon. In this case the charge in the absorber spring from low energy level to a high level, but since there is only a half energy, it is not enough. In this case, the charge will return the lower energy level, and hence the time reversal mutual energy will bring the energy of the half photon back to the emitter.

8.4 The not synchronized electromagnetic field is also allowed

In the subsection 6.4, we have said the two wave must synchronized. After we have the time-reversal wave, now we can allow that the wave only satisfy one Maxwell equations for example the retarded wave. In this situation, since there is no any mutual energy produced this retarded wave will send their energy to infinite far away in the space, however, thanks the exist of the time-reversal wave, which will cancel the energy of this wave. Hence, there is no any energy will be lost in the empty space. The energy sends from the emitter will return to the emitter. The energy or negative energy sends out from absorber will received by the absorber. The absorber sends out the advanced wave which have the energy with negative sign. The absorber actually receive energy not sends the energy out.

This kind waves is ghost wave, because it is sent out, but there is the time reversal wave which collapse back and hence the wave do not carry any energy. We can also call this wave as probability wave, the probability wave carry some probability but not energy. The probability is that this wave has the chance become a real thing, in that case this retarded wave has met an advanced wave and hence, two waves become synchronized. In that case the mutual energy flow will be built. And a photon is produced. The energy is sent form the emitter to the absorber.

8.5 Summary

Since now we have the self-energy principle Eq.(135-137), the self-energy terms Eq.(28) do not transfer energy, this will guarantee the mutual energy principle is established and also the energy conservation law Eq.(20) can be established. For this article, one of the purpose is the discussion of the superposition principle. Now we have proved that in order to obtain the superposition principle without test charge, the self-energy principle must be accept. In the next section we will begin discussion the superposition for details.

9 The superposition principle

9.1 Derive superposition principle form the mutual energy principle

This author will not apply the superposition principle as an axiom. The superposition principle will be derived from the mutual energy principle.

Assume there are charges N_1 which send the retarded wave and charge N_2 which send advanced wave. Assume the N_1 charge is inside the volume V_1 and the charge N_2 are at inside of volume V_2. We know that $N = N_1 + N_2$ for the N charges, $V_1 \subset V$, $V_2 \subset V$, $V_1 \cap V_2 = 0$. There is the mutual energy principle Eq.(29) which is rewritten as,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$

$$+ \iiint_{V} \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{144}$$

Consider the retarded wave is orthogonal to the retarded waves, and the advanced wave is orthogonal to the advanced waves, i.e.,

$$0 =$$

$$-\oiint_{\Gamma} \sum_{i=1}^{N_1} \sum_{j=1,j\neq i}^{N_1} \boldsymbol{E}_{1i} \times \boldsymbol{H}_{1j} \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N_1} \sum_{j=1,j\neq i}^{N_1} \boldsymbol{E}_{1i} \cdot \boldsymbol{J}_{1j} dV$$

$$+ \iiint_{V} \sum_{i=1}^{N_1} \sum_{j=1,j\neq i}^{N_1} (\boldsymbol{E}_{1i} \cdot \partial \boldsymbol{D}_{1j} + \boldsymbol{H}_{1i} \cdot \partial \boldsymbol{B}_{1j}) dV \tag{145}$$

$$0 =$$

$$-\oiint_{\Gamma} \sum_{i=1}^{N_2} \sum_{j=1,j\neq i}^{N_2} \boldsymbol{E}_{2i} \times \boldsymbol{H}_{2j} \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N_2} \sum_{j=1,j\neq i}^{N_2} \boldsymbol{E}_{2i} \cdot \boldsymbol{J}_{2j} dV$$

$$+ \iiint_V \sum_{i=1}^{N_2} \sum_{j=1,j\neq i}^{N_2} (\boldsymbol{E}_{2i} \cdot \partial \boldsymbol{D}_{2j} + \boldsymbol{H}_{2i} \cdot \partial \boldsymbol{B}_{2j}) dV \tag{146}$$

The above formula is because the second mutual energy flow theorem Eq.(59). Hence, only the term of retarded wave with the advanced wave need to be considered. Substitute Eq.(145-146) to Eq.(144), hence, we have,

$$- \iint_\Gamma \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (\boldsymbol{E}_{1i} \times \boldsymbol{H}_{2j} + \boldsymbol{E}_{2j} \times \boldsymbol{H}_{1i}) \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (\boldsymbol{E}_{1i} \cdot \boldsymbol{J}_{2j} + \boldsymbol{E}_{2j} \cdot \boldsymbol{J}_{1i}) dV$$

$$+ \iiint_V \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (\boldsymbol{E}_{1i} \cdot \partial \boldsymbol{D}_{2j} + \boldsymbol{H}_{1i} \cdot \partial \boldsymbol{B}_{2j} + \boldsymbol{E}_{2j} \cdot \partial \boldsymbol{D}_{1i} + \boldsymbol{H}_{2j} \cdot \partial \boldsymbol{B}_{1i}) dV \tag{147}$$

Writing,

$$\boldsymbol{E}_1 = \sum_{i=1}^{N_1} \boldsymbol{E}_{1i}, \ \boldsymbol{H}_1 = \sum_{i=1}^{N_1} \boldsymbol{H}_{1i} \tag{148}$$

$$\boldsymbol{E}_2 = \sum_{j=1}^{N_2} \boldsymbol{E}_{1j}, \ \boldsymbol{H}_2 = \sum_{j=1}^{N_2} \boldsymbol{H}_{2j} \tag{149}$$

We have,

$$- \iint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV$$

$$+ \iiint_V (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV \tag{150}$$

We get the mutual energy principle between two region, V_1 and V_2. We have assume in volume V_1 all charges are emitters. In the volume V_2 all charges are absorbers.

