The Unruh effect: insight from the laws of thermodynamics

2018 Annual Meeting of the APS Mid-Atlantic Section,
November 9-10, 2018,
University of Maryland, College Park,
College Park, MD

http://meetings.aps.org/Meeting/MAS18/Session/B01.18

Stephen J. Crothers
Tasmania, Australia
thenarmis@yahoo.com

Pierre-Marie Robitaille
Department of Radiology and Chemical Physics Program, The Ohio State University, Columbus, Ohio, 43210, USA
robitaille.1@osu.edu

ABSTRACT B01.00018

When an observer experiences uniform acceleration it has been postulated that empty space will appear as an infinite extent emitting a blackbody spectrum at a temperature proportional to the acceleration: the Unruh Effect. The Effect has been tied to the Hawking temperature of a black hole and is purportedly one account of the origin of black hole thermal emission, where the acceleration in the black hole case is due to gravity. The Unruh and Hawking temperatures have the same mathematical form: $T = ha/4\pi^2 ck_B$, were T is temperature, h is Planck's constant, a is acceleration, c is the speed of light in vacuo, and k_B is Boltzmann's constant. Acceleration for Hawking temperature is $a = GM/(r_S)^2$ where $r_S = 2GM/c^2$ (Schwarzschild radius) where G is the universal constant of gravitation, M the black hole mass. Temperature is always an intensive property. Acceleration is not an intensive property. The Unruh temperature, as with the Hawking temperature, although dimensionally balanced, is not thermodynamically balanced. Temperature cannot be equated to a term, or combination of terms, that is not intensive. The Unruh temperature therefore, violates the 0^{th} and 2^{nd} laws of thermodynamics, as does Hawking temperature. Consequently, they are invalid. The Unruh and Hawking effects do not exist.

I. INTRODUCTION

In thermodynamics, systems are described in terms of properties which are either intensive or extensive [1-5]. Intensive properties can be determined at every spatial location and are independent of any changes in the mass of a system by definition. Temperature, pressure, velocity, thermal conductivity, and density are examples. It is well recognized that temperature maintains the same value at all spatial locations within a system in thermodynamic equilibrium. However, it might take on varying values in a system out of equilibrium. In either case, temperature always remains intensive, as it can be defined at every spatial location. Conversely, extensive properties are defined over a certain spatial extent. Typical examples are mass, volume, internal energy, and heat capacity. Extensive properties are additive. The thermodynamic coordinates necessary and sufficient to describe any thermodynamic system are determined by experiment.

Consider a homogeneous system in thermodynamic equilibrium. Divide it into two equal parts, each having equal mass. Those properties of the original system that remain unchanged in each half of the original system are intensive. Those properties that are halved are extensive, those which change but not by half are neither intensive nor extensive [4]. It is also important to note that the quotient of two extensive properties is intensive. Density, $\rho = M/V$, is the best known example of such a quotient. Mass and volume are both extensive, but their quotient results in density, which is intensive. The quotient of two non-extensive properties, which behave identically with changes in spatial extent, is also intensive. However, the quotient (or the product) of an intensive property and an extensive property is always extensive.

Constants such as Boltzmann's constant, k_B, the Stefan-Boltzmann constant, σ, and Planck's constant, h, do not alter the intensive or extensive nature of a thermodynamic expression. For instance, the total energy of a simple monoatomic gas, E, can be given by the following simple expression, $E = 3Nk_BT/2$. In this expression, the extensive nature of E on the left hand side is imparted by the extensive nature of the number of particles, N, on the right, given that temperature must remain intensive. Boltzmann's constant does not contribute towards establishing the thermodynamic nature of energy as extensive in this expression. At the same time, variables that are neither intensive nor extensive, but have units, affect the thermodynamic balance of expressions. Variables without units have no influence on thermodynamic character. In the end, it is important to remember the following rules: 1) extensive and intensive properties exist, 2) some properties are neither extensive nor intensive, and 3) constants play no role in establishing whether a property is intensive, extensive, or neither. Landsberg [2] has argued that the nature of properties as intensive or extensive is so important to the study of thermodynamics that the concept should be adopted as the 4^{th} law.

