Refutation of superposition as glue in Matita theorem prover

Abstract: We evaluate the substitution lemma for the successor function, smart application of inductive hypotheses, and proof traces of a complex example in the Matita standard library. Results are *not* tautologous, hence refuting superposition.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

From: Asperti, A.; Tassi, E. (2009). Superposition as a logical glue. arxiv.org/pdf/1103.3319.pdf
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LET p, q, r, s, t, u, v, w: A, B, C, S, i, j, k, M;
~ Not; + Or; - Not Or; & And; \ Not And, /; = Equivalent;
> Imply, greater than; < Not Imply, lesser than.

The substitution lemma says that (where S is the successor function)

for all k;i A[B=i][C=i+k] = A[C=S(k+ i)][B[C=k]=i] (4.2.1)

(p&((q=#t)&(r=(#t+#v))))=(p&((r=(s&(#v+#t)))&(q&((r=#v)=#t)))) ; (4.2.2)

T**F**TT TTTT T**F**TT TTTT, TNTC TTTC TNTT TTTC,
TNTC TCTT TNTT TCTT, TNTT TTTC TNTT TTTT

Remark 4.2.2: Eq. 4.2.2 is not tautologous, hence refuting the substitution lemma for the successor function.

[T]he inductive hypothesis

Hind : ∀j.M[B/i][C/k+ j] = M[C/S(k+ j)][B[C/k]/ j] (4.3.1)

(w&((q\#t)&(r\(#v+#u))))=(s&((r\(s&(#v+#u)))&(q&((r\#v)\#u)))) ; (4.3.2)

TTTT TTTT TT**FF** TT**FF**, TTTT TTTT TTNN TTNN,
TTTT TTTT TT**FF** TTNN, TTTT TTTT TTNN TTNN,
FFFF **FFFF** **FF**TT **FF**TT, **FF**NN **FF**NN **FF**CC **FF**CC,
FFNN NNNN **FF**TT NNTT, **FFFF** NNNN **FF**TT NNTT,
FFNN NNNN **FF**CC NNTT, **FFFF** NNNN **FF**CC NNTT

It is evident that it is enough to instantiate j with i+1 but in order to unify (k+i)+1 with k+? j we have to use the associativity law for the sum! Hence the smart application of Hind succeeds where the normal application would fail.

Remark 4.3.2: Eq. 4.3.2 is *not* tautologous, hence refuting the smart application of inductive hypotheses.

LET p, q, r: j, k, n;
necessity, for all or every; % possibility, for one or some.
(%s>#s) ordinal 1; ~(y<x) (x≤y).

Proof traces: Since most of the time is spent in searching the right theorems composing the proof, a natural idea is to let the automation tactic return a trace of the proof consisting of all library results used to build the proof. ... Using these simple proof traces automation becomes extremely fast, and almost comparable to a fully expanded proof script.

This is a relatively complex example borrowed from the Matita standard library.
The goal to prove is $k \leq n-1$ under the assumption $H : j + k < n$. (5.1.1)

((p+q)<r)>~((r-(%s>#s))<q) ; `TNTT TTTT TNTT TTTT` (5.1.2)

Remark 5.1.2: Eq. 5.1.2 is *not* tautologous, hence refuting the goal and use of proof traces. The proof table diverges from tautology by two values for truthity.