Since $\boldsymbol{E}_1$ is retarded wave, $\boldsymbol{H}_2$ is advanced wave, $\boldsymbol{E}_1 \times \boldsymbol{H}_2$ cannot nonzero in the same time at Γ. This is also true to $\boldsymbol{E}_2 \times \boldsymbol{H}_1$ hence, we have,

$$\iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{151}$$

We also have,

$$\int_{t=-\infty}^{\infty} \iiint_V (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV dt$$

$$= \int_{t=-\infty}^{\infty} dU = U(\infty) - U(-\infty) = 0 \tag{152}$$

where

$$U = \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \boldsymbol{B}_2) dV \tag{153}$$

This further lead,

$$\int_{t=-\infty}^{\infty} \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt = 0 \tag{154}$$

or

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} (\boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t)) dV dt = \int_{t=-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t)) dV dt \tag{155}$$

This is the mutual energy theorem, it is also the Welech's reciprocity theorem. It also the energy conservation law for two systems $\zeta_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1, \boldsymbol{J}_1]$, $\zeta_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2]$. The two system satisfy the superposition formula Eq.(148, 149). From the above energy conservation law, this kind of superposition is the superposition with test charge. The test charge for the retarded wave is the absorber in the region V_2. The test charge for the advanced wave is the emitter in the region V_1. The retarded wave is only need to be calculated at the region V_2 and the advance wave is only need to be calculated at the region V_1. Hence for the above energy conservation law, actually we only need the superposition with test charges. We do not need the superposition without test charge.

This means we obtained the superposition principle from the mutual energy principle, but it is the superposition with test charge. The superposition with test charge is enough for the mutual energy theorem (energy conservation law). Since this kind of superposition can be derived from the mutual energy principle, hence, the superposition principle do not need to be as an axiom in the electromagnetic field theory of this author.

The above result can be further widened. Eq.(148,149) tell us, in the region V_1 all the retarded field can be superposed. In the region V_2 all the advance wave can be superposed. In this formula, we can see that the field, $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$,

$\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ can be defined on whole space. It is not defined only on the test charge. Even there is no the test charges. Actually in this situation, we have superposed all the field ether advanced waves or the retarded waves. Hence, seems that the all the retarded wave can be superposed. All the advanced wave can be superposed. The test charges do not need.

9.2 Further discussion for the superposition

Last subsection tell us, if started from the mutual energy principle we can derive the superposition principle with test charges. This kind of superposition can be widened to the situation in which all the fields are retarded or all the field are advanced. All retarded waves can be measured with an absorber hence they can be superposed. All advanced waves can be measured with an emitter, they also can be superposed. Hence we do not need a real test charge here to define the superposition. The only thing is this kind of field are same kind of field. Hence, if there is a charge, it can be measured.

What about the superposition without any restriction, that is the superposition without any test charge, the field can be retarded and also can be advanced? In this situation, From section 3 to section 7 we have proved, that if the self-energy principle is established, which means the formula Eq.(135-137) is established, we can obtain the mutual energy principle, and hence, we can obtained the same superposition as the last subsection. However if the formula Eq.(135-137) is not established, in general we get conflict between the superposition principle (superposition without test charge and the superposition with different waves the retarded and the advanced) and the energy conservation law.

Hence, if we believe the superimposition principle without test charge, include retarded wave and advanced wave, we have to accept the self-energy principle. That means the time-reversal waves mast exist.

In other hand, if we have accepted the self-energy principle, the superposition without any restriction also can be accept. Hence we have proved all 4 kinds of superposition which are all allowed. This permission of all kinds of superposition is not automatic, it is only true when we have modified the electromagnetic field theory, especially accepted the self-energy principle.

Perhaps now we can understand why the Wheeler and Feynman speak about the electromagnetic field has no its own freedom. After we have accept the self-energy principle, now we will allow the electromagnetic field to have its own freedom. The electromagnetic field can be defined without test charge. Even without test charge, the electromagnetic fields can be superposed.

We have choose the mutual energy principle as axioms. This is not necessary, if we do not apply mutual energy principle as axioms, we can apply the Maxwell equations as axioms. However in that case we have to speak there must have two group Maxwell equations which have to be synchronized. One group is for retarded wave, one is for the advanced wave. We need also to use the time-reversal Maxwell equation as axioms. The superposition principle cannot be derived from Maxwell equations that need to an additional axiom.

In case the the mutual energy principle is the axiom, the two groups Maxwell equations (for retarded wave and for the advanced wave) and the superposition principle plus a text descriptive the condition of synchronization of two kind of waves are replace by one formula that agree with the principle that an axiom should be as simple as possible. Another advantage is even the retarded wave and the advanced wave do not need to be defined in the axiom, this kind of concept can be derived. Apply the mutual energy principle as principle also will make the whole electromagnetic field theory much simpler. This is because the energy conservation law (or the mutual energy theorem Eq.(155)) and the mutual energy flow theorems can be easily derived from mutual energy principle. Most the real electromagnetic field problem can be solved with the mutual energy theorems. Maxwell equations also can be derived from the mutual energy principle. Started from Mutual energy principle two groups of Maxwell equations will automatically synchronized. We do not need a sentence to say that. Most radiation phenomenon can be derived from the mutual energy principle. The super position principle with test charges or the superposition for all retarded wave/advanced wave can also be derived form the mutual energy principle. That kind of superposition is enough in most situations.

The self-energy principle only will be applied to the more deep physical problem for example the interpretation of the quantum mechanics, the duality of the wave and particle, and also the problem like this article which kind of superposition can be allowed. The self-energy principle allows the superposition without the test charge. After we have the self-energy principle the restriction on the superposition of the electromagnetic fields is removed. This means any waves include retarded wave, advanced wave, with or without test charge can all be superposed.

