Refutation of two modern modal logics: "JYB4" and the follow-on "AR4"

Abstract: We evaluate the four-valued logic systems of J.-Y. Béziau and F. Schang as JYB4 and AR4. JYB4 named the four-valued modal logic Ł_4 as Łukasiewicz's nightmare because of the alleged absurdity of (◇p&◇q) → ◇(p&q). A model checking system is then framed based on 0± and 1±. We show (◇p&◇q) → ◇(p&q) is equivalent to (◇p&◇~p) → ◇(p&~p) with (◇p&◇q) = ◇(p&q) as a theorem. AR4 was a doxastic logic follow-on to JYB4. We name these modern modal logic systems as Béziau's nightmare and Schang's nightmare.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not; + Or, ∨; & And, ∧; > Imply, ⊢; < Not Imply, ⊣; = Equivalent, ⊣⊢;
(p=p) Tautology;
% possibility, possibly, for one or some; # necessity, necessarily, for every or all.

See: J.-Y. Béziau. (2011).“A new four-valued approach to modal logic”. Logique & Analyse, Vol. 54.
J.-Y. Béziau. (2005). “Paraconsistent logic from a modal viewpoint”. Journal of Applied Logic.

We name this system after its writer J.-Y. Béziau as JYB4. It is less a logic system and more of a model checking system based on 15 axioms for which p and q are assigned 0±, 1± to evaluate models by arithmetic. These are keyed to (Béziau, 2011).

For definitions and properties:

#p>p ;	TTTT TTTT TTTT TTTT	(2.1.11.2)
p<#p ;	**F**C**F**C **F**C**F**C **F**C**F**C **F**C**F**C	(2.1.12.2) x
p>%p ;	TTTT TTTT TTTT TTTT	(2.1.21.2)
%p<p ;	C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F**	(2.1.22.2) x
#p>%p ;	TTTT TTTT TTTT TTTT	(2.1.31.2)

Remark 2.1.31.2: This theorem supposedly "results from (11), (21) and transivity" as (#p>p)&(p>%p) ; TTTT TTTT TTTT TTTT (2.1.31.2.2)

%p<#p ;	CCCC CCCC CCCC CCCC	(2.1.32.2) x

Remark 2.1.32.2: This theorem supposedly "results from (11), (22) and transivity" as (#p>p)&(%p<p) ; C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F** (2.1.32.2.2)

For codi modal logics verifying conditions:

(#p&#q)=#(p&q) ;	TTTT TTTT TTTT TTTT	(4.1.1.2)
#(p+q)<(#p+#q) ;	**FFFF FFFF FFFF FFFF**	(4.1.2.2) x
(%p&%q)<%(p&q) ;	**FFFF FFFF FFFF FFFF**	(4.1.3.2) x

Remark 4.1.3.2: LET p, q=~p: rain tomorrow, not rain tomorrow.

[Eq. 4.1.3.2] is in fact the nightmare Łukasiewicz had to face all his life. This is a central feature of his systems and he was not able to give a satisfactory explanation in order to justify it. The absurdity appears clearly through the following example:
If it is possible that it will rain tomorrow and it is possible that it will not rain tomorrow, then it is possible that it will rain and not rain tomorrow."

(%p&%q)>%(p&q) ; TTTT TTTT TTTT TTTT (4.1.3.2.2)

However, invoking one variable and its negation to replace q produces compliance in *all* classical modal logics:

(%p&%~p)>%(p&~p) ; TTTT TTTT TTTT TTTT (4.1.3.3.2)

Hence, the alleged absurdity is contradicted in the contra-example by using one variable and its negation, instead of two variables.

The contra-example is amplified by removing the implication to replace with an equivalence. For example:

"It possibly will rain tomorrow and possibly will not rain tomorrow"
is equivalent to
"It possibly will rain tomorrow and not rain tomorrow"

(%p&%~p)=%(p&~p) ; TTTT TTTT TTTT TTTT (4.1.3.4.2)

(%p+%q)=%(p+q) ; TTTT TTTT TTTT TTTT (4.1.4.2)
(#p+#q)>#(p+q) ; TTTT TTTT TTTT TTTT (4.1.5.2)
%(p&q)>(%p&%q) ; TTTT TTTT TTTT TTTT (4.1.6.2)

Necessitation and replacement:

(p>p)>(p>#p) ; TNTN TNTN TNTN TNTN (7.1.2) x
(p=q)>(#p=#q) ; TTTT TTTT TTTT TTTT (7.2.1.2)
(p=q)>(%p=%q) ; TTTT TTTT TTTT TTTT (7.2.2.2)

For JYB4 Eqs. 2.-7., 5 of 15 or 33% are *not* tautologous. Consequently, we rename the alleged Łukasiewicz nightmare as Béziau's nightmare.

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We name this extension of JYB4 after its writer Fabian Schang as doxistic "deviant" logic system AR4.

#p=~%~p ; TTTT TTTT TTTT TTTT (14.2)
%p=~#~p ; TTTT TTTT TTTT TTTT (15.2)

Paracomplete negation:

~p=#~p ; NTNT NTNT NTNT NTNT ; (16.0.2) x

Paraconsistent negation;

~p=~#p ; TNTN TNTN TNTN TNTN ; (17.0.1.2) x
~p=%~p ; TNTN TNTN TNTN TNTN (17.0.2.2) x
~#p=%~p ; TTTT TTTT TTTT TTTT (17.0.3.2)

For AR4 Eqs. 14.-17., 3 of 6 or 50% are *not* tautologous. Consequently, we rename this subsequent work to Béziau's nightmare as Schang's nightmare.

We conclude that the statistics above remove JYB4 and AR4 from further serious consideration as viable and useful modern modal four-valued logics.