Shorter refutation of the Löb theorem and Gödel incompleteness by substitution of contradiction

Abstract: Löb's theorem $\Box(\Box X \rightarrow X) \rightarrow \Box X$ and Gödel's incompleteness as $\Box(\Box\bot\rightarrow\bot)\rightarrow\Box\bot$ are refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q: *X*; > Imply, →; @ Not Equivalent; # necessity, □; (p@p) **`F`** as contradiction, ⊥.

From: Gross, J. et al. (2016). Löb's Theorem. jasongross.github.io/lob-paper/nightly/lob.pdf jgross@mit.edu, jack@gallabytes.com, benya@intelligence.org

This, in a nutshell, is Löb's theorem: to prove *X*, it suffices to prove that *X* is true whenever *X* is provable. If we let □*X* denote the assertion "*X* is provable," then, symbolically, Löb's theorem becomes:

$$\Box(\Box X \rightarrow X) \rightarrow \Box X. \quad (1.1)$$

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (1.2)

Remark 1.2: Eq 1.2 as rendered is *not* tautologous, thus refuting Löb's theorem.

Note that Gödel's incompleteness theorem follows trivially from Löb's theorem: by instantiating *X* with a contradiction [⊥], we can see that it's impossible for provability to imply truth for propositions which are not already true. (2.1)

#(#(p@p)>(p@p))>#(p@p) ; `CCCC CCCC CCCC CCCC` (2.2)

Remark 2.2: Eq. 2.2, rendered as Eq. 1.2 with p substituted by (p@p), is *not* tautologous but consistently falsity as `C` for contingency. Hence Gödel's incompleteness theorem, as following trivially, is also refuted.

This means that the type of Löb's theorem becomes either □(□X→X)→□X [Eq. 1.1], which is not strictly positive, or

$$\Box(X \rightarrow X) \rightarrow \Box X, \quad (3.1)$$

#(p>p)>#p ; `CTCT CTCT CTCT CTCT` (3.2)

which, on interpretation, must be filled with a general fixpoint operator. Such an operator is well-known to be inconsistent.

Remark Fn. 2: Eq. 3.2 as rendered produces the same truth table result as Eq. 1.2 and as another trivial refutation.