9.3 The superposition without test charge

In this section we will prove that the superposition without test charge, this also means in this superposition the retarded wave and advanced wave are all included, in this situation the superposition can be derived from the self-energy principle and mutual energy principle.

We have proved this kind of superposition is OK if the self-energy principle is accept. That means this kind of superposition doesn't conflict with energy conservation law. This also means that if this kind of superposition used as axiom it will not conflict with energy conservation law. Now our task is changed, we assume we have the self-energy principle and the mutual energy principle, we would like to derive the superposition without any restrictions, i.e. without test charge, without limitation to to only retarded wave and only advanced wave. This means we will prove the superposition without a test charge and without restriction to only the retarded wave or advanced wave can be delete as axiom of electromagnetic field theory.

We have the mutual energy principle, which is,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$

$$+ \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{156}$$

We have know that started from the mutual energy principle we can obtained two group Maxwell equations which must synchronized. Any way we can obtained the Maxwell equations for all electromagnetic fields which can be retarded or advanced. It should be notice that this Maxwell equation is the Maxwell equation for single charge. Started from this Maxwell equations for single charge we can obtained the corresponding Poynting theorem which is,

$$-\iint_{S} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \iiint_{V} \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \iiint_{V} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV \tag{157}$$

This is also the Poynting theorem for single charge. We have the self-energy principle, which is,

$$-\iint_{S} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma + \iint_{S} (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = 0 \tag{158}$$

$$\iiint_{V} \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV + \iiint_{V} \boldsymbol{e} \cdot \boldsymbol{j} dV = 0 \tag{159}$$

$$\iiint_{V} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i) dV + \iiint_{V} (\boldsymbol{e} \cdot \partial \boldsymbol{d} + \boldsymbol{h} \cdot \partial \boldsymbol{b}) dV = 0 \tag{160}$$

This tell us that in the formula Eq.(157), each terms do not carry energy, it can be added to the mutual energy formula. The total energy transferred is not changed. Which means we have,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV$$

$$+ \iiint_{V} \sum_{i=1}^{N} \sum_{j=1}^{N} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_j) dV \tag{161}$$

Substitute,

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i, \qquad \boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \tag{162}$$

We have,

$$-\oint\!\!\!\oint_{\Gamma} \boldsymbol{E} \times \boldsymbol{H} \cdot \hat{n} d\Gamma = \iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV + \iiint_V (\boldsymbol{E} \cdot \partial \boldsymbol{D} + \boldsymbol{H} \cdot \partial \boldsymbol{B}) dV \tag{163}$$

This means that the superposed field satisfy the Poynting theorem. We have few time derive the Maxwell equation from the Poynting theorem, hence the superposed field also satisfy the Maxwell equations. It should be noticed that this Maxwell equation is for the superposed field.

We have derived the superposition principle without any restriction from the mutual energy principle. The superposed field can satisfy Maxwell equations.

It should be clear, that in the above derivation actually the self-energy principle do not really needed. This results can be also derived only from the mutual energy principle alone. However, self-energy principle is necessary in case we derive from energy conservation law form Maxwell equations and the superposition principle without test charge.

9.4 Example 1

In order to fully understand the concept of superposition without test charge and superposition and with a test charge, let us see an example. Many things we have already done before, but it is still worth to repeat it again.

Assume there are two charges, one is a emitter and another is an absorber. The emitter has the current $\boldsymbol{J}_1$ and the absorber has the current $\boldsymbol{J}_2$.

If we started from the superposition without test charge, we know that the fields of the two waves can be superposed, that means we have

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 \tag{164}$$

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 \tag{165}$$

The above superposition is the superposition without any restriction, the superposition can without any test charge and the superposition can be the retarded wave and also the advanced wave.

We can assume that the superposition of the two fields satisfy Maxwell equations, and hence also satisfy the Poynting theorem. Hence, we have,

$$-\oint\!\!\!\oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$$
$$= \iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV + \iiint_V (\boldsymbol{E} \cdot \partial \boldsymbol{D} + \boldsymbol{H} \cdot \partial \boldsymbol{B}) dV \tag{166}$$

Substituting Eq.(164,165) to the above Poynting theorem Eq.(166), we obtain,

$$-\oiint_{\Gamma}\sum_{i=1}^{2}\sum_{j=1}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V}\sum_{i=1}^{2}\sum_{j=1}^{2}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dV+\iiint_{V}\sum_{i=1}^{2}\sum_{j=1}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \qquad (167)$$

This is the Poynting theorem of the two charges. This formula have two parts, one is the mutual energy part which is,

$$-\oiint_{\Gamma}\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V}\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\boldsymbol{E}_i\cdot\boldsymbol{J}_j dV+\iiint_{V}\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_j)dV \quad (168)$$

another part is the self-energy part,

$$-\oiint_{\Gamma}\sum_{i=1}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V}\sum_{i=1}^{2}\boldsymbol{E}_i\cdot\boldsymbol{J}_i dV+\iiint_{V}\sum_{i=1}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_i)dV \qquad (169)$$

We can assume the self-energy part has no contribution to the energy transfer, since all self energy terms are canceled by the corresponding time-reversal waves. That means we have,

$$-\oiint_{\Gamma}\sum_{i=1}^{2}(\boldsymbol{E}_i\times\boldsymbol{H}_i-\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma=0 \qquad (170)$$

$$\iiint_{V}\sum_{i=1}^{2}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i+\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dV=0 \qquad (171)$$

$$\iiint_{V}\sum_{i=1}^{2}(\boldsymbol{E}_i\cdot\partial\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\partial\boldsymbol{B}_i+\boldsymbol{e}_i\cdot\partial\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\partial\boldsymbol{b}_i)dV=0 \qquad (172)$$

