

Refutation of alleged Łukasiewicz nightmare in L_4 logic: $(\diamond p \& \diamond q) \rightarrow \diamond(p \& q)$

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Abstract: In Prover9 the alleged Łukasiewicz nightmare of $(\diamond p \& \diamond q) \rightarrow \diamond(p \& q)$ is not tautologous. However, in Prover 9 the nightmare recast in one variable as $(\diamond p \& \diamond \sim p) \rightarrow \diamond(p \& \sim p)$ is tautologous. In Meth8/VŁ4, both propositions are tautologous. This speaks to Meth8/VŁ4, based on the corrected modern Square of Opposition as an exact bivalent system, opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q: Obama was born inside USA, Obama was born outside USA;
 ~ Not; & And; % possibility, for one or some; > Imply, greater than; = Equivalent.

From: Łukasiewicz, J. (1920). On three-valued logic in L. Borkowski (ed). 1970. 87-88.
 Łukasiewicz, J. (1953). A system of modal logic. *Journal of Computing Systems*. 1:111-149.

Remark: The term "nightmare" was attributed to J-Y. Béziau in 2011 for the purpose of his four-valued schema of paraconsistent logic to evaluate systems based on values such as -x, +x, -y, +y. The motivation was to discount the fact that the L_4 logic system was provably bi-valent (James, 2010, Estoril), and hence it was not mappable into a vector space which serves as the continuing basis of the field of paraconsistent logic.

This proposition is supposedly egregious to logic system L_4 : $(\diamond p \& \diamond q) \rightarrow \diamond(p \& q)$. (1.0)

If possibly Obama was born inside USA and possibly Obama was born outside USA, then possibly both Obama was born inside USA and Obama was born outside USA. (1.1)

$(\%p \& \%q) > \%(p \& q)$; TTTT TTTT TTTT TTTT (1.2)

Assumptions: $((\text{exists}(p) \& \text{exists}(q)))$.
 Goals $(\text{exists}(p \& q))$. Exhausted. (1.3)

Prover9 invalidates Eq. 1.0 to show L_4 is untenable as an alethic logic.

If we preload $p = \sim q$ as the antecedent to Eq. 1.0, then: (2.0)

If possibly Obama born inside USA is equivalent to Not (Obama born outside USA), then if possibly Obama was born inside USA and possibly Obama was born outside USA, then possibly both Obama was born inside USA and Obama was born outside USA. (2.1)

$\%(q = \sim p) > (\%(p \& q) > (\%p \& \%q))$; TTTT TTTT TTTT TTTT (2.2)

$$\begin{array}{ll}
\text{Assumptions: } (\exists(q \leftrightarrow \neg p)). & \\
\text{Goals: } (\exists(p) \& \exists(q)) \rightarrow (\exists(p \& q)). & \\
& \text{Exhausted.} \tag{2.3}
\end{array}$$

Prover9 invalidates Eq. 2.0 to show L_4 is untenable as an alethic logic.

Remark 2.3: Eq. 2.3 shows Prover9 does not distribute the existential quantifier.

We rewrite Eq. 2.1 using one variable and its negation as *inside USA* and *not inside USA*:

$$(\diamond p \& \diamond \neg p) \rightarrow \diamond(p \& \neg p). \tag{3.0}$$

If possibly Obama was born inside USA and possibly Obama was born not inside USA, then possibly both Obama was born inside USA and Obama was born not inside USA. (3.1)

$$(\%p \& \% \neg p) \> \%(p \& \neg p); \quad \text{TTTT TTTT TTTT TTTT} \tag{3.2}$$

$$\begin{array}{ll}
\text{Assumptions: } (\exists(p) \& \neg \exists(p)). & \\
\text{Goals: } (\exists(p \& \neg p)). & \text{Theorem.} \tag{3.3}
\end{array}$$

Prover9 validates Eq. 3.0 to show L_4 is tenable as an alethic logic.

We explain Eqs. 1.2, 2.2, and 3.2 as rendered as tautologous in Meth8, but 1.3 as exhausted in Prover9 in this way. For more than one variable, the vector space for arity with Prover9 diverges from the bivalence inherent in $V L_4$, in which modal operators and quantifiers are distributive. This speaks to Meth8/ $V L_4$, based on the *corrected* modern Square of Opposition as an exact bivalent system, opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space.

Remark 3.2: Meth8/ $V L_4$ distinguishes between Eqs. 2.0 and 3.0 by protasis and apodosis as:

$$\begin{array}{ll}
\%p \& \%q; & \text{CCCT CCCT CCCT CCCT} \tag{1.2.1.2} \\
\%(p \& q) = (p = q); & \text{CCCT CCCT CCCT CCCT} \tag{1.2.2.2} \\
\text{and} & \\
\%p \& \% \neg p; & \text{CCCC CCCC CCCC CCCC} \tag{3.2.1.2} \\
\%(p \& \neg p) = (p = \neg p); & \text{CCCC CCCC CCCC CCCC} \tag{3.2.2.2}
\end{array}$$