Quantum ontology suggested by a Kochen-Specker loophole

A. U. Thor*
The University of Uranus

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Abstract

We discuss a specific way in which the conclusions of the Kochen-Specker theorem may be avoided while, at the same time, closing the gap in a practical but usually neglected matter regarding scientific methodology in general. Implications of the possibilities of hidden variables thus defined are discussed, and a tentative connexion with cosmology is delineated.

1 Introduction

Carl Sagan, in the critically acclaimed series *Cosmos: A Personal Voyage*, made the following observation: "How could the rising of Mars at the moment of my birth affect me - then or now? [...] the only influence of Mars which could affect me was its gravity - but the gravitational influence of the obstetrician was much larger than the gravitational influence of Mars. Mars is a lot more massive, but the obstetrician was a lot closer." [1] While probably sounding perfectly trivial to the vast majority of scientists, this assertion contains an assumption that, we contend, is crucial to all the sciences, which we may refer to as the *principle of unlimited privacy*: that is, that we can always define a system of interest S (usually finite in spatial extension - perhaps including a surrounding "environment" of its own) such that we may study its dynamics completely independently from the rest of the Universe. Its justification is made seemingly trivial by Sagan's own remark: since the influence of Mars (as well as that of everything else "out there") is so small, we'd be justified, in a zeroth-order approximation, to ignore its effects entirely, and, in case higher accuracy was needed, we could, at least *a priori*, calculate higher-order corrections using, say, perturbation methods.

However, a moment's reflection will show that *ab initio* computation of even the first-order correction would be technically impossible - after all, we're not

*Email: glacatlan@hotmail.com

Laplacean demons. The implication then is that, if we wish to conserve the principle of unlimited privacy in our scientific methodology, we are forced to accept that *there is a fundamental uncertainty of sorts associated with the practical application of our theories*, which, at first, is seen to occur in the classical and quantal cases separately. But, as it's well-known, quantum theory already contains a fundamental uncertainty of its own, *regardless of how imperfect are the measuring apparata used in empirical investigations* [2]; this in turn suggests a way to provide closure for the consistency of our methodology, if we associate the *classical* uncertainty stemming from the abovementioned privacy-violating effects with the fuzzyness of the *quantum* formalism itself, which would then logically remain as the best complete theory available to Laplacean non-demons to explain Nature in a scientifically consistent fashion.

2 Discussion

To formalize the previous philosophical considerations, we introduce the original Kochen-Specker (KS) criteria for hidden-variable theories: i) there must exist a correspondence of quantum mechanical averages of observables with the phase space averages of hidden pure states, and ii) there must exist an imbedding of the algebra of quantum mechanical propositions into a classical algebra [3]. We must, however, adapt these to the general case described in the introduction: we define the phase space $\Omega = \Omega_x \times \Omega_y$, with $\Omega_x = \{x\}$ representing the degrees of freedom associated with S and $\Omega_y = \{y\}$ those of the "hidden variables", and to the former we associate the sets of observables $\mathcal{O}_x$ and of states $\mathcal{S}_x$; furthermore, we define maps $\mathcal{O}_x \ni A \longmapsto f_A : \Omega \to \mathbb{R}$ and $\mathcal{S}_x \ni \psi \longmapsto \rho_\psi$. The quantum expectation of an observable A is given by[1]

$$\langle \psi, A\psi \rangle = \int_{\sigma(A)} \lambda d\langle \psi, E^A(\lambda)\psi \rangle =: \int_{\sigma(A)} \lambda dw_\psi^A(\lambda); \tag{1}$$

what we wish to do here is to have this equal to the hidden space average $\int_\Omega f_A(x,y) d\rho_\psi(x,y)$ so that criterion i) is satisfied - however, differently from the original work, we must take notice that the y-variables are *not* quantized; therefore, it seems reasonable to assign maps only between quantities corresponding to the x-variables. One simple such correspondence follows from

$$\int_\Omega f_A(x,y) d\rho_\psi(x,y) = \int_{\Omega_x} \int_{\Omega_y} f_A(x,y) d\rho_\psi(x,y) =: \int_{\Omega_x} {}^y f_A(x) d^y \rho_\psi(x); \tag{2}$$

(notice the similarity with the partial trace in quantum mechanics). Differently than KS, we shall then impose

$$\int_{\sigma(A)} \lambda dw_\psi^A(\lambda) = \int_{\Omega_x} {}^y f_A(x) d^y \rho_\psi(x) \tag{3}$$

From this, we're ready to write down

[1] For details cf. [3] and [4]; we also adapt notation from those papers.

Criterion 1 $w_\psi^A(\lambda) = {}^y\rho_\psi({}^y f_A^{-1}(\lambda))$

Next, we need worry about the algebraic structure of the *functions* of the observable A: given a Borel function $b : \mathbb{R} \to \mathbb{R}$ we have

$$b(A) = \int_{\sigma(A)} b(\lambda) dE^A(\lambda) \equiv \int_{\sigma(A)} \lambda dE^A(b^{-1}(\lambda)) \tag{4}$$

so that it's natural to define

$$w_\psi^{b(A)}(\lambda) := w_\psi^A(b^{-1}(\lambda)); \tag{5}$$

imposing criterion 1 to this definition, we get

$${}^y\rho_\psi({}^y f_{b(A)}^{-1}(\lambda)) = w_\psi^{b(A)}(\lambda) = w_\psi^A(b^{-1}(\lambda)) =^y \rho_\psi({}^y f_A^{-1}(b^{-1}(\lambda))) \tag{6}$$