Hence, the self-energy part can be take away from the Poynting theorem of the two charges Eq.(167). Hence we derived the Eq.(168). The mutual energy part can be rewritten as,

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$
$$=\iiint_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV$$
$$+\iiint_{V}(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dV \qquad (173)$$

Assume Γ is infinite big sphere. Since we know that the two fields, if one is retarded field and another is advanced field, we can have,

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=0 \qquad (174)$$

We also know that,

$$\int_{-\infty}^{\infty}dt\iiint_{V}(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dV$$
$$=\int_{-\infty}^{\infty}\partial U dt=U|_{-\infty}^{\infty}=U(\infty)-U(-\infty)=0 \qquad (175)$$

where,

$$U=\boldsymbol{E}_1\cdot\boldsymbol{D}_2+\boldsymbol{H}_1\cdot\boldsymbol{B}_2 \qquad (176)$$

Here, $U(\infty)$ and $U(-\infty)$ are two stable states. The total energy should not changed. Hence we have $U(\infty)=U(-\infty)$.

Substituting Eq.(175) and Eq.(174) to Eq.(173) we obtain,

$$\int_{-\infty}^{\infty}dt\iiint_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV=0 \qquad (177)$$

This can be written as,

$$-\int_{-\infty}^{\infty}dt\iiint_{V_1}\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t))dV=\int_{-\infty}^{\infty}dt\iiint_{V_2}(\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t))dV \qquad (178)$$

This is Welch's reciprocity theorem, the mutual energy theorem or the energy conservation law, this formula tell us the field at the place $\boldsymbol{J}_2$ is $\boldsymbol{E}_1(t)$. The field at the place $\boldsymbol{J}_1(t)$ is only the field of $\boldsymbol{E}_2(t)$. We know that in the palace V_2, $\boldsymbol{J}_2$ is the test charge, $\boldsymbol{E}_1(t)$ is the field which has been test with this test charge. In the place V_1. $\boldsymbol{J}_1$ is the test charge, the field $\boldsymbol{E}_2(t)$ is the field measured with this test charge. Hence, actually we have applied the superposition principle with the test charge at the places V_1 and V_2. In the formula Eq.(178) we only need the superposition with a test charge.

In this example it is clear the superposition without test charge and the superposition with test charge can be united, because we started from the superposition without test charge and Maxwell equations, in the end we obtained the superposition with the test charge, the key is the self-energy principle Eq.(170-172).

This example is a situation for single photon. The emitter sends the retarded field, the absorber sends the advanced field. The two fields can be superposed. The energy is transferred from emitter to the absorber is through the mutual energy flow which can be written as,

$$-\int_{-\infty}^{\infty} dt \iiint_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t))dV = (\xi_1, \xi_2) = \int_{-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t))dV \quad (179)$$

where

$$(\xi_1, \xi_2) = \int_{-\infty}^{\infty} dt \iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \quad (180)$$

$Q = \iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1)\hat{n} d\Gamma$ is the mutual energy flow. (ξ_1, ξ_2) is the energy transferred through the mutual energy flow. The Γ can be taken on any closed surface between V_1 and V_2. (ξ_1, ξ_2) is the energy of the photon. Hence we can say that the photon is the mutual energy flow.

Single photon system needs the exist of the an emitter and an absorber. The photon "knows" its target which is the specific absorber in the very beginning.

9.5 Example 2

Assume we have two absorber charges and one emitter charge. The current of absorbers are $\boldsymbol{J}_{21}$ and $\boldsymbol{J}_{22}$. The current of the emitter is $\boldsymbol{J}_1$. The fields of the emitters are $\xi_{21} = [\boldsymbol{E}_{21}, \boldsymbol{H}_{21}]$ and $\xi_{22} = [\boldsymbol{E}_{22}, \boldsymbol{H}_{22}]$. The field of the emitter is $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$. We can have,

$$\xi_2 = \xi_{22} + \xi_{22} \quad (181)$$

where

$$\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2] = [\boldsymbol{E}_{21} + \boldsymbol{E}_{22}, \boldsymbol{H}_{21} + \boldsymbol{H}_{22}] \quad (182)$$

In this case we also have Eq.(178) which means,

$$-\iiint_{V_1} (\boldsymbol{E}_{21}(t) + \boldsymbol{E}_{22}(t)) \cdot \boldsymbol{J}_1(t)dV = \iiint_{V_2} \boldsymbol{E}_1(t) \cdot (\boldsymbol{J}_{22}(t) + \boldsymbol{J}_{22}(t))dV \quad (183)$$

This tell us, in the place of absorber V_1, the two advanced fields of the absorbers can be superposed. This kind of superposition is the superposition with a test charge. the test charge is $\boldsymbol{J}_1(t)$.

In most situation, this kind of superposition is enough. This kind of superposition can also be widen to the situation for all advanced waves can be superposed.

If we have considered the self-energy principle Eq.(170-172), the superposition without any restriction can also be allowed. This has been seen from the whole derivation process.

It should be notice that the superposition of two fields of the two absorbers only happened at the situation of the quantum entangled situation [9]. This is the situation when we in the beginning has one higher energy photon with 0 spin. This photon go through some nonlinear substance will become two entangled photons. This two photons have half energy as the original one. The two lower energy photon also have different spins, if the first lower energy photon is left spin, another lower energy photon must have the right spin, this will keep their angle moment conservative with the higher energy photon. Hence we have obtain the normal entanglement situation.

From my understand the superposition is only happened in the quantum entangle situation. We often speak about before the quantum measurement the system is taken as a superposition state, that is very wrong. Before the measurement, the mutual energy did not happen. The advanced wave has not released. Hence all the possibility can be possible. But here has no any physical fields are superposed.

