Refutation of superposition as glue in Matita theorem prover

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Abstract: We evaluate the substitution lemma for the successor function, smart application of inductive hypotheses, and proof traces of a complex example in the Matita standard library. Results are not tautologous, hence refuting superposition.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

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LET p, q, r, s, t, u, v, w: A, B, C, S, i, j, k, M;
~ Not; + Or; - Not Or; & And; \ Not And, /; = Equivalent;
> Imply, greater than; < Not Imply, lesser than.

The substitution lemma says that (where S is the successor function)

for all k; i A[B=i][C=i+k] = A[C=S(k+i)][B[k+i]]=i

(p&((q=#t)&(r=(#t+#v))))=(p&((r=(s&(#v+#t)))&(q&((r=#v)=#t))))

(4.2.2)

Remark 4.2.2: Eq. 4.2.2 is not tautologous, hence refuting the substitution lemma for the successor function.

The inductive hypothesis

Hind : \forall.M[B=i][C/k+j] = M[C/S(k+j)][B/C/k/ j]

(4.3.1)

(w&((q=#t)&(r'=(#v+#u))))=(s&((r'(s&(#v+#u)))&(q&((r#v)#u))))

(4.3.2)

Remark 4.3.2: Eq. 4.3.2 is not tautologous, hence refuting the smart application of inductive hypotheses.
Proof traces: Since most of the time is spent in searching the right theorems composing the proof, a natural idea is to let the automation tactic return a trace of the proof consisting of all library results used to build the proof. ... Using these simple proof traces automation becomes extremely fast, and almost comparable to a fully expanded proof script.

This is a relatively complex example borrowed from the Matita standard library. The goal to prove is $k \leq n-1$ under the assumption $H : j + k < n$. (5.1.1)

$((p+q)<r)>-(r-(%s>#s))<q) ; \quad \text{TNTT TTTT TNTT TTTT} \quad (5.1.2)$

**Remark 5.1.2:** Eq. 5.1.2 is not tautologous, hence refuting the goal and use of proof traces. The proof table diverges from tautology by two values for truthity.