from which we write

Criterion 2 ${}^y f_{b(A)} = b({}^y f_A)$

Now, it's not obvious at face value that criterion 2 embodies the physical intuition of KS's ii); in actuality, it's an ancilla towards that statement. The idea goes that, with the definition of a *partial algebra* in terms of a *commeasurability relation* (which, for quantum mechanics, is just pairwise commutativity: $[A_i, A_j] = 0,\ \forall i, j \in I$), we're able to translate the propositions that quantum mechanics permits us to test explicitly into classical language, provided we're able to imbed the partial algebra of the former into the latter. Then, the crux of the original argument rests in that the set $\mathbb{R}^\Omega$ of all (measurable) $f : \Omega \to \mathbb{R}$ forms a commutative algebra over $\mathbb{R}$, and that, in general, one can't imbed the quantum algebra into such an algebra. However, in this alternative context, we see no *a priori* reason to assume that the ${}^y f_A$ algebra is commutative - in fact, it's not even really a map $\Omega_x \to \mathbb{R}$; to appreciate that, let's digress to a simple quantum mechanical problem: suppose we have the Hilbert space $H = H_x \otimes H_y$, of which it's known the only physical observables available are of the form $\mathbf{A}_x \otimes \mathbf{1}_y$; then how are we to calculate averages of nonseparable states $|\psi\rangle_x \otimes |\psi\rangle_y \neq |\Psi\rangle \in H$? Expanding $|\Psi\rangle = \sum_{a,b} \Psi_{ab} |a\rangle|b\rangle$ and $\Psi_{ab}(x, y) = \sum_d c_{ab}^d(y)\psi_d(x)$, with $\mathbf{A}_x|a\rangle = A_a|a\rangle$, $\psi_a = \langle x|a\rangle$, and $\{|b\rangle\}$ some complete basis for H_y, we have

$$\begin{aligned}
\langle\Psi|\mathbf{A}_x \otimes \mathbf{1}_y|\Psi\rangle &= \sum_{a,b} A_a \left|\Psi_{ab}(x,y)\right|^2 = \sum_{d,d'} \left(\sum_a A_a \sum_b c_{ab}^{d'*} c_{ab}^d \right) \psi_{d'}^* \psi_d \quad (7)\\
&=: \sum_{d,d'} \left(\bar{A}_d \delta_{d'd}\right) \psi_{d'}^* \psi_d = \sum_a \bar{A}_a(y) \left|\psi_a(x)\right|^2; \quad (8)
\end{aligned}$$

we see, then, that the *definition* of the quantities $\bar{A}_a$ at the third equality allows us to talk of "effective" $\mathbf{A}_x$-averages in terms of H_x alone.

While we haven't been able to construct specific examples for the hidden space maps specifically, a few remarks are in order: first, from our previous arguments, on one hand, we see that it's not possible to choose both f_A and ρ_ψ separable (otherwise we'd retrieve the original KS formulation) - but, on the other hand, it seems plausible to take ρ_ψ as a product measure w.r.t. Ω and define ${}^y f_A(x) = \int_{\Omega_y} f_A(x,y) d^x \rho_\psi(y)$, which might be the simplest construction possible; however, be as it may, we feel it's desirable that their expressions should be as generic as possible, in the sense they shouldn't require detailed knowledge of the nature of the hidden variables (unless we had some kind of guidance from, say, cosmological data; vide infra). Second, we'd like to comment that the contradiction that validates the KS theorem in the proof mentioned by Straumann [4] relies on the fact that, in computing $\langle Q_j \rangle_\Psi$ and $\langle Q_1 Q_2 Q_3 \rangle_\Psi$, the f-maps inferred for each integral were assumed interchangeable - i.e., once the f_{Q_j}, etc. are figured out, they hold for *all* other observables in which they may appear; if this condition is lifted, no contradiction arises - which seems to be in agreement with the intuitions we tried to express with the modified criteria.

3 Conclusion

To the applied physicist that may roll eyes at this digression and politely refer to it as "an abject excursion into the faerie-realm of metaphysics", we remind an almost century-old embarassment: even though quantum mechanics is not some extraneous concoction brewed by a witch - it's a fundamental, hard-tested aspect of physical reality -, it seems to have become commonplace for even theoreticians to label it "weird" - but to allocate weirdness to the former is to alienate oneself from the latter, and that is unacceptable of a scientific mindset. Our perspective differs radically from the previous/current one in that, rather than assuming quantum theory is "incomplete" and that adding classical variables to the framework will somehow fix it, we take it to represent a fundamental reality of our condition and are looking for a way in which that reality may emerge from classical theory, such that both theories be complete ("private") in their respective domains; in this sense, perhaps a conceptual parallel could be drawn with stochastic interpretations such as Nelson's [5], and further exploration in this direction may prove interesting.

There are other features that may be worthy of consideration: for one, while the KS result is primarily concerned with algebras of pairwise commuting operators, it doesn't tell us anything additional about the generic case when at least some observables do *not* commute - which forms the basis of the uncertainty principle and the issue of counterfactual definiteness, as well as that of much discomfort in professional circles; however, once appropriate maps are defined for the criteria here proposed, extending analysis to those generic observable sets may shed some light on the nature of those quantal phenomena. Also, if correct, the present proposal opens a curious possibility to the field of cosmology: it seems to imply that, in case observations indicate the observable Universe to significantly follow some *completely* classical model, that could be taken as evidence in favor of there being little to no structure in the *non*observable Universe - whereas the appropriateness of a semiclassical/quantum model could lend support or provide a testbed to ideas such as the (cosmological) multiverse; at this point, however, it's still too early to draw any definitive conclusions, and the idea remains tentative. For now, we only wish to draw attention to the fact it suffices to define f_A and ρ_ψ such that the resulting partial algebra is homomorphic to the quantum one to avoid the main conclusion of the KS theorem, independently of the validity of the picture presented here.

References

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