9.6 Example 3

Assume we have two emitter charges and one absorber charges. The current of emitters are $\boldsymbol{J}_{11}$ and $\boldsymbol{J}_{12}$. The current of the absorber is $\boldsymbol{J}_2$. The fields of the emitters are $\xi_{11} = [\boldsymbol{E}_{11}, \boldsymbol{H}_{11}]$ and $\xi_{12} = [\boldsymbol{E}_{12}, \boldsymbol{H}_{12}]$. The field of the absorber is $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$. We can have,

$$\xi_1 = \xi_{12} + \xi_{12} \tag{184}$$

Where

$$\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1] = [\boldsymbol{E}_1 + \boldsymbol{E}_2, \boldsymbol{H}_1 + \boldsymbol{H}_2] \tag{185}$$

In this case we also have Eq.(178) which means,

$$-\iiint\limits_{V_1} \boldsymbol{E}_2(t) \cdot (\boldsymbol{J}_{12}(t) + \boldsymbol{J}_{12}(t)) dV = \iiint\limits_{V_2} (\boldsymbol{E}_{11}(t) + \boldsymbol{E}_{12}(t)) \cdot \boldsymbol{J}_2(t) dV \tag{186}$$

This tell us, in the place of absorber V_2, the two retarded fields of the emitters can be superposed. This kind of superposition is a superposition with test a charge.

In most situation this kind of superposition is enough. This kind of super position can also be widen to the situation for all retarded waves or all the advanced waves.

If we have considered the self-energy principle Eq.(170-172), the superposition without any restriction can be allowed. This has been seen from the whole derivation process.

The above is also a kind of entangled situation. This time the two retarded field from the emitter are superposed.

I believe this kind of the entangled situation should also exist. This can be proved by a future experiment. For example we can using lower energy photon to bombard the nonlinear material used in the Spontaneous parametric down-conversion experiment[9]. If the two lower energy photon can be convert to one higher energy photon, this situation has happened. Hence, this is the inverse process of the Spontaneous parametric down-conversion experiment[9].

9.7 Summarization

The above two examples tell us, even we started with a superposition without test charge, if we consider the self-energy principle, in the end we will obtain a result of the superposition with test charge, or we obtained the superposition of only one kind of field, either retarded or advanced.

However if we do not consider the self-energy principle, we cannot obtained the energy conservation law (which is the Welch's reciprocity theorem).

This also tell us, the Welch's reciprocity theorem is not only a reciprocity, it is a energy theorem, it is a energy conservation law for two charges. The author has call the Fourier transform of the Welch's reciprocity theorem as the mutual energy theorem[10], this is correct, but not enough, Welch's reciprocity theorem is energy conservation law. Eq.(20) can be seen as extension of the Welch's reciprocity theorem from charge 2 to N charges and which is a energy conservation law for N charges. It should be noticed that Eq.(20) is easy to been seen as a energy conservation law than Welch's reciprocity theorem. Since Welch's reciprocity theorem is usually not applied to two charges but two antennas. In the case of two antennas, Welch's reciprocity theorem discussed the energy transfer between the transmitting antenna to the receiving antenna. It is clear that is only a part of the energy of the transmitting antenna transferred to the receiving antenna, hence, is not a energy conservation law. This is also the reason when this author started the same problem, has called it as the mutual energy theorem.

It is only in the recent year, when this author move the problem from the antenna to the charges, In this case, we assume in the empty space there are only N charges, we can derive the energy conservation law. In the empty space only has N charges or two charges, this is a abstraction form. In the real situation, it is not possible to get a space where only two charges. Hence, this claim about a empty space with only N charges cannot be implemented in the real world. However the author think this kind of abstraction is very useful to make things clear.

10 The superposition for the traditional electromagnetic fields

The electromagnetic fields studied in the absorber theory[1, 2], and the author's theory[11], the retarded field and the advanced field are all assumed. The traditional field theory only the retarded field are assumed. We know that the traditional electromagnetic field is also very successful in most situations. Hence the author thought that the traditional electromagnetic field theory should be possible to be derived as approximate result of the author's new electromagnetic field theory.

10.1 Traditional electromagnetic field

In the last a few sections we consider the situation with the retarded wave and the advanced wave. The author believe that this is a more accurate theory. Then the traditional electromagnetic field theory should be possible to be derived from this new theory. We can assume the absorbers are uniformly distributed on a infinite big sphere, all the advanced fields contributed from the absorbers can be seen same as the retarded wave, in this situation, the traditional electromagnetic field will be two times as strong as the retarded field or advanced field. The traditional electromagnetic field can bee seen as retarded field also.

10.2 Wave guide situation

In case of the wave guide, for example a cylinder wave guide. Assume one end of cylinder there are emitters. Assume in the another end of the cylinder there are the absorbers. Hence there are transmitter and receiver on each end of the wave guide.

We assume the absorbers can absorb all the waves sent from the emitters and without any reflections. In this case we can assume the advanced fields send from these absorbers are nearly same as the retarded waves sent from the emitters. Hence we have,

$$\boldsymbol{E}_a \simeq \boldsymbol{E}_r \tag{187}$$

$$\boldsymbol{H}_a \simeq \boldsymbol{H}_r \tag{188}$$

$$\boldsymbol{E} = \boldsymbol{E}_r + \boldsymbol{E}_a \simeq 2\boldsymbol{E}_r \tag{189}$$

where $\boldsymbol{E}_r$ is the retarded field send from the end 1 which is the transmitter. $\boldsymbol{E}_a$ is the advanced field send by the end 2 which is a receiver. In the wave guide. $\boldsymbol{E}$ is the traditional electromagnetic fields which is consist of the retarded wave and the advanced wave. Usually we did not notice this, we will think this field is just produced only by the source which is the transmitter in side the wave guide.

