

Generalized definition of division in any field

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Abstract. A historical definition of division by zero is reconsidered.

Let F be a field. The definition of division $/$ in F may be stated as $a/b = ab^{-1}$ in the case $b \neq 0$, and a/b is not defined if $b = 0$. But we can easily define $/$ including the case $b = 0$, i.e., $a/b = ab^{-1}$ if $b \neq 0$, and $a/b = ba$ if $b = 0$. Notice that the definition is made in a natural way. Essentially the same definition was made by the Indian mathematician and astronomer Brahmagupta (598-665+ α).

Our definition is justified by Takahasi's theorem [2].

Theorem 1. *Let F be a field with characteristic different from 2. Let f be a mapping $f : F \times F \rightarrow F$ satisfying*

$$(i) f(a, b) = ab^{-1} \text{ if } b \neq 0,$$

$$(ii) f(a, b)f(c, d) = f(ac, bd).$$

Then we have $f(a, 0) = 0$ for any $a \in F$.

Since our generalized definition have not been considered until today, it works as something like an axiom when we consider over a field. Therefore we have to consider whether the definition gives as fruitful results or not. The definition should get canceled, if it do not give any interesting results. Experimenting works with this aspect have been made for more than four years under the leadership of Saburo Saitoh, and a listing with more than eight hundreds results is made, some of which can be seen in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

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