It is also true that any proposed thermodynamic equation must be thermodynamically balanced, as just demonstrated. If one side of the equation is intensive, or extensive, then the other side must also be intensive or extensive, respectively [5]. When a thermodynamic property in an expression is being defined in terms of other thermodynamic properties, the correct nature of the sought property must be obtained. Temperature cannot become non-intensive simply as a result of a mathematical expression. Temperature must always be intensive, in keeping with its role relative to defining the laws of thermodynamics.

The 0^{th} law of thermodynamics requires thermal equilibrium between objects in defining temperature. Consider two isolated systems[1], each in thermodynamic equilibrium. Remove a section of the thermal insulating material from the surface of each system and place them in contact via the uncovered sections. When there are no observable changes in any thermodynamic properties of either system, they are each at the same temperature. The 0^{th} law of thermodynamics not only makes a statement about thermal equilibrium of systems, it also includes the intensive character of temperature: "*when two systems are at the same temperature as a third, they are at the same temperature as each other*" [6], "*Two systems in thermal equilibrium with a third are in thermal equilibrium with each other*"[7].

Take two systems, A and B, at the same temperature in accordance with the foregoing method. The temperature of A is the same as that at every spatial location in B: as every part of B. Divide the system B into two parts, B_1 and B_2. Since A has the temperature of B, it has the temperature of B_1 and B_2: parts of B. Therefore, B_1 and B_2 are at the same temperature. Thus, the intensive nature of temperature is contained within the very definition of the 0^{th} law of thermodynamics. Such equilibrium cannot exist if temperature is no longer intensive. Similarly, entropy must always remain extensive, in order to preserve the 2^{nd} law.

If a system has spherical symmetry, its area can be expressed as $A = 4\pi r^2$. Clearly, r is neither intensive nor extensive, as it is not additive. This can also be established relative to a volumetric system with spherical symmetry. The radius is not extensive since volume, V, is given by $V = 4\pi r^3/3$. In this expression, it is the volume of a sphere which is an extensive property, along with r^3. It is clear that radius r is not additive. Hence, the radius of a sphere can never be considered as an extensive property.

Length is generally not extensive, as radius attests. However, length can become extensive in certain limited circumstances, as for example, in stretched wires [7], having the thermodynamic coordinates of tension (intensive), length (extensive), and temperature (intensive). The spatial extent of this system is length. It is extensive, in this case, as it is directly related to the mass of the system. Any change in length of the wire is directly associated with a change in its mass. It is also true that extensive properties in one system might not be extensive in another. A prime example is surface area. For a planar system composed of a single monolayer, area is extensive. Such systems arise when considering surface tension which, in turn, is an intensive property. However, the area of a sphere is never extensive, because such area is not

---

[1] An *isolated* system does not exchange any energy, either by mechanical work or flow of heat, with its surroundings.

additive. If one takes a sphere and divides it into two spheres of equal volume, the area of each sphere is not half of the initial.

As an additional example, consider the Stefan-Boltzmann law [8] describing a system in which area is a thermodynamic coordinate, $L = \varepsilon\sigma A T^4$. In this expression, L is the luminosity of the object, ε is emissivity of the material (a unitless property), σ is the Stefan-Boltzmann constant, A is the area, and T is temperature. In this case, note that neither luminosity nor area are extensive, because these properties are not additive. However, both luminosity and area change in identical fashion relative to spatial extent. Temperature is defined at every spatial location in the system and remains intensive. For any given temperature, the luminosity is directly proportional to the area. Hence, the luminosity per unit area (L/A), also known as the emissive power, is intensive and so is the temperature, as required by the laws of thermodynamics. The luminosity equation is therefore thermodynamically balanced.

