(pk mk qk) or an Unexpected Inconsistency

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Abstract. This note proves the inconsistency of the Peano arithmetic (PA) by deriving both a strengthened form of the strong Goldbach conjecture and its negation.

Notations. Let $\mathbb{N}$ denote the natural numbers starting from 1 and let $\mathbb{P}_3$ denote the prime numbers starting from 3.

Theorem. *The Peano arithmetic (PA) is inconsistent.*

Proof. We define the set S_g := { (pk, mk, qk) | k, m $\in \mathbb{N}$; p, q $\in \mathbb{P}_3$, p < q; m = (p + q) / 2 } and we consider the following two cases:

(G) The numbers m in the components mk take all integer values x ≥ 4.
¬(G) The numbers m in the components mk do not take all integer values x ≥ 4.

The case ¬(G) means that for each k ≥ 1 there is an nk, n ≥ 4, different from all the mk, where all pairs (p, q) of odd primes, which determine the numbers m, are used in S_g. For each k ≥ 1, such an nk can be written as some pk when n is prime, as some pk' when n is composite and not a power of 2, or as 4k' when n is a power of 2; p $\in \mathbb{P}_3$; k, k' $\in \mathbb{N}$.

The expression pk' for nk, k' = k or k' ≠ k, is a first component of S_g triples and the expression 4k' for nk is component of the triple (3k', 4k', 5k'). Thus, for each k ≥ 1, nk is a component of the same triples that exist when there is no such nk, i.e. in the case (G).

Therefore, we obtain that the triples of S_g are the same in both cases, (G) and ¬(G). So, the components mk are the same in both cases, (G) and ¬(G), which is a contradiction.

□

Actually, the above argument uses a strengthened form of the strong Goldbach conjecture and its negation:

Strengthened strong Goldbach conjecture (SSGB): *Every even integer greater than 6 can be expressed as the sum of two different primes.*

¬SSGB: *There is an even integer greater than 6 that cannot be expressed as the sum of two different primes*.

SSGB is equivalent to saying that all integers x ≥ 4 appear as m in a component mk of S_g. Therefore, SSGB is equivalent to the case (G) and the negation ¬SSGB is equivalent to the case ¬(G). We have seen above that the S_g triples are the same in these two cases. This means that both SSGB and ¬SSGB hold.