According to the mutual energy principle the retarded wave and the advanced wave satisfy the mutual energy principle, which is,

$$-\oiint_{\Gamma}(\boldsymbol{E}_r\times\boldsymbol{H}_a+\boldsymbol{E}_a\times\boldsymbol{H}_r)\cdot\hat{n}d\Gamma$$

$$=\iiint_V(\boldsymbol{E}_r\cdot\boldsymbol{J}_a+\boldsymbol{E}_a\cdot\boldsymbol{J}_r)dV$$

$$+\iiint_V(\boldsymbol{E}_r\cdot\partial\boldsymbol{D}_a+\boldsymbol{H}_r\cdot\partial\boldsymbol{B}_a+\boldsymbol{E}_a\cdot\partial\boldsymbol{D}_r+\boldsymbol{H}_a\cdot\partial\boldsymbol{B}_r)dV \qquad (190)$$

Assume the current $\boldsymbol{J}_r$ is inside the volume V_1. V_1 is the region close to $\boldsymbol{J}_r$. Γ_1 is the boundary surface of V_1. In the above formula the volume V can be chosen as an region. If we take the mutual energy theorem on the volume V_1, we have,

$$-\oiint_{\Gamma_1}(\boldsymbol{E}_r\times\boldsymbol{H}_a+\boldsymbol{E}_a\times\boldsymbol{H}_r)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V_1}\boldsymbol{E}_a\cdot\boldsymbol{J}_r dV$$

$$+\iiint_{V_1}(\boldsymbol{E}_r\cdot\partial\boldsymbol{D}_a+\boldsymbol{H}_r\cdot\partial\boldsymbol{B}_a+\boldsymbol{E}_a\cdot\partial\boldsymbol{D}_r+\boldsymbol{H}_a\cdot\partial\boldsymbol{B}_r)dV \qquad (191)$$

This is because inside the volume V_1, $\boldsymbol{J}_a=0$, and hence, $\boldsymbol{E}_r\cdot\boldsymbol{J}_a=0$. Consider Eq.(187) to the above formula we have approximately,

$$-\oiint_{\Gamma_1}(\boldsymbol{E}_r\times\boldsymbol{H}_r+\boldsymbol{E}_r\times\boldsymbol{H}_r)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V_1}\boldsymbol{E}_r\cdot\boldsymbol{J}_r dV$$

$$+\iiint_{V_1}(\boldsymbol{E}_r\cdot\partial\boldsymbol{D}_r+\boldsymbol{H}_r\cdot\partial\boldsymbol{B}_r+\boldsymbol{E}_r\cdot\partial\boldsymbol{D}_r+\boldsymbol{H}_r\cdot\partial\boldsymbol{B}_r)dV \qquad (192)$$

or

$$-2\oiint_{\Gamma_1}(\boldsymbol{E}_r\times\boldsymbol{H}_r)\cdot\hat{n}d\Gamma$$

$$=\iiint_{V_1}\boldsymbol{E}_r\cdot\boldsymbol{J}_r dV$$

$$+2\iiint\limits_{V_1}(\boldsymbol{E}_r\cdot\partial\boldsymbol{D}_r+\boldsymbol{H}_r\cdot\partial\boldsymbol{B}_r)dV \tag{193}$$

Considering Eq.(189)

$$-\oiint\limits_{\Gamma_1}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\iiint\limits_{V_1}\boldsymbol{E}\cdot\boldsymbol{J}dV$$

$$+\iiint\limits_{V_1}(\boldsymbol{E}\cdot\partial\boldsymbol{D}+\boldsymbol{H}\cdot\partial\boldsymbol{B})dV \tag{194}$$

This is the Poynting theorem. Hence, even we started with mutual energy principle and assume there are retarded wave and the advanced wave, but we still can obtained the Poynting theorem for traditional electromagnetic field $\xi=[\boldsymbol{E},\boldsymbol{H}]$. In this Poynting theorem, the field $\boldsymbol{E}=\boldsymbol{E}_r+\boldsymbol{E}_a$ are superposition of the retarded wave and advanced wave and hence two times as the field of only retarded wave. The above result is obtained inside a wave guide, but it can be widen further, see next sub-section.

10.3 Widen the result to the case with a cone-beam wave guide

The above result can be easily widen to the situation instead of the cylinder wave guide, but a cone-beam wave guide. We can assume that the emitters are all at the vertex of the cone. In the other end there is uniformly distributed absorbers. In this situation the Eq.(187-189) still can be established. This will lead the same result as last sub-section. Hence the traditional electromagnetic fields also satisfy the Poynting theorem for cone beam wave guide situation. It should notice we have assume in one end of the cone-beam wave guide the absorbers has been distributed uniformly. This assumption will lead the advance wave of the absorber close to equal to the retarded wave sent from the vertex of the cone.

10.4 Further widen the result to free space

The free space can be also see as a cone beam wave guide, the only difference is this kind of the wave guide has the cone-beam angle as 4π. The normal come-beam wave guide the beam span angel is less 4π. In case we have assume the absorbers are uniformly distributed on the infinite big sphere, we can assume the advanced wave of the absorber is approximately equal to the retarded wave.

It should be noticed that the condition above for widening the result is the absorbers uniformly distributed on the infinite big sphere. This condition is not easily to be met in any situation. In case there are two antenna, one is a transmitting antenna, another is a receiving antenna, there are absorbers in the back ground of the receiving antenna. The back ground can also receive the electromagnetic field which can be seen as a uniformly distributed on infinite sphere.

