# (pk mk qk) or an Unexpected Inconsistency

Ralf Wüsthofen

**Abstract.** This note proves the inconsistency of the Peano arithmetic (PA) by deriving both a strengthened form of the strong Goldbach conjecture and its negation.

**Notations.** Let $\mathbb{N}$ denote the natural numbers starting from 1 and let $\mathbb{P}_3$ denote the prime numbers starting from 3.

**Theorem.** *The Peano arithmetic (PA) is inconsistent.*

*Proof.* We define the set $S_g$ := { (pk, mk, qk) | k, m $\in \mathbb{N}$; p, q $\in \mathbb{P}_3$, p < q; m = (p + q) / 2 }.

Then, $S_g$ is the same under these two conditions:

(i) For each k ≥ 1, there is an nk, n ≥ 4, different from all the mk.
(ii) For each k ≥ 1, there is no nk, n ≥ 4, different from all the mk.

The reason is that for each k ≥ 1 such an nk from (i) can be written as some pk when n is prime, as some pk' when n is composite and not a power of 2, or as 4k' when n is a power of 2; p $\in \mathbb{P}_3$; k, k' $\in \mathbb{N}$. As each of these expressions pk, pk' and 4k' for nk is a $S_g$ triple component, $S_g$ does not change regardless of whether (i) or (ii) applies.
We note that in the definition of $S_g$ all pairs (p, q), p < q, of odd primes are used. This excludes the possibility of an nk from (i) where nk = ($p_n$k + $q_n$k) / 2 with a pair of primes ($p_n$, $q_n$) not used in $S_g$.

So, if all the triples of $S_g$ are the same in both cases, then all the mk are the same in both cases. This is a contradiction since the case "nk exists" means that the numbers m do not take all integer values x ≥ 4 and the case "nk does not exist" means that the numbers m take all integer values x ≥ 4. □

Actually, the above argument uses a strengthened form of the strong Goldbach conjecture and its negation:

**Strengthened strong Goldbach conjecture (SSGB):** *Every even integer greater than 6 can be expressed as the sum of two different primes.*

**¬SSGB:** *There is an even integer greater than 6 that cannot be expressed as the sum of two different primes*.

SSGB is equivalent to saying that all integers x ≥ 4 appear as m in a component mk of $S_g$. Therefore, SSGB is equivalent to the case (ii) and the negation ¬SSGB is equivalent to the case (i). We have seen above that the $S_g$ triples are the same in these two cases. This means that both SSGB and ¬SSGB hold.