II. THE UNRUH EFFECT

According to this theoretical effect:

"*From the point of view of an accelerating observer or detector, empty space contains a gas of particles at a temperature proportional to the acceleration*" [9].

From the Newtonian relation for gravitational force, gravitational acceleration g is,

$$g = \frac{GM}{r^2}. \tag{1}$$

The mass M is extensive and r^2 is neither intensive nor extensive. Supposing gravity a thermodynamic relation, the gravitational acceleration g is therefore neither intensive nor extensive. The uniform acceleration associated with the Unruh Effect is tied to gravitational acceleration at the event horizon of a black hole:

"*At a small distance (close to the black hole's event horizon, which is well defined without reference to accelerated worldliness), the thermal effects can, however, be attributed to the acceleration of the curves of constant Schwarzschild radial position, whereas a freely falling observer there sees, approximately, cold empty space (Unruh, 1977b; Fulling, 1977) This is the origin of the thermal emission or ambience, as viewed from afar, of black holes, as already emphasized in Unruh's original paper (Unruh, 1976)*" [9].

"*These results are independent of the means used to accelerate the detector, but depend only on the acceleration itself. … Applying these results on particle detectors to the black-hole evaporation problem, one finds that for a detector stationed near the horizon of the black hole, the transition probability of the detector per unit time can be calculated in a similar way to that for a static detector …*" [10].

Comparison of the temperature equations for the Unruh Effect and for a black hole reveals the close association:

$$T_{Unruh} = \frac{ha}{4\pi^2 c k_B}, \qquad T_H = \frac{hg}{4\pi^2 c k_B}. \tag{2}$$

Both a and g are uniform accelerations, the latter due to gravity from the Newtonian relation. The 'Schwarzschild radius' is $r_S = 2GM/c^2$. Then the 'surface gravity' of a black hole is,

$$g = \frac{GM}{r_S^2} = \frac{c^4}{4GM}. \tag{3}$$

Then from Eqs.(2),

$$T_H = \frac{hc^3}{16\pi^2 k_B GM}, \tag{4}$$

the familiar form of the Hawking black hole temperature equation. The right side of Eq.(4) is not intensive, owing to the factor $1/M$, so it is invalid. In both of Eqs.(2), temperature is intensive as required by the laws of thermodynamics. The accelerations a and g are not intensive. All other terms in these equations are constants. Thus, Eqs.(2) violate the laws of thermodynamics.

As with Hawking Radiation, the Unruh Temperature is purported to be that from a blackbody spectrum produced by vacuum particles:

"*… an accelerated detector even in flat spacetime will detect particles in the vacuum*" [10].

"*When a detector, coupled to a relativistic quantum field in its vacuum state, is uniformly accelerated through Minkowski spacetime, with proper acceleration a, it registers a thermal black body radiation at temperature* $T = \frac{\hbar a}{2\pi c k_B} \sim 10^{-19} a$. *In other words, it detects a thermal bath of particles*" [11].

However, a thermal spectrum can only be produced by a physical lattice [12]. Neither "*a gas of particles at a temperature proportional to the acceleration*" [9], nor "*a thermal bath of particles*" [11], nor "*a thermal bath of scalar photons*" [10], possess a lattice structure.

III. CONCLUSIONS

From this simple analysis, it has been demonstrated that the equation advanced for Unruh temperature, T_{Unruh}, stands in violation of the laws of thermodynamics. In analogous fashion, the expressions advanced for the formation of a gaseous star from the gravitational collapse of a gaseous mass are also thermodynamically invalid [13-16]. It is not appropriate to utilise the virial theorem and introduce temperature through kinetic theory, when balancing kinetic energy with potential energy. Such an approach results in direct violations of the laws of thermodynamics. Gravitational collapse of an ideal gas produces a perpetual motion machine of the first kind [13-16]. Similarly, the Bekenstein-Hawking black hole entropy and Hawking black hole temperature violate the laws of thermodynamics, so they are invalid [17-21].