But since we have added a receiving antenna to the uniformly distributed absorbers, the total advanced fields send out from the receiving antenna and the absorbers in infinite big sphere together can not be seen as uniformly distributed. The receiving antenna are close to the transmitting antenna, it can offer big influence to the emitter than other absorbers on the infinite big sphere. If the receiving antenna are very far away from the transmitting antenna, the influence of the receiving antenna to the transmuting antenna can be omit. In this situation we can also say that the absorbers are uniformly distributed on the big sphere when we discussion the problem of transmitting antenna.

In case we need to calculate some thing for example the directivity diagram of the receiving antenna. In this case we cannot assume the absorber are uniformly distributed. And hence, the traditional electromagnetic field theory will fail. This is also the reason we cannot directly calculate the directivity diagram of the receiving antenna with the traditional electromagnetic field theory. Normally in this situation we have to apply the Lorentz reciprocity theorem[3, 4] to find the the directivity diagram. The correct way to calculate the directivity diagram of a receiving antenna is to apply the mutual energy theorem or Welch's reciprocity theorem. Since Lorentz reciprocity theorem is only a transform of the mutual energy theorem or Welch's reciprocity theorem, it can also obtained correct directivity diagram of a receiving antenna. However this is not mean the Lorentz reciprocity theorem is correct. It is wrong because it assume the receiving antenna also sent the retarded wave. Now it is clear that the receiving antenna sends the advanced field. Abort the wrong doing of the Lorentz reciprocity theorem I will discuss it more detail in other article. Here, we only need to know that the absorbers are uniformly distributed this condition is often to be violated. Only when the absorber can be seen as uniformly distributed we can thought the advance wave same as the retarded wave and hence the traditional electromagnetic field theory can be applied.

10.5 Further widen the super position principle to the traditional electromagnetic fields

In case there is uniformly distributed absorbers, then the advanced wave can be seen approximately equal to the retarded wave. We known that the retarded waves can be superposed, If the absorber are uniformly distributed on the infinite big sphere, the advanced wave is approximately equal to the retarded wave. Hence the total field which is the traditional electromagnetic field will be two times of the retarded waves. The retarded wave can be superposed, since this traditional electromagnetic wave, similar to the retarded wave, can also be superposed.

Hence the traditional electromagnetic field can be superposed. Please notice, according to our discussion before, when we discussion the directivity diagram of the receiving antenna, this result is not suitable. In the case of directivity diagram of the receiving antenna, the concept of the traditional electromagnetic field will fail.

11 Energy of the superpositions

In this section we discussion the energy of the different superpositions. The field can be superposed or not is not all we need to be discussion. The energy of the different superpositions should be also discussed. In this article we have discussed 5 different superpositions. Assume there two fields each have the same energy transferred, the question is how much the energy can be transferred by these 5 different superpositions?

11.1 The superposition of the retarded waves

Assume there are two antenna each send the retarded waves, assume the two antenna are very close to each other, in this situation, the field of the retarded wave can be superposed, hence the retarded wave has doubled, in this case the wave doubled, energy will 4 times as one antenna works alone.

This result is also true for the traditional field which are the retarded field together with the advanced field from uniformly distributed absorbers. If this traditional field doubled, the power after the superposition will 4 times larger. This is because in case the absorber has uniformly distributed on infinite big sphere we have proved that the traditional field satisfy Poynting theorem. From the Poynting theorem if the current $\boldsymbol{J}$ is doubled, the radiation energy will 4 times as before.

11.2 The superposition of a retarded field send from the emitter and the advance field send from the absorber

We have said the superposition with retarded wave and advance wave is allowed, if we have accept the self-energy principle. However how much energy can be transferred for this kind of superposition? Think that the retarded field transfer the energy is positive. The advanced field transfer the energy is negative. Here the negative I mean that the advance field actually receiving energy is not really sending energy out. The advance wave sends from current time to the past time, but the source of the advance field actually receive energy from past. What will happen when this two kinds of fields superposed? If we think this situation should same as two retarded waves, the transferred energy will 4 times as there is only one wave. If we think the advance wave receive negative energy and hence, the retarded wave receiving positive energy, together should be 0. I thought the result perhaps is between 0 to 4, but what should be the correct answer?

Assume the $\boldsymbol{J}_1$ sends retarded wave. $\boldsymbol{J}_2$ sends advanced wave. Assume these two wave are synchronized and hence the mutual energy principle are satisfied. Hence we have the mutual energy flow theorem,

$$-\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt$$

$$= \int_{t=-\infty}^{\infty} \iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \tag{195}$$

This formula offer us the energy transferred from the emitter to the absorber. This energy is the energy of the photon. The self-energy items,

$$- \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \iiint_{V_1} \boldsymbol{E}_1 \cdot \boldsymbol{J}_1 dV + \iiint_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_1) dV \tag{196}$$

is the self-energy of the retarded wave.

$$- \oiint_{\Gamma_2} (\boldsymbol{E}_2 \times \boldsymbol{H}_2) \cdot \hat{n} d\Gamma$$

$$= \iiint_{V_2} \boldsymbol{E}_2 \cdot \boldsymbol{J}_2 dV + \iiint_{V} (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_2) dV \tag{197}$$

is the self-energy of the advance wave.

