Refutation of Metamath theorem prover

Abstract: We evaluate six conjectures and one theorem, as proffered by Metamath staff. The conjectures are *not* tautologous. The Tarski-Grothendieck theorem is also *not* tautologous. Metamath fails our analysis.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of
128-tables for more variables. (See ersatz-systems.com.)

[C]onsider the theorem

-. A. y E. z x = y (1.1)

which is provable in FOL.

LET p, q, r: x, y, z;
~ Not, -.; & And, /\; Or; = Equivalent, <->; @ Not Equivalent;
> Imply, ->, greater than; < Not Imply, lesser than;
necessity, for all or every, A.; % possibility, for one or some, E.;
(p=p) Tautology.

(~#q&(%r&p))=q ; TN**F**C T**FF**C TN**F**C T**FF**C (1.2)

Remark 1.1: Eq. 1.2 as rendered is *not* tautologous, hence not provable in FOL per Meth8/VŁ4.

The Exists.z part is trivial because z is not in the statement, so it says that not every y is equal to some fixed variable x.

We consider Eqs. 1.0, 1.1 without the z as r:

(~#q&p)=q ; T**FF**C T**FF**C T**FF**C T**FF**C (1.2.2)

Remark 1.2.2: Table results for Eq. 1.2.2 differ from Eq. 1.2 by the lines TN**F**C, so the exists z part is not so trivial (even though z isn't in the statement).

(If the free x is uncomfortable, [one] can also bind it [Eq. 1.1] as

A. x -. A. y E. z x = y.) (2.1)

((#p&~#q)&(%r&p))=q ; TT**FF** TC**FF** TT**FF** TC**FF** (2.2)

Remark 2.2: Table results for Eqs. 1.2, 1.2.2, 2.2 are different, so the binding in 2.1 is not equivalent to 1.1.

... [one] would translate this [Eq. 2.2] to:

~#%(p=q) (3.2)

~(#(%(p=q)=(p=p))) = (p=p) ; CTTC CTTC CTTC CTTC (3.2.2)

Remark 3.2.2: Eq. 3.2.2 does not produce the same table result as Eq. 2.2, meaning this translation of equivalence is mistaken.

and

%(p=q) evaluates to T, (4.1)

%(p=q) = (p=p) ; TCCT TCCT TCCT TCCT (4.2)

Remark 4.2: Eq. 4.1 is *not* tautologous, hence mistaken as an evaluation.

so

~#T evaluates to F. (5.1)

~(#(p=p)=(p=p)) = (p@p) ; NNNN NNNN NNNN NNNN (5.2)

Remark 5.2: The table result for Eq. 5.2 is truthity, and hence at a single value stage closest to tautology.

The Tarski–Grothendieck [*ax-groth*] is rendered as:

((-. (x -> E. y) /\ -. (A. z -> E. y)) /\
(((z -> A. w) -> -. (A. w -> E. y)) /\
(-. (E. w -> E. y) /\ (-. (-. z -> A. v) -> -. (A. v -> w))))) /\
(-. (-. y -> A. z) -> ((A. z <-> y) /\ -. (A. z -> y))) (6.1)

LET w, x, y, z: w, x, y, z.

((~(x>%y)&~(#z>%y)) &
(((z>#w)>~(#w>%y)) &
(~(%w>%y)&(~(~z>#v)>~(#v>w))))) &
(~(~y>#z)>((#z=y)&~(#z>y))) ;
TTFF TCFF TTFF TCFF (6.2)

Remark 6.2: Eq. 6.2 is *not* tautologous, meaning that *ax-groth* is refuted.