REFERENCES

[1] P. Atkins, The Law of Thermodynamics: A Very Short Introduction, (Oxford University Press, Oxford, UK, p.3, 2010).

[2] P. T. Landsberg, Thermodynamics with Quantum Statistical Illustrations, (Interscience Publishers, New York, p.142, 1961).

[3] Y. A. Cengel and M. A. Boles, Thermodynamics - An Engineering Approach, 8th ed., (McGraw-Hill, New York, p.15, 2015).

[4] G. N. Hatsopoulos, and J. H. Keenan, Principles of General Thermodynamics, (John Wiley & Sons, Inc., p.1, 1965).

[5] S. G. Canagaratna, J. Chem. Educ., v.69, no.12, 957-963, (1992).

[6] F. W. Sears, An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics, 2nd ed., (Addison-Wesley Publishing Company, Inc., Cambridge, MA, 1955).

[7] M. W. Zemansky and R. H. Dittman, Heat and Thermodynamics, 6th ed., (McGraw-Hill Book Company, New York, NY, 1981).

[8] J. Stefan, Sitzungsberichte der mathematischnatur wissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften, Wien, v.79, 391-428, (1879).

[9] S. A. Fulling, G. E. A. Matsas, Scholarpedia, 9(10):31789, (2014), http://www.scholarpedia.org/article/Unruh_effect

[10] W. G. Unruh, Phys. Rev. D, **14**, 4, (1976).

[11] S. De Bièvre, M. Merkli, Class. Quantum Grav., **23**, (2006), http://math.univ-lille1.fr/~debievre/Talks/unruhrolla09A.pdf

[12] P.-M. Robitaille, IEEE Trans. Plasma Sci., v.31(6), (2003).

[13] P.-M. Robitaille, Thermodynamics and the Virial Theorem, Gravitational Collapse and the Virial Theorem: Insight from the Laws of Thermodynamics. Prog. Phys., v.14, no.3, 179-181, (2018), http://ptep-online.com/2018/PP-54-13.PDF

[14] P.-M. Robitaille, How are Stars Formed? The Standard Model: Gravitational Collapse, Black Holes, and The Big Bang! (Sky Scholar, 2017), https://www.youtube.com/watch?v=LoqsG7V13G8

[15] P.-M. Robitaille, Does Gravitational Collapse Occur? Insight from the Laws of Thermodynamics! (Sky Scholar, 2018), https://www.youtube.com/watch?v=DtMee3rrHDY

[16] P.-M. Robitaille, Gravitational Thermodynamics - Is it Science? (Sky Scholar, 2018), https://www.youtube.com/watch?v=mZ2F2Kw5-nQ

[17] P.-M. Robitaille, Hawking Radiation: A Violation of the Zeroth Law of Thermodynamics. Spring 2018 Meeting of the APS New England Section, March 16–17, 2018, Boston, MA. http://meetings.aps.org/Meeting/NES18/Session/D01.3

[18] P.-M. Robitaille, Black Hole Entropy: A Violation of the Second Law of Thermodynamics, Fall 2018 Meeting of the APS, Ohio Section, Sept. 28-29, 2018, Toledo, Ohio. http://meetings.aps.org/Meeting/OSF18/Session/C01.18

[19] B. H. Lavenda, J. Dunning-Davies, On the law of error for mass fluctuations in black holes, Class. Quantum Grav. **5**, L149-L154, (1988).

[20] B. H. Lavenda, J. Dunning-Davies, The essence of the second law is concavity, Foundations of Physics Letters, V.3, No.5, (1990).

[21] B. H. Lavenda, J. Dunning-Davies, Stefan-Boltzmann law for black bodies and black holes, International Journal of Theoretical Physics, V.29, No.5, (1990).