These two self-energy flow do not create any energy transfer. If it transfer any energy, that means these waves are collapsed. However we have point out the wave are not collapsed but collapsed back, which means the self-energy principle should be accept which is,

$$- \oiint_{\Gamma} \sum_{i=1}^{2} (\boldsymbol{E}_i \times \boldsymbol{H}_i - \boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma = 0 \tag{198}$$

$$\iiint_{V} \sum_{i=1}^{2} (\boldsymbol{E}_i \cdot \boldsymbol{J}_i + \boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV = 0 \tag{199}$$

$$\iiint_{V} \sum_{i=1}^{2} (\boldsymbol{E}_i \cdot \partial \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \partial \boldsymbol{B}_i + \boldsymbol{e}_i \cdot \partial \boldsymbol{d}_i + \boldsymbol{h}_i \cdot \partial \boldsymbol{b}_i) dV = 0 \tag{200}$$

This means all self-energy flow are canceled with the corresponding time reversal self-energy flow. All self-energy terms are canceled. Hence the two formula of Poynting theorem Eq.(196, 197) do not transfer any energy! This looks good, if they transfer energy, this energy will send to the whole space the energy eventually will be lost in our universe. The wave collapse can not be accepted. The reason is that, the wave collapse none can offer a formula to describe it.

The another reason is the mutual energy flow theory has offered a correct energy conservation law for the two charges, there is no any room for some any other energy terms. If the self-energy flow also collapse, which will contributed some additional energy on the top of the energy conservation law, which will destroy the energy conservation law. And hence, cannot be accept.

Hence Eq.(195) offer us the energy transferred by this kind of superposition. This energy is not related to the self-energy flow but only the mutual energy flow, it is also nothing to do with concept the field is 2 times the energy will be 4 times.

11.3 The superposition of fields of two absorbers

Assume there are two absorber receive the retarded field send from one emitter. This is the situation of the quantum entanglement. Assume there is a high frequency photon run to the nonlinear substance, this nonlinear substance can be seen as the emitter which send two low frequency photons out[9]. Here we speak about two photons that means this emitter has send a retarded wave in the same time. There are two absorbers which send two advanced waves to the emitter. Assume this two advanced waves are synchronized, that means they reach the emitter at the same time. We know that the emitter has obtain the energy from the input photon which is a higher energy photon. Since now there are two waves from two absorbers come to the emitter, The energy of the emitter is divided by two as two parts. Each absorber can obtained only one half of the energy. Since the two lower energy photons should have same angle moment with the original higher frequency photon. If the original photon have 0 angle moment the two new photons can only have one is left spin and the other is right spin. Hence the two photons are entangled. This means if we have measured one of the photon which is left spin, we should know that the another photon is right spin. This way we can make the two photons with the total self-rotation angle moment as 0. In this case the angle moment of two photons must be entangled.

In this situation there are two photon has been received. Hence, the two absorbers can only double the transferred energy.

11.4 Can the time reversal field superpose to the normal electromagnetic field

In this article we have introduced the time reversal electromagnetic fields. The time reversal electromagnetic fields do not satisfy the same Maxwell equations. The electromagnetic field satisfy the Maxwell equations, the time-reversal electromagnetic field satisfy the time reversal Maxwell equations. Hence, the author does not assume that this two kind of fields can be superposed. i.e. $\boldsymbol{E}$ cannot superposed with $\boldsymbol{e}$. $\boldsymbol{H}$ cannot superposed with $\boldsymbol{h}$. However they transfer same amount of energy and, hence, canceled each other.

11.5 Summary

According to the above discussion that if the retarded wave sends from the emitter is doubled, since in this case the advanced wave send back from the environment will also be doubled, the energy transfer will be 4 times as original situation.

If the retarded wave sends from an emitter and the advanced wave sends from an absorber are superposed. The two wave become a energy transfer pair, it can only transfer energy of 1 photon.

If the advanced wave sends from an absorber is doubled the transferred energy can only have two times as before.

Hence according to author's electromagnetic field theory there are different superposition, the energy transferred by the different superposition cannot be simply thought the two equal fields superposed, the energy transferred is 4.

12 Conclusion

12.1 United the two superposition principles

In this article we started two kinds of superposition principle, the first is the superposition without a test charge. The second is the superposition with a test charge. Since in the author's electromagnetic field theory the advanced wave is accept as physical wave, the superposition with the retarded wave alone or advanced wave alone are also considered. The superposition for the traditional electromagnetic fields are also considered. The traditional electromagnetic field which is obtained by assume the absorber uniformly distributed on the infinite big sphere, and hence the retarded wave and the advanced wave can be merged together become the one field.

The author has proved if we accept the self-energy principle, we can proved that the above 5 different superpositions are all allowed. But if the self-energy principle is denied. The superposition without test charge and the Maxwell equations will conflict with energy conservation law. The super position with test charge has different meaning with the superposition without test charge. Hence self-energy principle is the key to united all kinds of the superpositions. This also tell us that the self-energy principle must to be accepted.

This article we started from the Maxwell equations and superposition principle without test charge, we first obtained the Poynting theorem of N charges. If self-energy principle are accepted we obtained the mutual energy formula (The formula of the the mutual energy principle, here we started from Maxwell equations and the superposition principle, and hence this is referred as formula instead of principle). From mutual energy formula we derived the Maxwell equations must be synchronized. In order to derived the energy conservation law, we need the fields of the two Maxwell equations must one is retarded and another is advanced. Hence the the advance wave have to be accepted. If self-energy principle isn't accepted, we cannot obtained energy conservation law. Hence all kinds superpositions cannot be derived. Hence, the self-energy principle must be accepted.

The advance wave also need to be accepted at the electromagnetic field theory. Without advance wave we cannot prove the energy conservation law from mutual energy principle. After we accepted the advanced wave the superposition with retarded wave alone or with only advanced wave alone have different meaning. Hence, this two kinds of superpositions need to be distinguished. Since we have retarded waves and advanced waves, the traditional electromagnetic field in which only the retarded wave are allowed also need to be distinguished. This way we obtained 5 different superpositions. However, if we accepted the self-energy principle, this 5 different superpositions are all allowed.

Without self-energy principle, we can still have the mutual energy principle. From mutual energy principle we can obtained the superposition with test charge or the superposition with the retarded wave alone/the superposition with the advanced wave alone.

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